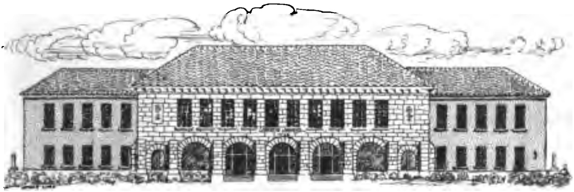


NATURAL PHILOSOPHY FOR BEGINNERS.

PART I.

SCHOOL CLASS BOOKS

CAMBRIDGE.

NATURAL PHILOSOPHY FOR BEGINNERS

WITH NUMEROUS EXAMPLES

BY

I. TODHUNTER, M.A., F.R.S.

HONORARY FELLOW OF ST JOHN'S COLLEGE, CAMBRIDGE.

PART I.

THE PROPERTIES OF SOLID AND FLUID BODIES.

SECOND EDITION.

London:

MACMILLAN AND CO.

1881

SCHOOL CLASS BOOKS

CAMBRIDGE.

NATURAL PHILOSOPHY FOR BEGINNERS

WITH NUMEROUS EXAMPLES

BY

I. TODHUNTER, M.A., F.R.S.

HONORARY FELLOW OF ST JOHN'S COLLEGE, CAMBRIDGE.

PART I.

THE PROPERTIES OF SOLID AND FLUID BODIES,

SECOND EDITION.

Cambridge:
PRINTED BY C. J. CLAY, M.A.
AT THE UNIVERSITY PRESS.

PREFACE.

The design of this work is to furnish a simple and trustworthy manual for those who are beginning the study of Natural Philosophy; and it ventures to claim a distinct position among the numerous publications which have appeared with somewhat similar aims. On the one hand great pains have been taken to render the book intelligible to early students; the amount of mathematical knowledge assumed is merely a familiarity with the elements of Arithmetic. On the other hand the subject is presented, it may be hoped, with adequate fulness; so that a person who has mastered the work will have gained considerable acquaintance with the principles of Natural Philosophy. Moreover a collection of Examples for exercise is supplied.

The present volume consists of four parts. The first part extends to Chapter III. inclusive; this is of a preliminary character, recalling to the student's attention some things with which he is already familiar, indicating the various branches of knowledge, and giving an outline of that with which we are here concerned. The second part extends to Chapter XXIV. inclusive; this treats of the mechanical properties of *solid* bodies. The third part extends to Chapter L. inclusive; this treats of the mechanical properties of *fluid* bodies. The fourth part extends to the end of the volume; this consists of various Chapters which illustrate and apply the principles already established. Thus the present volume is devoted to the

Mechanical properties of solid and fluid bodies; the second volume, completing the work, will treat on what Dr Whewell has called the *Secondary* Mechanical Sciences, namely those relating to Sound, Light, and Heat; this volume is already written and will soon be sent to press.

As in former elementary works the plan is adopted of breaking up the subject into numerous short Chapters which are to a great extent independent of each other; thus the attention of the student is required for only a moderate portion at one time, and if he finds a difficulty in thoroughly mastering a particular Chapter he may pass on to the following Chapters, and afterwards recur to the passages not understood at first.

The Examples, which are above 500 in number, form an important part of the work; many of them are original, while the rest have been selected from the Examination papers published by the Universities and other Examining bodies. These Examples will be found, it is believed, not too difficult for the use of an early student; while they will afford real exercises to test his knowledge and his power of application. Both the text and the Examples have been arranged with the view of meeting fairly all the difficulties that may occur in the course of study; it is quite possible to give to a work a fallacious appearance of simplicity by omitting every point that requires close attention, and by constructing examples which all resemble a few familiar types, and so may be solved almost without thought. The student however who wishes to master any science, or to pass an examination in it, must be willing to make the exertion which is necessary in order to comprehend the whole of it, and not merely easy selections from it; and he must be prepared to encounter a wide variety of examples and problems.

An introduction to Natural Philosophy may be used by different classes of readers; some may intend hereafter to devote themselves to the earnest study of the subject either on its theoretical or its experimental side; some may be looking forward to professional occupation with the numerous practical applications of science; while others may seek for such knowledge as will give them an intelligent interest in the phenomena of the world, and in the discoveries and inventions which proceed from the regular cultivators of the subject. To all these classes it is important that the notions at first acquired should be accurate; and I venture to repeat with respect to the present work the hope expressed ten years since with respect to another, namely that the beginner will here find a satisfactory foundation for his future studies, so that afterwards he will only have to increase his knowledge, without rejecting what he originally acquired. An elementary writer may well propose to himself as one of his main objects that those who use his work should have nothing afterwards to unlearn; and this has been recently explicitly recognized by more than one eminent authority.

I. TODHUNTER.

JANUARY, 1877.

CONTENTS.

I. INTRODUCTION.

1. The late Dr Whewell, congratulating a friend famous for his knowledge and ability on the birth of a son, said, "Young as he is he will learn more than you in the next twelve months." The remark may appear simple but it is striking from its truth; for it is curious to notice how soon a child placed under reasonably favourable circumstances gains the rudiments of all the science which the wisest men can teach. At a very early age the child begins to arrange and classify; he sees that some of the objects around him can move themselves, and that others cannot, suggesting the broad distinction between things which have life and things which have not life. Again, further subdivisions soon become clear; thus for example among living things he learns to bring together in his thoughts many that *fly*, and to call them by the name of *birds*. Even if he does not use a common name for a class of things which in some respects are like each other, he can hardly fail to notice the fact of likeness. Thus the water in which he is bathed, the milk he drinks, the ink he is forbidden to touch, must seem to him in some respects like each other, and different from the chairs and tables and toys of his nursery; though he has not learned to call the former *fluids* and the latter *solids*.

2. One of the most important words to be found in our language is *Law*. The original sense of the word is that of a rule or command which must be obeyed. Thus it is the duty of all people to obey the Law of the land; and it is the duty of children to obey the Law of their parents. In another sense the word *Law* is used to denote the unwavering constancy with which certain results will follow

when the circumstances are the same. Thus, for example, we say it is a law that a stone will fall down again if it be thrown up into the air; by this we mean that from repeated observation we are certain this result will happen. There are many such facts, which are called *Laws of Nature;* and from our infancy we begin to learn these laws, and to shape our conduct according to them. These are the *Laws* with which we shall be occupied in the present work; Laws not in the sense of *duties* but of *facts*.

3. That Laws in the sense here adopted extend throughout his little world soon becomes obvious to a child. He discovers and remembers that if he falls against a hard body he hurts himself, that if he pulls his toy cart with a string it follows him. He observes too the prevalence of Law in one of the most important fields of his early education, namely *language;* and at first he even exaggerates the range of this principle, and assumes that there are no exceptions to it. Thus he learns by habit that the past time of an English verb is usually made by adding *d* or *ed* to the present; and accordingly he constructs such forms for himself. For example a child says, "he fighted me"; thus conjugating the verb according to the general law, before he has learned by experience that *to fight* is an irregular verb, the past time of which is *fought.*

4. Those who wish to have a profound knowledge of Natural Philosophy must acquire considerable familiarity with Mathematics; but it will be possible to understand the elementary principles of the subject with the aid of a little skill in the operations of Arithmetic, and some acquaintance with the figures of Geometry. It will be convenient to mention the most important particulars which we shall assume to be known.

5. There are certain signs which are used as very convenient abbreviations in Arithmetic; among these are $=$ for *equal to,* $+$ for *added to,* and $-$ for *diminished by.* The use of these signs is exemplified by such statements as $12-5=7=4+3$. We may say that $=$ is the sign of *equality,* $+$ the sign of *addition* and $-$ the sign of *subtraction.*

6. The sign $\times$ denotes *multiplication;* thus $6 \times 7 = 42$.

7. If a number be multiplied by itself the product is called the *square* of the number; thus $6 \times 6 = 36$, so that 36 is the *square* of 6. The product of the square of a number into the number is called the *cube* of the number; thus $36 \times 6 = 216$, so that 216 is the *cube* of 6. The *square root* of any assigned number is that number which has the assigned number for its *square*. The *cube root* of any assigned number is that number which has the assigned number for its *cube*. Thus 7 is the square root of 49, and 5 is the cube root of 125.

8. A *fraction* means a part or parts of some whole or unit. Thus $\frac{5}{8}$ is a fraction; it means that some whole or unit is to be divided into eight equal parts and five of them taken. If the whole or unit is a weight of one pound, that is of sixteen ounces, an eighth part is two ounces, and five such parts are ten ounces. If the whole or unit is a shilling, that is twenty-four halfpence, an eighth part is three halfpence, and five such parts are fifteen halfpence, that is sevenpence-halfpenny. In the fraction $\frac{5}{8}$ the 5 is called the *numerator*, and the 8 the *denominator*.

9. The *product* of two fractions is obtained by multiplying the two numerators for a new numerator, and the two denominators for a new denominator. Thus $\frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$. It is explained in books on Arithmetic that the term *product* is conveniently and naturally used in this case, although the meaning of the term may seem somewhat different from that which it has in the multiplication of whole numbers. Thus $2 \times 3 = 6$, that is 6 is the product of 2 and 3; so that the product is *greater* than either of the *factors* 2 and 3. But $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$, that is $\frac{1}{6}$ is the product of $\frac{1}{2}$ and $\frac{1}{3}$; and in this case the product is *less* than either of the *factors* $\frac{1}{2}$ and $\frac{1}{3}$.

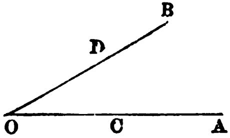
10. The notion of *proportion* is one of the most important of those which are illustrated in Arithmetic. Suppose that a man walks four miles in one hour, and we have to find how far he can walk in two hours and a half. The answer can be obtained without any explicit reference to proportion, but the most instructive mode of regarding

the question is as an example of proportion: ten miles bear the same proportion to four miles as two hours and a half bear to one hour. The notion of proportion is suggested by innumerable circumstances of ordinary life, as well as by the questions proposed in books on Arithmetic. For example take a map of England; the distance between London and Cambridge on the map bears the *same proportion* to the distance between London and Manchester on the map, as the real distance between London and Cambridge bears to the real distance between London and Manchester. Similarly in the plan of a building the lengths of the straight lines on the plan will be in the same proportion as the lengths of the corresponding straight lines of the building.

11. We pass now to some of the rudiments of Geometry. The meaning of most of the common terms is probably known to the reader, but we will draw attention to them.

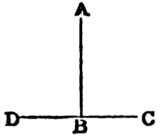
12. An *angle* is the inclination of two straight lines to one another which meet together, but are not in the same straight line.

Thus the two straight lines AO, BO, which meet at O form an angle there. The angle is not altered by altering the lengths of the straight lines which form it; thus CO and DO form the same angle as AO and BO. The angle may be denoted in various ways, as the angle AOB, or the angle AOD, or the angle COB, or the angle COD: all mean the same angle.



13. When one straight line is *upright* to another the angle which the straight lines form is called a *right angle*, and each straight line is said to be *perpendicular* to the other. This is put into a more precise form in the following manner: when a straight line standing on another straight line makes the adjacent angles equal to one another, each of the angles is called a right angle, and the straight line which stands on the other is called a perpendicular to it.

Thus in the figure if the angle *ABC* is equal to the angle *ABD* each of them is a right angle, and *AB* is perpendicular to *DC*.



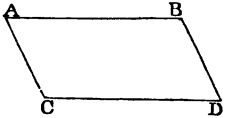
A right angle is divided into 90 equal parts called *degrees*, and a degree is divided into 60 equal parts called *minutes*.

14. *Parallel* straight lines are such as are in the same plane, and which being produced ever so far both ways do not meet.

15. A *triangle* is a figure formed by three straight lines. If one of the angles of the triangle is a right angle, the triangle is called a *right-angled triangle*, and the side opposite to the right angle is called the *hypotenuse*.

16. A *parallelogram* is a four-sided figure which has its opposite sides parallel.

Thus *AB* and *CD* are parallel, and *AC* and *BD* are parallel in the parallelogram *ABDC*.

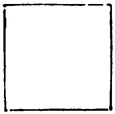


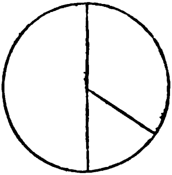
It is a property of such a figure which may be verified by measurement that the *opposite sides are equal;* thus *AB* is equal to *CD*, and *AC* is equal to *BD*.

A straight line joining two opposite corners of a parallelogram is called a *diagonal*. Thus if *AD* and *BC* are drawn each of them is a *diagonal*.

17. A *rectangle* is a parallelogram with all its angles right angles.

18. A *square* is a rectangle with all its sides equal.





19. A *circle* is a plane figure bounded by one line which is called the circumference, and is such that all straight lines drawn from a certain point within the figure to the circumference are equal to one another: this point is called the *centre* of the circle. A *radius* of a circle is a straight line drawn from the centre to the circumference. A *diameter* of a circle is a straight line drawn through the centre and terminated both ways by the circumference. An *arc* of a circle is any part of the circumference.

20. In Arts. 15...19 we have spoken of certain *plane* figures which present themselves very frequently to our notice. We are now about to mention some *solid* figures which are also of great importance.

21. A *cube* is a solid bounded by six equal squares of which every opposite two are in parallel planes. A cube is not an object which comes often under our observation; but an idea of it may be readily obtained. A common brick is usually $8\frac{1}{2}$ inches long, 4 inches broad, and $2\frac{1}{2}$ inches thick; now it is easy to *imagine* a brick in which the *length*, the *breadth*, and the *depth* should all be equal: the brick would then be a cube.

22. A *sphere* is a solid having every point of its surface equally distant from a certain point called the *centre* of the sphere. A *radius* of a sphere is a straight line drawn from the centre to the surface. A *diameter* of a sphere is a straight line drawn through the centre and terminated both ways by the surface. A *sphere* is sometimes called a globe: marbles and billiard balls are familiar examples of spheres.

23. A *right circular cylinder* is an upright column standing on a circular base; it is frequently called briefly a *cylinder*. An uncut lead pencil is an example of this solid. The straight line which joins the centres of the circular ends is called the *axis* of the cylinder. This is the *geometrical* axis of the cylinder: in practice the word often

means not a straight line, but a slender cylinder having the same geometrical axis as the other, but projecting beyond it at the ends.

24. A *pyramid* is a solid bounded by three or more triangles which meet at a point, and by another rectilineal figure. The point is called the *vertex* of the pyramid, and the rectilineal figure opposite to the vertex is called the *base* of the pyramid. When three triangles meet at the vertex the base of the pyramid is a triangle; when four triangles meet at the vertex the base of the pyramid is a four-sided figure; and so on. The bases of the famous pyramids of Egypt are squares.

25. A *right circular cone* is a solid having a circle for its base, and its vertex on a straight line at right angles to the base through the centre: for a strict definition the reader should consult the *Elements of Euclid*, or the *Mensuration*. It is frequently called briefly a *cone*. The straight line which joins the vertex to the centre of the base is called the *axis* of the cone.

26. The *centre* of a circle or of a sphere is a well-known point in connection with them. It is found convenient to extend the use of the word *centre*. In some plane figures a point can be found such that every straight line drawn through it and terminated by the figure is *bisected* at that point. Thus for a parallelogram the intersection of the diagonals is such a point; and it may be called the *centre* of the figure. Likewise a cube and a cylinder have each a *centre* in such a sense.

27. A vast body of important knowledge has been formed in the course of more than two thousand years out of these and a few other definitions and notions. We shall refer the reader for an elementary account of them to the *Mensuration*, and for fuller information to the *Elements of Euclid*. Here it will be sufficient to notice a few facts.

28. We often require to find the length of the circumference of a circle when the length of the diameter is known; and this we can do, though not with perfect accuracy, yet with sufficient exactness for any practical pur-

pose. The following Rule may be used: *multiply the diameter by* $3\frac{1}{7}$. This Rule makes the circumference a little greater than it ought to be, about a foot too great in a circumference of half a mile. Another Rule which is more accurate is the following: *multiply the diameter by* 3·1416. This Rule also makes the circumference a little greater than it ought to be; but the error is very small, being less than a foot in a circumference of 75 miles.

29. To find the *area* of a circle we must take half the product of the radius of the circle into the circumference, or we may multiply the square of the radius by 3·1416.

30. The reader is supposed to be familiar with the general principle which applies to every measurable thing, namely that it must be *measured by a unit of its own kind.* For example when we wish to measure *lengths* we fix on some length for a standard or unit; thus we may take a foot as the unit, and then any length is measured by finding how many times it contains the unit. So also when we wish to measure the *areas* or sizes of plane figures we fix on some area as the standard or unit; thus we may take a square of which the side is one inch as the unit; such an area is called a *square inch.* Or we may take a *square foot*, that is a square of which the side is one foot. In like manner when we wish to measure the bulk of solid figures we fix on some solid as the standard or unit; thus we may take a *cubic inch*, that is a cube of which the edge is one inch long; or we may take a *cubic foot*, that is a cube of which the edge is one foot long.

31. The following are the rules for finding the bulk or volume in the case of some solid bodies.

Cylinder. Multiply the area of the base by the perpendicular distance between the two ends.

Pyramid or Cone. Multiply the area of the base by one-third of the perpendicular from the vertex on the base.

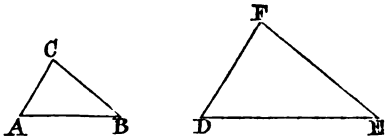
Sphere. Multiply the cube of the diameter by ·5236.

32. The following proposition in Geometry is probably the most important fact in the whole range of human

science; tradition says that it was discovered by Pythagoras about 2500 years ago, and that he offered 100 oxen in sacrifice to shew his gratitude: *In any right-angled triangle the square described on the hypotenuse is equal to the sum of the squares described on the sides.*

33. In the *Mensuration* it is explained how the truth of the preceding statement is rendered visibly self-evident. It is also shewn that when we know the lengths of the two sides we can deduce that of the hypotenuse; and that when we know the lengths of the hypotenuse and of one side, we can deduce that of the other side. It is obvious that the hypotenuse is longer than either of the sides; because the square on the hypotenuse is greater than the square on either side.

34. The important notion of *proportion* makes its appearance in Geometry. Thus, to take a single example, let there be two triangles ABC and DEF, such that the angle A is equal to the angle D, the angle B equal to the angle E, and the angle C equal to the angle F; *then the corresponding sides are in proportion.*



That is to say, whatever may be the proportion of DE to AB, the proportion of EF to BC is the same, and so is the proportion of FD to CA. Thus, for example, if DE is twice AB then EF is twice BC, and FD is twice CA.

35. As a good example of proportion we may take the case of the relation between the heights of two objects and the lengths of their shadows in sunlight. It will be plain to a reader who has a little acquaintance with Optics that the two heights are in the same proportion as the two corresponding lengths.

36. The two triangles in Art. 34 are said to be *similar*. And in general two plane figures are said to be similar when one is exactly a copy of the other on a larger or smaller scale. It is an important property of similar figures that their areas are in the same proportion as the *squares* of the numbers which denote corresponding lengths. For instance if EF is 2 times BC then the area of the triangle DFE is to that of the triangle ABC in the same proportion as the square of 2 to the square of 1, that is in the proportion of 4 to 1.

37. In like manner when one solid is exactly a copy of another on a larger or smaller scale the solids are said to be *similar*. It is an important property of similar solids that their volumes are in the same proportion as the *cubes* of the numbers which denote corresponding lengths. Thus if a brick were made $4\frac{1}{2}$ inches long, 2 inches broad, and $1\frac{1}{2}$ inches thick, it would be similar to the ordinary brick of Art. 21. And as the length of an edge of the ordinay rbrick is 2 times the corresponding length on the smaller brick, the volume of the ordinary brick is to the volume of the smaller in the same proportion as the cube of 2 to the cube of 1, that is in the proportion of 8 to 1.

38. We can always draw straight lines the lengths of which shall be in any *proportion* we please, and thus by the means of straight lines we may often conveniently bring such a proportion before the eye. Take for example 14 pounds and 112 pounds; the latter is eight times the former, and so the proportion may be represented to the eye if we draw one straight line of any length we please and another straight line eight times as long.

39. Philosophers have occasionally speculated on the possibility of constructing some universal language which should serve as a medium of intercourse between all mankind. Up to the present time the signs and diagrams of mathematics seem the nearest approach to the realization of such a scheme; for by their aid, almost without any vocabulary, truths may be rendered intelligible to nations of the most different languages. And even a still wider range

has been suggested for the prevalence of this mode of communication. "We can conceive occurrences which would give us evidence that the Moon, as well as the Earth, contains geometers. If we were to see, on the face of the full moon, a figure gradually becoming visible, representing a right-angled triangle with a square constructed on each of its three sides as a base; we should regard it as the work of intelligent creatures there, who might be thus making a signal to the inhabitants of the earth, that they possessed such knowledge, and were desirous of making known to their nearest neighbours in the solar system, their existence and their speculations." *Plurality of Worlds*, Chapter IV.

40. In asking the beginner to give his attention to the work on Natural Philosophy now put into his hands, it will be well to remind him that the knowledge which he gains from the book should be confirmed and extended by carefully watching the phenomena which spontaneously offer themselves to his observation, and also by attending good experimental lectures if such be within his reach. Some attempt might be made to supersede the advantage of external observation and experiment by elaborate drawings, but it is difficult to make these easily intelligible without familiarity with the objects they represent, and after such familiarity they become superfluous. While it may be readily admitted that *books* on Natural Philosophy alone do not make a sufficient impression on the mind and memory, it is equally certain that a *book* in which the principles are recorded and explained, is a necessary accompaniment to the oral and visible teaching of the lecture room. It must not be forgotten that in the course of life books are always and everywhere accessible, but lectures by no means so certainly; hence too much stress cannot be laid on the importance of early acquiring the habit of learning from books. In these days of diffused knowledge it is curious to observe how many persons of respectable education are practically unable to read; though they may be fluent in conversation and quick to appreciate what is made audible or tangible, they have never accustomed themselves to apply with close attention to the silent and unobtrusive teaching of the printed page.

41. It has been the singular honour of some elementary books intended mainly for youth that they have fallen under the notice of persons of maturer power, and have thus indirectly influenced the history of science. Thus it has been stated that Mrs Marcet's *Conversations on Chemistry* "first opened out to Faraday's mind that field of science in which he became so illustrious, and at the height of his fame he always mentioned Mrs Marcet with deep reverence." (Mrs Somerville's *Personal Recollections*..., page 114.) The same book had the honour of Dr Whewell's attention; he read it and made a short analysis of it in 1817. A sentence in Mrs Somerville's *Connection of the Physical Sciences* incited a living astronomer to undertake the laborious investigation which finally enabled him to ascertain the existence of the planet Neptune, then unknown.

II. VARIOUS BRANCHES OF KNOWLEDGE.

42. Many eminent philosophers have turned their attention to the subject of the classification of the various branches of knowledge, and though no solution of the difficult problem has been obtained which is entirely satisfactory, yet the attempts have been interesting and instructive. We shall not give here any elaborate discussion of the subject, but a few remarks will be advantageous, as they will furnish a general idea of the range of the present work.

43. It will be sufficient for our purpose to consider that there are *five* main branches of knowledge; these may be called Mathematical, Physical, Chemical, Vital and Mental.

44. The Mathematical sciences relate to number and to figure. They have as their foundation Arithmetic and Geometry. They are sometimes called *abstract sciences*, being to a great extent independent of all that takes place in the world around us, and derived by the human mind

from its own resources. These sciences have been cultivated from the days of the ancient Greeks to our own, and as, from their nature, whatever has been once established in them remains as a permanent truth, an enormous mass of striking and valuable results has been accumulated by the labour of successive generations. As we have already said, the amount of mathematical knowledge assumed for the purposes of the present work is very slight.

45. The Physical sciences are often called *Natural Philosophy.* Such sciences might have originally included the knowledge of everything which the world of Nature contains; but at present the term is somewhat restricted in its application. *Natural Philosophy* now may be said to include a group of sciences which has grown up round Astronomy, the oldest and most perfect of them all. Astronomy at first involved only observations of the situations of the heavenly bodies, and predictions of their future course from the records of the past; but Newton by his theory of gravity extended the subject and deduced the motions of the moon and the planets from one general law. Then the whole science of *Mechanics* in its widest sense was gradually formed; this treats of the connexion between *force* and the *motion* which it produces or changes or arrests, and it has different names according as it relates mainly to motion or to rest, to solid or to fluid bodies. With Astronomy is naturally connected the science which treats on *Light,* the medium by which so much of our knowledge of the skies is obtained; and *Navigation* which is closely connected with *Astronomy* introduces *Magnetism* in virtue of the *Mariner's Compass.* Light may be said to draw with it the kindred subject of *Heat,* and Magnetism all the train of sciences which in modern times have sprung from this and *Electricity.* The progress of every science and of every part of a science resembles that of Astronomy; it is traced back to more simple and more general principles as its origin, and carried forward to more numerous and more varied applications and extensions.

46. The Chemical sciences take their rise from the fact that there is more than *one kind* of substance in nature. Had there been only *one kind,* what is called Chemistry

would have been absorbed in Natural Philosophy. But there are more than sixty different kinds of substance, as gold, silver, charcoal, sulphur, and others, which, at least according to present knowledge, are believed to be quite distinct. Now all the Laws of Natural Philosophy hold with regard to each of these kinds of substance separately, so that no new science is introduced as yet. Moreover observation and record of the special properties of each kind of substance would be included in what is popularly known as *Natural History*. But it is found that when two or more of these different kinds of substance are brought together under certain circumstances, then special phenomena are seen. Thus, for example, fine sand and powdered soda exposed to heat and melted together become glass, which differs from each of its components as to its distinctive properties. Again, the metal sodium is poisonous, if swallowed, and the gas chlorine, if breathed; if these are brought together they explode and burst into a flame, and the result of the combustion is common salt, which is very wholesome. Chemistry then treats of all the phenomena which are connected with the combination of two or more kinds of substance to form a new body, or with the separation of any body into the simple kinds of substance of which it may consist. The science had its origin in the attempts made by enthusiasts to convert the more common metals into gold, the most valuable: these men received the name of *alchemists*.

47. Next we have to consider the sciences which involve the idea of *Life*. The bodies of men and of animals, and the vegetable structures, consist of various remarkable collections of tubes and cavities, in which fluids circulate and produce constant change. In addition to the laws which prevail and the forces which act in Natural Philosophy and Chemistry, others of a peculiar kind here present themselves; instead of the permanence which belongs more or less to the objects of the two former divisions of knowledge, we have here the changes involved in birth, growth, and decay.

48. In the last place we have sciences which relate to the *mind* itself, as Logic and Metaphysics. These have

been cultivated from the origin of civilization, and though they have passed through various fluctuations of influence have never lost their charm. "Even in ages the most devoted to material interests, some portion of the current of thought has been reflected inwards, and the desire to comprehend that by which all else is comprehended has only been baffled in order to be renewed." In this division we may place various studies which bear, at least indirectly, on the mind—as that of Languages which has long been held of great value as a training, History which teaches by example the lessons of duty and prudence, and Moral Philosophy which gathers these lessons into a system, and seeks to enforce them by adequate sanctions.

49. It is easy to see that the rough division which we have given of the branches of Human Knowledge is open to the objection of a failure in distinctness; some sciences may claim to appear under more than one of the five classes. Thus we make Mathematics a distinct class, and yet it must be allowed that this science enters largely into all the elaborate works on Natural Philosophy. The aid of Mathematics is absolutely necessary in order to develop fully the principles which are discovered in operation throughout nature; and not unfrequently the wish to penetrate further into the constitution of the earth and the heavens has led men to the construction of new methods in Mathematics. The sciences which we include under the title Natural Philosophy have sometimes been called *Mixed Mathematics*, while the title *Pure Mathematics* has been adopted as more strictly appropriate to the first of our five classes. Up to the present time Chemistry has not been annexed to Mathematics; but eminent men, among whom Faraday may be named, have pointed with satisfaction to that as the destiny of their science.

50. Again, some of the subjects which we have included under Natural Philosophy are closely connected with Chemistry. Thus, many of the changes which Chemistry investigates are produced by the agency of *Heat;* so that this subject belongs both to the third and to the second class of our arrangement. A similar remark holds with respect to *Electricity* and the kindred sciences. Again,

Photography claims the attention of the Natural Philosopher as falling under the head of Light, and is at the same time of great interest to the Chemist by reason of the sensitive materials on which the impressions of the sun's rays are received.

51. Some sciences may appear to be without a place in our classes. Thus for instance, Geology and Mineralogy are not very happily put under Chemistry, though this seems to be the least unsuitable station for them. Moreover subjects which in many respects it would be very convenient to associate become separated by our arrangement. Thus all the components which make up the tolerably well defined aggregate called *Natural History* are broken up and distributed over the third and fourth classes. The arrangement in five classes does however agree reasonably well with the schemes proposed by some philosophers who have paid special attention to the classification of knowledge. The late Dr Whewell, whose great and varied attainments rendered him specially qualified to deal with such a matter, gives implicitly in his *Philosophy of the Inductive Sciences* a scheme of classification. The subjects which are included in our first three classes are arranged by him in the following manner: Pure Sciences; Mechanical Sciences; Secondary Mechanical Sciences, namely, Sound, Light, and Heat; Mechanico-Chemical Sciences, namely, Magnetism and Electricity; and Chemistry. Besides these he treats of the Natural History sciences under various titles.

52. With the *Sciences* are connected *Arts* in which the lessons of theory are applied to purposes of utility and ornament. Thus Engineering, Architecture, and Navigation are all Arts connected with the Sciences of Natural Philosophy. Dr Whewell was fond of calling *Art* the lovely mother, and *Science* the daughter of far loftier and serener beauty; there is no doubt that in the main this is historically correct. Some simple processes of engineering must have preceded the science of mechanics, and some rough comparisons of size and figure must have been made before the truths of geometry were arranged in a system. Not only the ruder arts relating to food and clothing which

sustain our lives, but even those of a more refined character such as music, painting, and sculpture, which *adorn* them, must have made some progress before the sciences arose to explain the principles on which such arts depend. Still in recent times science has more than repaid all that she ever borrowed from art; such important applications as the steam-engine, photography, and the electric telegraph were, we know, derived almost exclusively from science.

53. Controversy has been often maintained with respect to the subjects which are most valuable for the education of youth, and to the order in which they should be presented for study. The older theory in England was that the languages of Greece and Rome were the best instruments for cultivating the powers of the mind, and that the literature preserved in these languages was the most valuable treasury that could be found of history and poetry and mental science. In one University Mathematics have long been highly valued as a discipline for maturer years, and recently they have gained some position in the course of school instruction. Other studies however in the present day urge their claims to attention; modern languages are cultivated principally on the ground that they give access to rich stores of information; Chemistry and the Natural Sciences demand and receive considerable regard, on account of their important practical applications. Perhaps the acquisition of knowledge is appreciated more highly now than in former days, and the mere training of the powers of the mind less exclusively considered. However we must not forget that many able and enthusiastic men attribute as much merit to the favourite modern studies as our forefathers assumed to belong to the Classical Languages and Mathematics. Thus Dr Arnott says, "Reverting to the importance of Natural Philosophy as a general study, it may be remarked that there is no occupation which so much strengthens and quickens the judgment. This praise has often been awarded to the Mathematics, although a knowledge of abstract Mathematics existed with all the absurdities of the dark ages; but a familiarity with Natural Philosophy, which includes fundamental Mathematics, and gives tangible and pleasing illustrations of the abstract truths, seems incompatible with the admission of any gross absurdity."

III. NATURAL PHILOSOPHY.

54. It is the design of the present work to consider an important part of the second of the five classes into which we have divided knowledge in Art. 43; and it will be convenient here to offer a few preliminary remarks which will bring the more important facts into view. The reader will probably not fully comprehend at first all that this chapter contains, but he can hardly fail to obtain from it some general notions which will be of assistance to him as he proceeds through the rest of the work. The advance in knowledge which an individual student obtains by the devotion of time and attention to a science is similar in character to the progress which the science itself makes in the course of ages; the student can trace his way backwards to a clearer view of the first principles, and forwards to more extensive developments and applications.

55. In such a sketch as we are now about to give, the reader enters into possession of knowledge which has been accumulated by centuries of thought and labour. The tendency of this long series of investigations has been to produce a firm conviction that order and law prevail throughout nature; and that often apparently contradictory phenomena result from the operation of one general principle. Thus, for instance, most things fall to the ground when unsupported, while a few, like smoke, or bubbles, or balloons rise. Hence it might seem that there is a difference in the structure of bodies or in the substance of which they consist, in virtue of which some will descend and some will ascend, when set free. The notion is embodied in the well-known witticism respecting a man who gained an eminent position by a discreet sobriety of manner, and then lost respect by his want of official decorum: "contrary to the laws of physics he rose by gravity and sank by levity." But we know now that all the bodies with which we are concerned are really heavy, though some are much heavier,

bulk for bulk, than others; so that if some bodies rise when they are set free, it is not because they have no weight, but because other bodies have more weight, and so sink below them and force them up.

56. Nature then is really an exhibition of the regular and constant operation of Laws. These by their combination or by their conflict give rise to countless phenomena; it is the business of Natural Philosophy to seek for the laws amidst the endless variety of the phenomena. There are two great agents of investigation, namely sense and thought. By sense, such as that of our eyes and ears, we note what goes on around us; we in fact employ *observation* and *experiment*. By *thought* we conjecture explanations of what has been presented to us by sense; that is, we suggest laws and theories, and by the aid of mathematical calculation we trace the consequences of these suggestions: then by a new appeal to observation and experiment we determine how far these consequences really correspond to truth in nature. It is by the combination of theory on the one hand with experiment and observation on the other that the present stock of knowledge has been acquired and is daily augmented. The perpetual striving after principles which will connect and interpret what we see around us is at the same time one of the most necessary tasks and one of the highest pleasures of the human intellect. For so vast and bewildering is the range of phenomena presented to us, that the memory could not retain them and the mind could not apprehend them unless they were connected and illustrated by tracing them up to the operation of a few invariable laws. And no gratification is more intense than that of the philosopher when he has succeeded in uniting a group of facts hitherto apparently isolated, and shewing that they are all consequences of some principle which he has himself discovered: while even to understand that which it has been the privilege of another to reveal is in some measure to share in his satisfaction.

57. The elementary statements of Astronomy are received by the world at large with a confidence which may be called remarkable, when we reflect that the direct evidence for them is very slight. This confidence is an

unconscious tribute to the surpassing genius of Newton, and to the mathematical powers of the great astronomers who, following in his steps, have demonstrated the results now universally believed. The earth is known to be at a distance from the sun of about 90,000,000 miles, and to go round the sun once in the course of a year, tracing out a path which does not differ much from a circle; the earth is retained in this perpetual journey by the attractive power of the sun. The earth is nearly a sphere in form, and turns once round on its axis in a day; this rotation does not require the operation of any external power to keep it up when once it has been started. The moon goes round the earth once in the course of about 29 days, being always at the distance of about 240,000 miles from the earth, and turns once round on its axis in the same time.

58. With a few apparent exceptions, which will be explained hereafter, all bodies when set free are observed to fall to the ground; and so long as we keep near to the same place on the earth's surface the directions of falling bodies are all *parallel* straight lines. The word *vertical* is used to denote the common direction, and a strict definition is easily given. Suppose a pond or small lake in which the water is at rest: then the plane surface of the water is called a *horizontal plane*, and the *vertical* direction is that which is perpendicular to the horizontal plane. The floors of our rooms are *horizontal* planes, and the surfaces of the walls are *vertical* planes. The straight line which is formed where the surfaces of two walls meet is a vertical straight line. Bricklayers who have often to determine accurately the vertical direction make use of a string held at one end and having a weight at the other end; this is called a *plumb-line*, and it hangs when at rest in a vertical direction.

59. We proceed to notice some of the general properties which we observe in the bodies around us, or which we gather by reflection from what we directly see.

60. Take any substance whatever, and we find that we can by mechanical or chemical action divide it into ex-

tremely minute pieces. Thus, to draw our examples from solid bodies, a lump of marble may be crushed into powder, a piece of gold may be beaten into leaves of extreme fineness, sugar may be crumbled or it may be dissolved in water; in the last case we know that the sugar becomes diffused through the whole of the water, because the peculiar taste is present throughout. Hence we learn that matter possesses the property of *divisibility* in an extreme degree.

61. Illustrations of the extreme divisibility of matter are given by various authors. Thus we read "Goldbeaters, by hammering, reduce gold to leaves so thin, that 360,000 must be laid upon one another to produce the thickness of an inch. Eighteen hundred of them occupy only the space of a single leaf of common paper; yet they are so perfect or free from holes, that one of them laid upon any surface, as in gilding, gives the appearance of solid gold." Let us suppose, as we may without extravagance, that by photography, or in some other way, visible impressions can be made on gold leaf; then it might be possible to have a copy of a folio page taken off on a duodecimo page, and yet easily legible by the aid of a small magnifying glass. Thus the contents of a thousand large volumes might be reproduced in the bulk of the little work now in the reader's hands; and therefore all the treasures of a national library packed into a moderate book-case.

62. As further illustrations of the same subject we read that: "A grain of blue vitriol, or carmine, will tinge a gallon of water, so that in every drop the colour may be perceived. A grain of musk will scent a room for twenty years, and will have lost but little of its weight."

63. Notwithstanding the extreme divisibility of matter philosophers have come to the conclusion that there is a limit to the property. Thus, taking a *simple* substance, as gold, they believe that it consists of excessively minute particles which cannot be further divided; and these they call *atoms*. In the case of a *compound* substance they consider that it consists also of excessively minute particles which they call *molecules:* each molecule contains a defi-

nite number of the *atoms* of the simple substances which form the compound. Thus a molecule of sand is supposed to consist of one atom of silicon and two atoms of oxygen. These doctrines have long been received with more or less favour, but they have probably never been held so firmly as at the present time. Some calculations have been made as to the size of the atoms of a substance, and the distance between two neighbouring atoms.

64. It is supposed that the *atoms* cannot be hurt or destroyed; and moreover it is usually held that they possess *extension* and *impenetrability*. These are terms which involve some difficulty, but as they are of frequent occurrence they must be noticed.

65. By *extension* is meant definite *size*, and therefore definite *figure*. Thus the *atom* of Natural Philosophy is not the same as the *point* of Geometry; for the latter has no extension.

66. Most works on Natural Philosophy devote some space to what they call the *Impenetrability of Matter*. Thus we are told that when we attempt to push one ball of wood or metal into another we cannot accomplish it; that if we drive a nail into wood the nail certainly pushes aside the fibres of the wood, but we do not get the wood and the nail into the same place. But the illustrations are not very decisive; there are cases in which two gases or two fluids are mixed together where they seem to blend so intimately that the doctrine of the impenetrability of matter is not confirmed even if it is not somewhat shaken.

67. But the impenetrability of *matter* is not quite the same thing as the impenetrability of the *atoms* of which we believe that simple substances consist. The latter doctrine can scarcely admit of any direct experimental evidence; though it may be conceived to follow from our notion of an atom. Sir J. Herschel, alluding to the extension and impenetrability of matter, says, "at least in the sense in which those terms have been hitherto used by metaphysicians...probably few will be found disposed to maintain either the one or the other."

68. Weight is a property which belongs to all bodies with which we are familiar; that is to say, although in popular language we speak of *heavy* bodies and *light* bodies, yet the difference is one of degree and not of kind. All bodies have weight, although some have much more than others, bulk for bulk. Whether weight is an *essential* property of bodies, as according to some persons extension and impenetrability are, is uncertain; most writers hold that it is not; the late Dr Whewell held that it is, and he wrote a paper the object of which was to demonstrate that *all matter is heavy.*

69. That bodies differ in weight, bulk for bulk, is a matter of common experience: we all know that a piece of lead is very much heavier than a piece of cork of the same size. Now tables have been drawn up which tell us the proportion of the weights of quantities of the same size of different substances. It is usual to take pure water as the standard; and then in such a table we may find against platinum the number 22, and against gold the number 19. This means that a quantity of gold is 19 times as heavy as a quantity of water of the same size, and that a quantity of platinum is 22 times as heavy as a quantity of water of the same size.

70. The words *attract* and *attraction* are of frequent occurrence in Natural Philosophy, and it is important to notice the various cases in which they are employed. Every person has observed what takes place when a piece of iron is put near a magnet; the iron moves towards the magnet, and the magnet is said to *attract* or *draw* the iron. We cannot explain how this is done; we merely see that an effect is produced something like that which occurs when a man or an animal draws a burden along. This may be called *magnetic attraction.*

71. Newton discovered that the earth attracts the moon and that the sun attracts the earth, in a way apparently resembling that in which a magnet attracts iron. The process is mysterious and inexplicable; we see no bands connecting the earth and the moon, and we cannot make any reasonable conjecture as to the agency by which

the result is produced, but we cannot doubt the fact that the earth does draw the moon. It is found also that this attraction is universally a property of matter; that the moon also attracts the earth, and that the earth and the moon attract the sun. The earth's attraction moreover gives rise to the *weight* of a body which is supported, and to the *fall* of an unsupported body: these two results are in a slight degree modified by the earth's rotation on its axis, but in the main they depend on the earth's attraction. This attraction, which, as far as we can see, prevails throughout the universe, is called *attraction of gravitation*, or simply *gravitation*.

72. Another kind of attraction is that which exists between the particles of a solid body. We know that if we want to break up a solid body we must make an effort to separate the parts, and it may happen that a very considerable effort is necessary; hence we are led to ascribe the union of the parts to the existence of some mutual attraction between them. This is called the *attraction of cohesion*, or simply *cohesion*.

73. If the surfaces of two bodies are made smooth, and pressed together with a moderate force, they will sometimes stick very closely. For example, let two cylinders of lead have their ends scraped very smooth, and let the ends be pressed and turned against each other until they are in close contact; then it will require some effort to separate the cylinders. Glass plates can be made so even that when once in contact they cannot be separated without breaking. Contact is frequently ensured by putting some soft substance between the surfaces to be joined; when this dries it is in close contact with both surfaces, and much effort is required to effect a separation. This kind of attraction is called *adhesion;* and in accordance with this and the definition of the preceding Article we may say that particles of the *same* body *cohere*, and that particles of *different* bodies in suitable circumstances *adhere*.

74. Again in Chemistry we often find that if two substances of different kinds are brought together under favourable conditions they will unite and form a new sub-

stance. This is spoken of as *chemical attraction*, or sometimes as *chemical affinity*. A large part of chemistry consists of illustrations of this kind of attraction. For an example: "Sulphuric acid will unite with copper and water, and form a beautiful translucent blue salt, with iron it will form a green salt; and if a piece of iron be thrown into a solution of the copper salt, the acid will immediately let fall the copper, and take up or dissolve the iron. Sulphuric acid will not unite with or dissolve gold at all."

75. *Capillary attraction* is the name given to the action between fluids contained in slender glass-tubes and those tubes themselves; the term has been extended to include some other cases of action between fluids and solids, as we shall see hereafter.

76. *Electrical attraction* is the name given to some cases resembling magnetic attraction, in which electricity is the agent.

77. Of the various kinds of attraction which we have mentioned that called *gravitation* is the one which has been most carefully studied, and of which we know the most. Here we have that very important principle or law called the *law of gravitation*, which tells us how the amount of the attraction changes when the distance of the attracting bodies changes. Suppose we have a sphere composed of one substance, as lead or iron; and suppose we have also another sphere composed of one substance, which may be the same as that of the first sphere or different. Each sphere may be of any size we please. By the *distance between the spheres* let us understand the distance between their *centres*. Put the spheres at any distances apart; then there is a certain attraction exerted between them, so that unless the spheres were in some way kept apart they would move towards each other. Now suppose that the spheres are put at *double* the former distance, then the attraction will be only a *quarter* of what it was at first; suppose the distance is made three times as great as at first, then the attraction will be only one-ninth of what it was at first; suppose the distance is made four times as great as at first, then the attraction will be only one-

sixteenth of what it was at first; and so on. This principle or law is easily understood for all cases; it is technically expressed by saying that the *attraction varies inversely as the square of the distance:* thus, if the distance is made ten times as great, since the square of 10 is 100, the attraction will be only one-hundredth of what it was before.

78. The law stated in the preceding Article is true of the spheres whatever be the size and the substance of each. It is not strictly true of other bodies; but there are cases in which it may be extended to bodies not spherical. Thus, if the bodies are excessively small it will be true without regard to the shape of them; and therefore it is frequently given as the law of attraction of *particles.* If the bodies are not particles, still, if the distance between them is *very great* compared with the size of the bodies, the law will be practically true.

79. As to the kind of attraction called *cohesion* we do not know accurately in what way the change of distance is connected with the change in the amount of the attraction. The attraction appears to be very intense between particles that are very close together, and to become feeble as soon as the distance between the particles is large enough to be practically sensible. It is possible that science may hereafter shew that cohesion and gravitation are really attractions of the same kind; the law being that established by Newton when the particles are at a sensible distance, and taking some other form when the particles are extremely close.

80. Many substances which occur in nature present themselves under three forms, namely, the solid, the liquid, and the aeriform. Thus it is one and the same substance with which we are familiar under the names of ice, water, and steam. Mercury also, which is usually a liquid, can be frozen, and can be turned into a vapour. There are grounds for believing that every substance can take these three forms, and that the change from the solid state to the liquid state is produced by the application of heat, and the change from the liquid state to the aeriform state by the further application of heat.

81. In solids the cohesion is strong, and keeps the particles in contact; in liquids the cohesion is very weak, and indeed scarcely sensible, so that the particles may be separated by the slightest effort; in aeriform bodies there is no cohesion whatever, but on the contrary the particles *repel* each other, and some external force is required in order to keep them near each other. The distinction between the three forms of matter is sometimes expressed with technical precision as follows. Solid bodies have an independent volume and an independent shape. Their parts do not move easily among themselves; it requires always more or less effort to disturb them or to separate them. When once separated they do not unite by being merely placed again in contact. Liquids have an independent volume, but not an independent shape. They take the shape of the vessel in which they are placed. The least effort can move or separate the parts; but after separation the parts unite again when placed in contact. Aeriform bodies have neither an independent volume, nor an independent shape; they spread themselves through any space open to them, until restrained by some external obstacle.

82. It is, as we have said, by the application of heat that the cohesion of solid bodies is destroyed and the liquid state assumed; and by a further application of heat the cohesion is changed into repulsion and the aeriform state assumed. Hence some writers have been inclined to consider heat mainly as a repulsive power opposite in character to that attractive power of which we have already spoken.

After this general notice of our subject, of its connection with other parts of science, and of the necessary preliminary mathematical knowledge, we proceed in this volume to treat in detail of the various mechanical properties of solid and fluid bodies; that is of properties which belong to all such bodies, connected with the operation of force.

IV. MOTION. FALLING BODIES.

83. Objects in motion present themselves readily to our notice as we look around us; and moreover we soon learn that motion is a thing which may be *measured*. Thus we are told that a man whom we know walks four miles an hour, that a certain horse trots nine miles an hour, and that a railway train in which we made a journey moved through thirty miles in an hour. In these cases we understand that the motion is *uniform*, that is the motion is kept up steadily, not becoming sometimes faster and sometimes slower. It will then be an easy question in arithmetic to find the distance moved through by any of these bodies in whatever interval of time may be mentioned; as, for instance, to find the distance moved through by the railway train in one second. In 30 miles there are 30 times 5280 feet, that is 158,400 feet; and in an hour there are 60 times 60 seconds, that is 3600 seconds. Divide 158,400 by 3600; the quotient is 44: so that the railway train moved through 44 feet in one second.

84. There are two words much used when we speak and write about motion, of which the meaning is perhaps familiar, but for clearness should be mentioned; these words are *space* and *describe*. The word *space* is used as equivalent to length or distance; thus we talk about a *space* of 44 feet, meaning a length or distance of 44 feet. The word *describe* is used in the sense of *moving through;* thus we say a body *describes a space of* 44 *feet*, meaning that it moves through a space or distance of 44 feet. Sometimes we omit the word *space* and say that a body *describes* 44 *feet*.

85. When a body describes 44 feet in a second we say that its *rate of motion* is 44 feet a second; or we may say that its *velocity* is 44 feet a second: it is very customary to employ a Latin preposition and say 44 feet *per* second. We see that *velocity* is a thing which admits of exact measurement. Thus, if a child can walk at the rate of one mile in an hour, and his father at the rate of four miles in an hour, the *velocity* of the father is four times that of the child. If a railway train goes 45 miles in an hour, and a

steamer 15 miles in an hour, the *velocity* of the railway train is three times that of the steamer.

86. *Uniform* motion is the simplest kind of motion, and that with which we first become familiar; but we soon find that it is not the only kind of motion. Thus, if an arrow be shot straight upwards the eye can easily see that as the arrow gets nearer to the highest point which it reaches it moves more slowly than when it first left the bowstring; and if a cricket-ball be driven a long way over the ground by a stroke from a bat it moves more slowly at last than at first. Such motion is called *variable* motion.

87. One of the simplest cases of variable motion, and at the same time one of the most important, is that of *falling bodies*. The *fact*, that bodies if not supported will fall to the ground, must have been known from the earliest ages, but what we call the *laws* of falling bodies were not discovered by any person before Galileo, a famous Italian philosopher who lived from 1564 to 1642. We proceed to state these laws.

88. The motion of a falling body is not uniform; the longer a body falls the more quickly it moves at the end of the time. In the following Table the first column gives the number of seconds since the beginning of the motion, and the second column gives the space through which the body has fallen since the beginning of the motion:

In one second	16 feet;
in two seconds	$2 \times 2 \times 16$ feet;
in three seconds	$3 \times 3 \times 16$ feet;
in four seconds	$4 \times 4 \times 16$ feet;

and so on.

Thus we have an easy Rule for finding the *number of feet* fallen through since the beginning of the motion: take the number of seconds, multiply that number by itself, and the product by 16. If we remember the meaning of the word *square* in Arithmetic, as stated in Art. 7, we may put the Rule more briefly thus: *multiply the square of the number of seconds by* 16. For example, to find the number of feet fallen through in 10 seconds: the square of 10 is 100, and $100 \times 16 = 1600$.

89. A reader of a cautious turn of mind may perhaps think that no person can ever have dropped a stone down 1600 feet and observed the time of motion to be 10 seconds; and thus he may suppose that we are here saying more than we strictly know to be true. And indeed it must be confessed that this precise experiment never has been made and probably never will be made: but still we may feel confident that if it could be made the result would be just what has been stated: the grounds of this confidence will become to some extent known as we proceed. We may say briefly here that by observation and experiment we gain the conviction that there are laws of nature, and that these laws are permanent and universal; then when by long investigation we have discovered such a law, we believe that it will hold even in circumstances which do not admit of obvious trial by experiment.

90. The Rule given in Art. 88 will also apply if we wish to find the space fallen through in a time which is not a *whole* number of seconds. For example, required the space fallen through in two seconds and a half. We have $2\frac{1}{2} = \frac{5}{2}$; the square of $\frac{5}{2} = \frac{5}{2} \times \frac{5}{2} = \frac{25}{4}$; and $\frac{25}{4} \times 16 = 100$; thus the space fallen through is 100 feet.

91. The Rule will also enable us to determine the space fallen through in any time which may be specified, even when not beginning with the beginning of the motion. For example, suppose we want to know the space fallen through during the *fourth* second of the motion. In four seconds the space fallen through is $4 \times 4 \times 16$ feet, that is 256 feet; in three seconds the space fallen through is $3 \times 3 \times 16$ feet, that is 144 feet. Hence, to find the space fallen through in the *fourth* second we must subtract 144 feet from 256 feet; the result is 112 feet. Again, suppose it required to find the space fallen through in one-tenth of a second, occurring at the end of three seconds after the beginning of the motion. We have $3\frac{1}{10} = \frac{31}{10}$; the square of $\frac{31}{10} = \frac{31}{10} \times \frac{31}{10} = \frac{961}{100}$; and $\frac{961}{100} \times 16 = 153\frac{19}{25}$. Thus in three

seconds and one-tenth the space fallen through is $153\frac{19}{25}$ feet; and we know that the space fallen through in three seconds is 144 feet; subtract 144 feet from $153\frac{19}{25}$ feet, and the remainder is $9\frac{19}{25}$ feet. This is therefore the space fallen through in one-tenth of a second occurring at the end of three seconds after the beginning of the motion.

92. Thus we can determine the *space* through which a body falls in any time which may be specified; we shall now determine the *velocity* of the falling body at any instant which may be specified; or in other words the rate at which the body is then moving. In the following Table the first column gives the number of seconds since the beginning of the motion, and the second column gives the velocity at that instant.

At the end of 1 second......32 feet per second;
at the end of 2 seconds......2×32 feet per second;
at the end of 3 seconds......3×32 feet per second;
at the end of 4 seconds......4×32 feet per second;
and so on.

Thus we have an easy Rule for finding the velocity, *expressed in feet per second*, at the end of any number of seconds since the beginning of the motion: *multiply the number of seconds by* 32. For example, required the velocity at the end of seven seconds; $7 \times 32 = 224$; thus the falling body at the end of seven seconds is moving at the rate of 224 feet per second. Again, required the velocity at the end of two seconds and a half; we have $2\frac{1}{2} = \frac{5}{2}$; and $\frac{5}{2} \times 32 = 80$; thus the falling body at the end of two seconds and a half is moving ta the rate of 80 feet per second.

93. It will be seen that the number 32 which occurs in the Table and Rule of the preceding Article is *double* the number 16 which occurs in the Table and Rule of Art. 88.

94. The case of falling bodies offers a very simple example of *variable motion;* the velocity, that is the rate of motion, increases just as fast as the time increases, so that, for example, at the end of five seconds the velocity is five times as great as at the end of one second.

95. But here a very important point requires to be explained. As the velocity of a falling body is continually changing, how can we speak of the velocity at any specified instant? When we say that at the end of three seconds the velocity is 96 feet per second, we mean that if no change of motion took place afterwards the motion *would be* uniform, and at the rate of 96 feet per second. This is the *exact* meaning of the statement; and we may illustrate it by putting it in another form which is perhaps easier for a beginner, though not quite so exact. Suppose we ask what space is really fallen through in a *very short time* directly after the end of the first three seconds, say in one-tenth of a second. If the motion were uniform and at the rate of 96 feet per second, the space fallen through in one-tenth of a second would be $\frac{1}{10} \times 96$ feet, that is $9\frac{6}{10}$ feet, that is $9\frac{15}{25}$ feet. We found in Art. 91 that the space through which the body really falls in this tenth of a second is $9\frac{19}{25}$ feet, which is somewhat greater than the result obtained on the supposition of uniform motion at the rate of 96 feet per second. In this way we are led to another answer to the question, what is meant by saying that the velocity of a falling body at the end of three seconds is 96 feet per second: we may say that for a very short time the body without sensible error may be considered to be moving uniformly at the rate of 96 feet per second.

96. The beginner should illustrate the important statement at the end of the preceding Article by other numerical examples. For instance, take a shorter interval of time than one-tenth of a second, say one-twentieth of a second, or one-hundredth of a second; then calculate by the method of Art. 91 the space actually fallen through in this interval directly after the end of the first three seconds: it will be found to differ but very slightly from the space which would have been described in the interval by a body moving uniformly at the rate of 96 feet per second. The smaller we take the interval the more closely will the two results agree. Similarly we can illustrate the statement that at the end of four seconds the velocity of a falling body is 128 feet per second; and so on.

97. We have now stated the *laws* which hold with respect to the motion of falling bodies, but, as will often happen in the course of treatises on Natural Philosophy, we must next indicate some slight modifications and corrections of the general statements which have been made. It will be seen that the matters to which we proceed, though of considerable interest as to theory, do not sensibly impair the practical accuracy of what has gone before.

98. We have taken 16 feet as the space through which a body falls during the first second of its motion; but the number is really rather different for different places, and at London it is about 16 feet and 1 inch. It increases as the distance from the equator increases, and is about an inch greater at the poles than at the equator. So also the velocity at the end of the first second, which we denoted by 32 in Art. 93, is really different in different places, being for any place whatever double the number which denotes at that place the space fallen through during the first second.

99. Again, we have spoken of *falling bodies*, without any distinction, as if the motion were precisely the same for all bodies whatever; but strictly speaking this is not true. The air which surrounds the earth resists the motion of falling bodies, and the resistance is more influential for light bodies, like cork or paper, than for heavy bodies, like stone or lead: the reason of this will be explained hereafter. But it is found by experiment that, if the air be removed, what is called a heavy body and what is called a light body fall down equal spaces in the same time. This can be shewn by the aid of the air-pump, an instrument to be described hereafter; the experiment is very impressive, and has been ascribed to Newton. But even without an air-pump it is easy to shew that the difference in the motions of falling bodies is not due to the kind of substance of which the body is composed. For gold, which in the case of a sovereign falls as fast as anything which we have commonly in view, may be beaten out to a thin leaf which almost floats in the air; and on the other hand a sheet of paper when open falls very slowly, but when rolled up into a tight ball falls like wood or stone. It must be

observed that the resistance of the air increases very greatly as the velocity of the moving body increases; and thus the laws which we have given as those of falling bodies would require some practical correction if they were to be applied to cases of bodies falling during long times.

100. Finally, the laws which we have stated apply to falling bodies near the surface of the earth. If we had the power of ascending to a distance of hundreds of miles from the earth, and dropping a body from such a point, the motion would be of a different kind: this will be noticed hereafter. See Art. 301.

101. As an example of the laws of falling bodies it is sometimes proposed to find the depth below the ground of the surface of the water in a well. Suppose, for instance, that a stone is dropped into a well, and that in two seconds it is heard to strike the water. Since in two seconds a body falls through 64 feet, we may take this for the required depth. This is really a trifle too great, because it takes *some* time, though very little, for the sound of the splash to reach the ear; and thus the real time of the motion of the stone to the water is somewhat less than two seconds. But it need scarcely be said that dropping stones into a well is a practice to be avoided for fear of choking the well. There can, however, be no objection to pouring back into the well some of the water drawn from it; and this is sometimes done for the amusement of visitors in the case of wells which are famous for their depth. But when the well is very deep the resistance of the air on the drops of falling water will be so great as to render the experiment worthless with regard to any numerical result. In the fortress of Königstein, in Saxony, there is a well which is known to be 640 feet deep, so that the splash, if there were no resistance of the air, would reach the ear in about seven seconds after the water is poured back; but practically the time is about fifteen seconds.

102. The *direction* in which a body falls is that which is called the *vertical* direction, which is perpendicular to a horizontal plane; and thus it is different at different places on the earth; but the difference is not sensible to

common observation so long as we keep within a few miles of the same spot. See Art. 58.

103. We have hitherto spoken of falling bodies as phenomena which are observed, without referring to the *cause* of the phenomena. We will now briefly allude to this point. The earth in fact draws bodies to itself, somewhat in the same way as a magnet draws a piece of iron towards itself. This *attraction* of the earth, as it is called, gives rise to the *weight* of a body which is supported, and makes a body *fall* which is unsupported. The effect of the attraction is slightly diminished by the rotation of the earth on its axis; so that, for instance, if there were no rotation a body at the equator would fall in the first second through about two-thirds of an inch more than it actually does. The word *gravity* is used to denote this power which the earth possesses, as shewn in the *weight* and the *fall* of bodies; the effects are said to be produced by the *force of gravity*, or simply by *gravity*.

V. RELATIVE MOTION. COMPOUND MOTION.

104. We know from Astronomy that the earth is not at rest, but really possesses two different motions at the same time; it moves nearly in a circle about the sun once in a year, and it turns round its axis once in a day. Speaking roughly, we may say that in consequence of the motion about the sun the earth moves through somewhat more than one and a half millions of miles in a day; or we may say that it moves through a space about equal to two hundred times its own diameter. In the former mode of statement the velocity seems almost inconceivably great; in the latter it seems more moderate. In consequence of the earth's turning round its axis a place on the equater describes in one day a circle of which the circumference is about twenty-five thousand miles. Now all people and things on the earth have these two motions, and hence

when in common language we say that an object is at rest, we mean that it is at rest *so far as the earth is concerned*, and not that it is really at rest. Or we may express our meaning by saying that the object is *relatively* at rest, and not *absolutely* at rest.

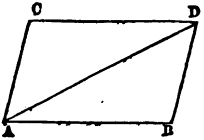
105. In like manner when bodies *fall* the motion which we observe does not constitute the *whole* motion, but only the motion *relative* to the earth. The notions of *relative* rest and motion as different respectively from *absolute* rest and motion are very important. The fact that we may be in motion and scarcely conscious of it, can be established by a common observation. Let there be, as is frequently the case, two railway trains side by side at a station, and let one of them move gently. A person sitting in either of them can see that there is a motion of one of them; but if he merely looks at the other train without noticing the objects beyond it, he may be puzzled to decide which of the trains is really moving. He in fact sees that there is *relative* motion, but is uncertain whether his own train or the other is at rest with respect to the station.

106. The consideration of *relative* motion leads naturally to that of *compound* motion. Suppose a steamer going uniformly through very smooth water; then persons on the deck find that they can perform all such actions as involve motion in the same manner as if the steamer were at rest. Thus a ball can be thrown up and caught again in the hand in the same manner as by the same person on land. Also if a stone is dropped from the top of the mast of the steamer it falls at the foot of the mast, just as it would do if the steamer were at rest. A railway carriage in motion would give the means of observing the same class of facts if the space within the carriage were rather larger. As it is any person may see that the movements of a fly inside such a carriage seem precisely the same as those of a fly in a room; and some observers have noticed that insects keep up with a railway carriage, alternately flying out and in at the windows, in the same way as if the train had been at rest. The general principle in all these cases is that when a vehicle is in motion persons and things in the vehicle all have that motion.

Neglect of this principle has sometimes led to strange blunders; thus a person proposed to reach America in a balloon by ascending in the air, waiting there until the earth's rotation brought America beneath him, and then descending; he forgot that his balloon on starting from the ground had, like all other terrestrial objects, the motion of rotation which the earth has.

107. Return to the case of the steamer. Suppose the steamer moving directly towards the North; let a marble be shot from a point on one side of the steamer towards the corresponding point on the other side, say towards the East. Then the marble goes with the ship towards the North, and goes towards the East in virtue of the special motion which is given to it. Its motion on the whole is said to be *compounded* of the two motions, namely, that of the ship towards the North, and the special motion towards the East.

108. The following is the exact statement of the principle of the *composition of motions.*



Let AB represent in magnitude and direction one velocity given to a body, and let AC represent in magnitude and direction another velocity given to the same body at the same time. Complete the parallelogram of which AB and AC are adjacent sides, and draw the diagonal AD; then *AD will represent the whole velocity in magnitude and direction.* The velocities represented by AB and AC are called *component velocities*, and the velocity represented by AD is called the *resultant velocity.* The principle is for brevity called the *Parallelogram of Velocities.*

109. For an example return to the case of the steamer. Suppose it moving at the rate of 30 feet per second towards the North, and let AC denote this velocity. Also suppose we give to the marble a velocity of 40 feet per second towards the East, and let AB denote this velocity. Then AC and AB must be in the proportion of 30 to 40; so that if AC is 30-tenths of an inch AB will be 40-tenths

of an inch. Also the angle CAB will be a right angle, since that is the angle between the Northern and Eastern directions at any place. Thus AD will represent the whole velocity; and it will represent 50 feet per second. The length of AD follows from the proposition of Art. 32; for it is a property of the figure, since CAB is a right angle, that ABD is a right angle; and the square of 50, which is 2500, is equal to the sum of the squares of 30 and 40, that is to the sum of 900 and 1600. Also the angles CAD and BAD are the angles which the resultant velocity makes with the Northern and Eastern directions respectively.

110. By measuring the lengths on a scale, and drawing the figure carefully, we can by the principle of Art. 108 always find the resultant velocity when the component velocities are known, assuming that the principle is true. That the principle is true will be seen by reflecting on the experimental fact stated in Art. 106, that the motions on the deck of the steamer proceed as if the steamer were at rest. It is a property of the parallelogram $ABDC$ that BD is equal and parallel to AC. Now when we say that AB represents one velocity given to the body, we mean that if this were the only velocity the body would move from the point denoted by A to the point denoted by B, along the direction denoted by the straight line AB, in the unit of time; and when we say that AC represents another velocity given to the same body at the same time, we mean that if this were the only velocity the body would move from the point denoted by A to the point denoted by C, along the direction denoted by the straight line AC, in the unit of time. Then in virtue of both velocities we say that AD represents the whole velocity, meaning that the body will really move from the point denoted by A to the point denoted by D, along the direction denoted by the straight line AD, in the unit of time. The position of the body at D is the same as if it had first moved from A to B, and then from B to D; and as BD is equal and parallel to AC, we see that the final *position* is the same whether we suppose the two component velocities simultaneous or successive.

VI. MOTION CAUSED BY FORCE.

111. When by our own strength we put a body in motion, or change or stop the motion which a body already has, we feel that we make some *effort*, that we exert *force*. We soon learn that we can assist the power of our own bodies by the aid of tools or machines, as of a bat to strike a ball, or of a bow to discharge an arrow. Again, we avail ourselves of the stores which nature presents to us in the services of animals to draw our burdens, and we have no difficulty in believing that the force which they put forth in their tasks resembles that which we ourselves are conscious of exerting in similar circumstances. Next we employ the resources of inanimate nature, as of the wind to waft our ships or of the falling waters of a stream to turn our mills. Finally, there are more subtle agencies, not presented by nature of her own accord, but gained from her by the inventive skill of man; such as the explosive force of gunpowder and the expansive force of steam.

112. It is the nature of man to seek for the *causes* of the *effects* which he experiences, and though in many cases his search leads to little direct result yet he gains much indirectly; for the effects become better known by the attempt to trace them to their causes, and sometimes we may ascertain many of the laws according to which a cause acts though the cause itself may remain unknown.

113. We cannot explain how the earth has the remarkable property of attracting bodies, though we are sure of the fact; see Art. 71. Indeed so familiar are we with the fact that we cease to wonder at it; and yet there is something very strange in the communication of motion by one lifeless mass, as the earth, to others, as stones and logs. The marvel will be increased when we learn from Astronomy that this power of attraction belongs apparently to all the bodies of the universe, and that it reaches through the long distances which separate the earth from the sun

and the planets. A very able man, the late Professor Vince of Cambridge, held that this wonderful power of attraction could be explained in no other way than by ascribing it to the immediate and ever present action of the Deity.

114. Although forces differ as to their origin, yet they all agree as to the way in which they produce or change motion. There are certain *Laws* which hold with respect to the connection between force and motion, which are called briefly *Laws of Motion*. These Laws are not presented in quite the same form and number by all writers, although there is in general substantial agreement among those who have carefully discussed the subject. We shall follow Newton in making *three* Laws of Motion, and also in the mode of stating them.

115. *First Law of Motion.* Every body continues in a state of rest, or of uniform motion in a straight line, except in so far as it may be compelled to change that state by force acting on it.

The law may be said to assert that every body is of itself passive and inert, and cannot of itself either begin to move or change its motion if it has any. The body is of course to be understood as inanimate; nothing is said as to the complex operation of the will and the muscles by which a man moves himself, or of the instinct and the muscles in the case of an animal.

116. We must now consider what evidence we have of the truth of the Law. That a body at rest will continue in that state unless force acts on it may be held to be obvious from observation and trial. But that a body in motion, if left to itself, will continue to move uniformly in a straight line seems not so obvious. For we cannot devise any means of preserving a body which is in motion from the action of force, and so we cannot obtain that perseverance in uniform rectilinear motion of which the Law speaks. If a stone is made to slide along the ground it is soon reduced to rest; but we can easily admit that this destruction of motion is due to the roughness of the ground. Accordingly we find that if the same stone is

started in the same manner to slide on a smooth sheet of ice it will go much further before it is reduced to rest. And we may imagine that if we could remove all obstacles arising from the roughness of the surface on which the stone slides, and from the resistance of the air, the motion might go on for ever unchanged.

117. Still it would be wrong to suppose that the Law can be readily accepted by the beginner on the ground of such rude experiments as he may make or imagine. The history of science shews distinctly that the Law is not one of those truths which present themselves obviously and are easily believed. The ancient Greeks, who made great progress in some branches of knowledge, never reached the simple Laws of Motion; and the honour of laying the foundation of this part of Natural Philosophy was reserved for Galileo.

118. Observation will however supply facts which are quite consistent with the Law. Thus, for instance, if a railway train in motion is suddenly stopped a passenger seated on the back seat of a carriage finds himself *thrown forwards*; this arises from the fact that he continues in his state of motion after the carriage itself is stopped. So also if a man is riding rapidly on horseback and the horse stumbles the man is thrown over the horse's head.

119. But since the direct evidence which can be produced in favour of the first Law of Motion is slight, it may be asked how can we be confident of its truth. The answer is complete and decisive. The oldest and most eminent of human sciences is Astronomy; and the theory of Astronomy rests on the three Laws of Motion as a foundation. By the aid of this theory astronomers are able to predict years beforehand the occurrence of striking phenomena in the heavens, such as eclipses of the sun and moon, and the return of a comet after an unseen journey of three quarters of a century; and these predictions are found to be fulfilled with minute accuracy. Thus, for example, it was predicted long in advance, that on December 9th, 1874, there would be the remarkable appearance called a transit of the planet Venus over the Sun's disc;

accordingly all the civilized nations of the world sent observers at great expense and trouble to various places to watch the appearance: and it occurred at the appointed day and hour. Now it is impossible to suppose that the three Laws of Motion can be false when astronomers have deduced from them numerous and various results which are found to be accurately true; and thus we may say briefly that the unfailing certainty with which the predictions of Astronomy are always verified supplies abundant evidence of the truth of the Laws of Motion. We need not repeat this remark in connection with the second and third Laws.

120. *Second Law of Motion.* Change of motion is proportional to the acting force, and takes place in the direction of the straight line in which the force acts.

This Law requires to be explained before the beginner will receive all that its statement includes; at present we will take only a portion of it. Suppose then that a body is moving in a straight line, and that a force acts on the body in the same direction; then the Law says that the change of motion is proportional to the acting force. This implies that the change of motion does *not* depend on the velocity which the body already has. Now this may be very well exemplified by the case of falling bodies which we considered in Chapter IV. Thus, for instance, according to our statement in Art. 92, the velocity of a falling body at the end of three seconds is 96 feet per second; so that if gravity were suddenly to cease the body would fall through 96 feet in the next second. But gravity continues to act, and according to the second Law of Motion it will affect the motion in the same way as if the body, instead of starting with the velocity of 96 feet per second, started from rest. Thus throughout the fourth second fresh velocity is constantly communicated to the falling body, just as it was during the first second; so that by virtue of this action 16 feet are fallen through, besides the 96 feet fallen through by virtue of the velocity at the beginning of the fourth second. Hence on the whole 112 feet are fallen through in the fourth second. This agrees with the result obtained in Art. 91.

121. The facts to which we have called attention in Arts. 106 and 107 are in agreement with the second Law of Motion. The steamer and every thing on it move from South to North; then the motion or change of motion which is produced in any thing on the steamer by the action of force is the same as that force would produce if the steamer were at rest. Newton himself deduces from the second Law of Motion the principle called the *Parallelogram of Velocities* which we have stated in Art. 108.

122. So long as we consider only the *same body*, change of motion is measured by *change of velocity;* then the second Law of Motion asserts that any force will communicate velocity in the direction in which the force acts, and it is implied that the amount of the velocity so communicated does not depend on the amount of the velocity which the body already has. When we consider different bodies the Law implies something more than this, as we shall see hereafter; we defer the discussion of this, and of the third Law of Motion until we have explained what is meant by *Mass.*

123. The beginner must not expect to become at once familiar with the full meaning of the Laws of Motion; by watching the application made of them in trustworthy books, and by reflexion, he will gradually gain a firm hold of them, and learn to use them with confidence to explain what he sees around him. One caution is necessary with respect to the kind of motion which we have in view. We mean such motion as that of a falling body, or of a body carried by a railway train; motion in which one point of the body moves just like any other point which may be on the side of it, or in the front of it, or behind it. We mean in fact to leave out of consideration the motion of *rotation*, such as that of the sails of a windmill, or of a child's top. The motion of rotation may exist together with the other kind of motion, as we know from the case of the earth as stated in Art. 104; and a more obvious example is furnished by the wheels of a carriage which turn round while at the same time they move forward with the body of the carriage. The motion of rotation is more difficult to treat

than that to which we here confine ourselves. The caution as to the kind of motion we are considering is expressed by some writers by speaking in the Laws of Motion of *particles* instead of bodies.

124. A few more remarks relating to falling bodies will be useful in further illustration of the second Law of Motion. Suppose a body to fall from a certain height to the ground; then if the body be sent straight upwards, starting with a velocity equal to that with which it reached the ground, it will just reach the height from which it fell, taking the same time in the ascent as it did in the fall. This may be easily seen from a particular instance. Suppose that the time of the fall is four seconds; then by Art. 92 the body reaches the ground with the velocity of 128 feet per second. Start the body straight upwards with this velocity; then if gravity did not act the body would move through 128 feet in one second, and still retain at the end of the second the velocity with which it started: this follows from the first Law of Motion. But during this second gravity acts, and in the direction just *contrary* to the motion; and in virtue of this a *downward* velocity of 32 feet per second is given to the body, and also the body is drawn *down* through 16 feet. In consequence of this the body really ascends upwards through 112 feet, and has at the end of the second an upward velocity of 96 feet per second. That is the body has ascended through just the space which a body would fall through during the *fourth* second of its motion, and it has a velocity upwards just equal to the downward velocity of a body at the end of the *third* second of its fall. In precisely the same manner we can shew that in the next second the body will ascend through 80 feet, and will have at the end of the second an upward velocity of 64 feet per second; that is it will ascend through just the space which a body would fall through during the *third* second of its fall, and it has a velocity upwards just equal to the downward velocity of a body at the end of the *second* second of its fall. Proceeding in this way we find that the body just reaches in four seconds the height from which it fell, and it has then no velocity, so that it goes no higher.

125. In the same way as the example of the preceding Article was treated we may treat any similar example. It will be observed that the following proposition becomes evident from the course of the discussion. Suppose a body to fall from a certain point to the ground and to be started upwards with a velocity equal to that with which it reached the ground, then the velocity on reaching to any height is equal to that of the falling body at the same height, though in the opposite direction. We may also shew that the following Rule will give us the height which the body will reach in any assigned time: *Calculate the space through which a body would have moved uniformly in that time and from it subtract the space through which a body would have fallen from rest in that time; the remainder gives the height required.* For instance, returning to the Example of Art. 124, find the height reached in two seconds. A body moving uniformly with the velocity of 128 feet per second, will in two seconds describe 128×2 feet, that is 256 feet. And in two seconds a body would fall through $16 \times 2 \times 2$ feet, that is through 64 feet. Now $256 - 64 = 192$; and $192 = 112 + 80$, that is 192 feet is the height of the body at the end of two seconds by Art. 124. It must be remembered that in this and the preceding Article we neglect the influence of the resistance of the air.

126. While a body is falling the only force acting on it is gravity, and so also while a body is rising the only force acting is gravity, which acts in the direction contrary to motion. It is quite true that in the latter case some force must have acted just at the beginning of the ascent to start the body, but still *during* the ascent the only force acting is gravity. This may appear a simple remark, but it is necessary to draw attention to it, because some popular books are very erroneous as to the matter.

127. If we know how long a body has been falling we can immediately determine the space through which it has fallen, by Art. 88; and we can determine the velocity which it has at the end of that time by Art. 92. From the Rules given in these two Articles various others can be deduced by processes which do not require more than common Arithmetic. Thus from Art. 88 we may deduce the following

Rule for finding the number of seconds occupied in the fall: *Divide the number of feet fallen through by* 16 *and extract the square root of the result.* Then if we multiply the result thus obtained by 32 we obtain the velocity at the end of the time. Or we may obtain this velocity by the following Rule which will be found on trial to agree with the former: *Multiply the number of feet fallen through by* 64, *and extract the square root of the product.* The last Rule is important and often wanted in practice.

VII. MASS AND MOMENTUM.

128. Suppose we take two bodies of the same size and shape, say a cricket ball and an iron ball just as big; we find that the iron ball presses more strongly than the cricket ball on the hand which holds them: in fact the iron ball *weighs more* than the cricket ball. Now we use the word *matter* to express the substance, material, or stuff of which bodies are composed; and we use the word *mass* as an abbreviation for *quantity of matter*. We also take it for granted that at the same place on the earth's surface the *mass* of bodies is proportional to their weight.

129. The reader will naturally be led to think that as *mass* is proportional to *weight* it is unnecessary to introduce the word *mass*. But as we proceed it will be found very convenient to have this word expressing something which belongs to the body, and remains unchanged when the body is taken from one place to another. We have said that at the *same place* on the earth's surface the mass is proportional to the weight, and it is important to bear in mind this condition, *for the weight of a body is not the same at all places*. If we use a pair of scales to weigh a body in the ordinary manner, we shall find no difference in the number of pounds and ounces which we call the weight of the body when we take the scales to various places. A foolish person being told that bodies weighed less when taken to a height above the earth's surface than they did at the surface, declared that the statement was untrue, for he had weighed a body most carefully in the cellar and in the attic of his house, and found no difference in the two cases. He had misunderstood what he had been told, and

which we may explain as follows. Suppose a string just strong enough to bear at London without breaking a certain piece of lead fastened to the end of it; then at the equator it would bear a rather larger piece without breaking, while at the pole it would not bear quite so much. If a string could bear at the pole the weight of 200 coins all exactly alike, then at the equator it would bear the weight of about 201 of them; or in other words the weight of any body is diminished by about $\frac{1}{200}$ in passing from the pole to the equator. The diminution is another consequence of the same cause as that which operates in Art. 98; where the result is a diminution of about 1 inch in 16 feet with respect to the fall of a heavy body in a second. Instead of weighing a body in *scales* we may make use of one of the contrivances by which the result is ascertained by noticing how far the body will bend a spring; then it will be found, if we employ a very delicate spring, that the weight is less in places which are nearer to the equator than in those which are further from it.

130. In examining questions about motion we soon learn that we have to pay attention to two things, the *mass* in motion, and the *velocity* with which it is moving. Thus the mischief and destruction which a cannon ball produces increase both as the mass of the cannon ball increases and as the velocity with which it moves increases; and a similar remark holds with respect to the disaster of a collision between ships, or between railway trains. An iceberg, though moving with very small velocity, may produce a great effect by its vast mass. Accordingly we are led to the important idea which we express by the word *momentum;* this means the *product of the mass into the velocity.* Thus if one body has a mass which we denote by 2, and a velocity which we denote by 3, the momentum is 2×3, that is 6; hence another body which has a mass 3 and a velocity 2 will have the *same* momentum; and a third body which has a mass 4 and velocity 6 will have a momentum 24, which is four times as great as in the former cases. The word *momentum* is one of those which unscientific people employ in various senses, so that the reader must bear in mind the strict meaning which we give to it.

131. We will now repeat the *second Law of Motion.* Change of motion is proportional to the acting force, and takes place in the direction of the straight line in which the force acts. By motion here we are to understand motion as measured by *momentum;* and with this explanation we need not restrict ourselves to the case of one body and one force, but may if we please take more complex cases in which different bodies and different forces occur.

132. The effect of force then is to give velocity to bodies, and we measure the effect by the *momentum* produced. Hence if we have a certain force at our disposal we can produce only a certain amount of momentum; if we operate on a heavier body we produce a less velocity than if we operate on a lighter body. Thus if a blow will give a certain velocity to a ball, the same blow applied to a ball of double weight will give half the former velocity. Now it will be seen that the force of gravity differs remarkably in one respect from the forces of men, of animals, of wind, of water, and of steam with which we are familiar. In all the latter cases we are accustomed to see a less velocity produced according as the body in which it is produced is greater. But when bodies fall to the ground, whether they are large or small they acquire equal velocities in falling for the same time. The fact is that the force of gravity is not of a *fixed amount* for all bodies, but varies in proportion to the mass moved. If a double mass has to be moved the force of gravity puts forth as it were a double energy; or in other words the force of gravity acts on each of the two equal halves of the double mass just as if the other half did not exist.

133. We can now give some explanation of the fact noticed in Art. 99, that the resistance of the air interferes more with the motion of light bodies than with the motion of heavy bodies. Let us suppose a hollow ball made of very thin iron, and a solid ball of the same size also made of iron. As we have just remarked, the force of gravity will give the same velocity to one ball as to the other in the same time, so that, setting aside the resistance of the air, the two balls would fall through equal spaces in the

same time. Now consider the resistance of the air; this cannot depend in any way on the nature of the inside of the balls, and so must be the same on two balls of the same size, shape, and texture of surface, if they move with the same velocity. But by Art. 132 this force would produce less velocity in the solid ball than in the hollow ball; and so in the actual case we may readily suppose that it will take away much less of the downward velocity of the solid ball than of the hollow ball.

134. An example will illustrate the difference of the influence of the resistance of the air on bodies differing only in size. Suppose, for example, two cannon balls, one 4 inches in diameter and the other 5, but formed of the same material. The masses of the balls are in the same proportion as the cubes of the diameters, that is in the proportion of 64 to 125. It appears by experiment and theory that the resistances of the air are in the same proportion as the squares of the diameters, that is in the proportion of 16 to 25, that is in the proportion of 80 to 125. Hence we see that the resistance on the smaller ball bears to that on the larger ball a greater proportion than the mass of the smaller ball bears to that of the larger; and so the resistance exercises more influence on the smaller than on the larger ball, supposing the velocities equal.

135. We have not given a very full account of the influence of the resistance of the air because we have not attended to the way in which the resistance depends on the velocity of the moving body. In reality the resistance increases very rapidly as the velocity of the moving body increases. For an example, it has been found that under certain circumstances the range of a cannon ball would be 23000 feet if there were no such resistance, while it was in fact about 6400 feet. Another example is furnished by an experiment with a railway engine. The engine was started down an inclined plane with a velocity of 45 miles an hour; the velocity gradually diminished until it became 32 miles an hour and remained at that. Thus the resistance of the air together with that caused by the want of perfect smoothness in the wheels and iron rails just balanced the influence of the force of gravity in urging the engine down the plane and so the velocity was maintained uniform.

VIII. THIRD LAW OF MOTION.

136. *Third Law of Motion.* To every action there is always an equal and contrary reaction: or the mutual actions of any two bodies are always equal and oppositely directed in the same straight line.

Newton gives three illustrations of this Law:

If any one presses a stone with his finger, his finger is also pressed by the stone.

If a horse draws a stone fastened to a rope, the horse is drawn backwards, so to speak, equally towards the stone.

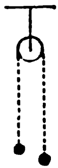
If one body impinges on another body and changes the motion of the other body, its own motion experiences an equal change in the opposite direction.

In the third illustration *motion* is to be measured by momentum as in all cases. We shall return to the discussion of this illustration hereafter.

137. One of the most important examples of this Law is furnished by the attraction of bodies. The earth, for instance, attracts a body, and that body attracts the earth again with equal power. Thus when the earth produces velocity in a falling body that falling body also produces velocity in the earth, although the latter velocity is so small as to be imperceptible. For, according to the third Law of Motion, the stone gives to the earth as much *momentum* as the earth gives to the stone, and as the *mass* of the earth is incomparably greater than that of the stone the *velocity* given to the earth is incomparably less than that given to the stone. In the science of Astronomy the mutual attraction of bodies is a principle of supreme importance; the earth, for instance, attracts the moon, and the moon attracts the earth again with equal power.

138. The fact that not merely the earth as a whole attracts, but that distinct portions of the earth also do so, has been made obvious by noticing the action of moun-

tains on plumb-lines hanging at places near them. It is thus discovered that the weight at the end of a plumb-line is drawn a little towards a neighbouring mountain; so that the plumb-line does not hang quite in the direction in which it would hang if there were no mountain. In very accurate surveys of the earth made for the purpose of determining its exact size and shape, it is necessary to pay great attention to the deviation which the action of mountains produces in the direction of the plumb-line.



139. A very interesting example of motion is furnished by a contrivance of which the essential part is indicated by the diagram. Two heavy bodies are connected by a string which passes over a smooth peg. Here the force of gravity tends to draw each body *down*, while the force exerted by the string tends to draw each body *up*. The force exerted by the string is the same on the two bodies in agreement with the third Law of Motion, which makes the *action* of one body on the other equal to the *reaction* of the latter on the former. Experiment will shew that if the two bodies are of unequal weight and are left to themselves the heavier will descend; so that the force exerted by the string is less than the weight of the heavier body, but greater than the weight of the lighter body. When the case is examined by the aid of a little mathematics it is found that the motion is just like what would take place if a force equal to the *difference* of the two weights were employed to move a mass equal to the *sum* of the two masses. Thus if one body weighs 13 pounds, and the other weighs 12 pounds, the motion will be just like that of a body which weighs 25 pounds acted on by a force of 1 pound. Therefore the motion will be like that of a falling body but much slower, namely, at the rate of 1 foot for every 25 feet of the body falling freely.

140. The preceding example is one of those which justify our confidence in the truth of the laws of falling bodies: see Art. 89. We have here a case of motion which by the aid of sound theory we can shew to be of the same *kind* as that of falling bodies; while the motion is so much

less rapid that it can be easily observed. A machine is made, named after its inventor, Atwood, which is furnished with appliances for performing the experiment easily, but which in principle is the contrivance of the preceding Article. The results are very satisfactory, and the student will be pleased when he has the opportunity of seeing them exhibited in a lecture-room.

141. It is usual to call the force exerted by a string, as in Art. 139, the *tension* of the string. There is nothing special in the nature of the force exerted in this way, but it is convenient to give it a name.

142. The solution of the problem of motion noticed in Art. 139 involves more mathematics than we assume in the reader; but it may be instructive to verify by an example the result which is asserted to hold, at least so far as to shew that it is reasonable and consistent with itself. It will be seen that *both* bodies move, and that by the nature of the contrivance the weights of the two bodies are set in *opposition* as it were; so that the motion may naturally be that which would be produced in the *sum* of the masses by the *difference* of the weights. Now in the example we say that the heavier body will descend at $\frac{1}{25}$ of the rate of a body falling freely; thus in fact $\frac{24}{25}$ of the weight of the body is taken away by the tension of the string. Again, the lighter body *rises* at $\frac{1}{25}$ of the rate of a body falling freely; thus in fact the weight of the body is taken away by the tension of the string and besides a force equal to $\frac{1}{25}$ of the weight exerted upwards. Thus the tension of the string must be $\frac{24}{25}$ of the weight of the heavier body, and must also be $\frac{26}{25}$ of the weight of the lighter body; so that our statement will not be consistent unless these two results are equal: it is easily found by trial that they are equal, each of them being $12\frac{12}{25}$ pounds.

IX. COMPOSITION OF FORCES AT A POINT.

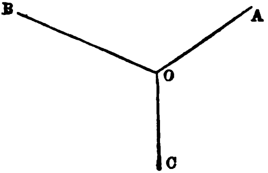
143. In Chapters IV. to VIII. we have discussed the motion of falling bodies, and also the Laws which relate to the connexion between force and the motion produced by it; we must now devote some Chapters to the consideration of forces not producing motion but checking the action of other forces. It is a matter of observation that forces may act on a body without putting it in motion. A man may try to lift a body and find it too heavy for him: in this case the body is acted on by the force of gravity downwards, by the resistance of the ground on which it is placed which acts upwards, and by the effort of the man which also acts upwards; and the body remains at rest. When a body remains at rest though acted on by forces, it is said to be in *equilibrium;* and the forces are said to *counteract* each other or to *balance* each other.

144. There are three things to consider with respect to a force acting on a body; the *point of application*, that is the point of the body at which the force is applied; the *direction* of the force; and the *magnitude* of the force. It is necessary for simplicity to confine ourselves for some time to the case of a *very small* body, which we call a *particle.* In this case the forces which we have to consider all act at one point, namely that at which the particle is situated. The direction of any force is the straight line along which it tends to move the particle. We have seen in Art. 123 that a similar restriction as to the size of the bodies we consider is advantageous in treating the subject of motion.

145. The magnitudes of forces are conveniently measured by the weights which they will support. Thus we speak of a force of 5 pounds; by this we mean a force which will just support a weight of 5 pounds, that is a force which will just counteract the force exerted by gravity on a body weighing 5 pounds.

146. Forces may be conveniently represented by *straight lines.* For we may take a point to denote the

point of application of the force, and may draw a straight line from that point in the direction of the force, and of a length proportional to the magnitude of the force. Thus, for example, suppose a particle acted on by three forces in three different directions; and let these forces be of 3, 4, and 2 pounds respectively. Draw straight lines OA, OB, OC in the directions of these forces, and take the lengths of these straight lines proportional to the forces; that is take OB in the same proportion to OA as 4 is to 3, and take OC in the same proportion to OA as 2 is to 3: then OA, OB, and OC respectively completely represent the forces. In saying that OA represents the force we suppose that the force acts *from* O *towards* A; if the force acts *from* A *towards* O we shall say that AO represents it.



147. Now suppose we have two or more forces acting at once on a particle, we may ask if we can find a *single* force which will produce the same effect as the two or more do jointly. For simplicity we will suppose *two* forces to be acting at once, and consider various cases.

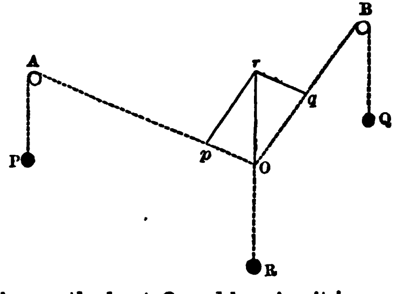
148. Suppose two forces to act in the *same* direction; then they are equivalent to a single force in this direction represented by their *sum*. Thus if a weight of 8 pounds be hung at the end of a string, and also a weight of 10 pounds, the effect is the same as if a single weight of 18 pounds were hung at the end. Again, suppose two forces to act in *opposite* directions; then they are equivalent to a single force in the direction of the greater represented by their *difference*. Thus if a force of 10 pounds act in one direction, and a force of 8 pounds in the opposite direction, the effect is the same as if a force of 2 pounds acted singly in the former direction.

149. When two or more forces are equivalent to a single force that single force is called the *resultant* of the others, and they are called *components*.

105. The method of finding the resultant of *two* forces acting on a particle, not in the same straight line, is given by the following proposition. *If two forces acting on a particle be represented in magnitude and direction by straight lines drawn from the particle, and a parallelogram be constructed having these straight lines as adjacent sides, then the resultant of the two forces is represented in magnitude and direction by that diagonal of the parallelogram which passes through the particle.* This proposition is called the *Parallelogram of Forces*; it is one of the most important in our subject, and we shall shew how it may be verified by experiment.

151. Let A and B be smooth horizontal pegs fixed in a vertical wall. Let three strings be knotted together; let O represent the knot. Let one string pass over the peg A and have a weight P attached to its end; let another string pass over the peg B and have a weight Q attached to its end; and let a weight R be hung from O. Let the system be allowed to adjust itself so as to be at rest.

The effects of the weights P and Q are not changed as to *magnitude* by the passing of the strings which support them over the smooth pegs A and B. We have thus three

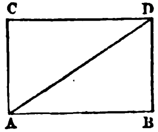


forces acting on the knot O, and keeping it in equilibrium; so that the effect of P along OA and of Q along OB are together just counteracted by the effect of R acting vertically downwards at O. Therefore the *resultant* of P along OA and of Q along OB must be equal to a force R acting

upwards at O. Now on OA take Op to contain as many inches as P contains pounds; and on OB take Oq to contain as many inches as Q contains pounds; and complete the parallelogram $Oqrp$. Then it will be found by trial that Or contains as many inches as the weight R contains pounds, and that Or is a vertical straight line. We may change the positions of the pegs and the magnitudes of the weights employed in order to give due variety to the experiment; and the general results will afford sufficient evidence of the truth of the *Parallelogram of Forces.*

152. Besides the experimental verification, modes of establishing the proposition by mathematical reasoning have been given; but these are unsuitable for the present work. As we have already said in Art. 121, Newton deduces from his Laws of Motion a principle called the *Parallelogram of Velocities;* and from this he considers the Parallelogram of Forces to follow immediately.

153. The case in which the directions of the two forces include a right angle deserves especial notice. Here the *magnitude* of the resultant force can be found by Arithmetic when the magnitudes of the components are known. Thus, if AC represents a force of 3 pounds, and AB a force of 4 pounds, and the angle BAC is a right angle, then AD, the resultant, will represent a force of 5 pounds. For the square of 5 is equal to the sum of the squares of 3 and 4: see Art. 32.



154. If more than two forces act together on a particle we can find their resultant by repeated use of the *Parallelogram of Forces.* For instance, suppose there are three forces. Find the resultant of two of them by the Parallelogram of Forces; then the two may be removed and their resultant placed instead of them. Again, take this resultant and the third force, and find their resultant by the Parallelogram of Forces; we thus obtain finally a single force equivalent to the three which act together.

155. The proposition called the *Parallelogram of Forces* may be put in another form which expresses substantially the same fact; in this form it is called the *Triangle of Forces*. We may state it thus: *If three forces acting on a particle keep it in equilibrium and a triangle be drawn having its sides parallel to the lines of action of the forces, the sides of the triangle will be proportional to the forces which are respectively parallel to them.* Thus, for instance, in Art. 151 we have the triangle Orq; now Or is in the line of action of R, and Oq is in the line of action of Q, and rq is parallel to Op, which is in the line of action of P. And since rq is equal to Op, by Art. 16, it follows that the triangle Orq has its sides proportional to the three forces R, P, and Q, which act on the knot at O and keep it in equilibrium. Any other triangle drawn so as to have its sides parallel to those of Orq would be similar to Orq, and so its sides would be in the same proportion: see Art. 34.

156. As we may substitute for two or more forces a single resultant, so on the other hand we may replace a single force by two or more forces which are equivalent to it. We shall not have much occasion to use this process of *resolving a force into components*, as it is called, but it is of great importance and value in the higher treatises on mechanics. The most common case is that illustrated by the diagram of Art. 153; instead of any force represented by AD we may substitute the two forces represented by AC and AB, the angle BAC being a right angle.

157. It is found by experiment that a force acting on a body may be supposed applied at any point of its line of action. As a simple case suppose a heavy body hung up by a string to a support. The string may be fastened to the body on the side nearest to the support. Suppose a hole bored through the body exactly in the direction of the string, and instead of being fastened at the point of the hole nearest the support let the string be put through the hole and fastened to the point furthest from the support. The tension of the string, that is the force exerted by the string, will be found the same in the two cases: it is in fact just equal to the weight of the body. This principle

of the transmissibility of a force to any point in its line of action is frequently of great use.

158. If three forces keep a body in equilibrium and the directions of two of them meet at a point the direction of the third must pass through that point. For, consider the two forces of which the directions meet at a point; then by Art. 157 they may be supposed to act at that point: consequently they will have a resultant acting at that point, and they may be replaced by that resultant. Now it is obvious that this cannot be counteracted by the third force unless the direction of this force is exactly opposite that of the resultant. Hence the direction of the third force must pass through the point at which the directions of the other two meet. The proposition is important as affording a notion of the way in which results obtained with respect to *particles* are extended to the case of *bodies:* see Art. 144.

159. If more than two forces act on a body we may find the resultant of all the forces by the aid of the principles explained. Suppose, for example, that *three* forces act on a body. Take two of the forces; they may be supposed by Art. 157 to act at the point where their directions meet: find the resultant of these two forces by Art 150, and substitute the resultant in the place of the two. Then produce the direction of this resultant to meet that of the third force, and find the resultant of these two by Art. 150. Thus we obtain a *single* force which is equivalent to the original three forces.

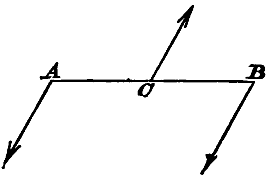
X. PARALLEL FORCES. CENTRE OF GRAVITY.

160. In the preceding Chapter we spoke of forces acting on a *particle*, that is on a body so small that the forces might be supposed to be applied at the same point. But it is obvious that forces may be applied at various points of a body which is too large to be considered as a particle, and we may want to know if we can find a single force equivalent to them. The question in its widest form

we shall not attempt to discuss; we have just alluded to it in Art. 159, and shall now confine ourselves to the case in which the forces act in *parallel* directions and towards the same part: this case leads to some very important applications.

161. We must explain what we mean by acting towards *the same part*. A body, for example, may be acted on by two forces in parallel directions which both tend to urge it from the South towards the North; in this case the forces are said to act towards the *same* part. They may be called briefly *like* parallel forces. Or a body may be acted on by two forces in parallel directions, one of which tends to urge it from the South towards the North, and the other from the North towards the South; in this case the forces are said to act towards *opposite* parts. They may be called briefly *unlike* parallel forces. We shall be concerned with *like* parallel forces.

162. We begin with the simplest case. *Let there be two equal and like parallel forces; their effect will be the same as that of a single force equal to the sum of the two, acting in the straight line which is parallel to the directions of the two and equidistant from them.* This seems very reasonable, and may be easily verified by experiment. For instance, let AB represent a rod, and suppose that at A and B forces act which are equal, like, and parallel. Let C be the middle point of AB; it will be found that a force equal to the sum of the two at A and B, acting at C parallel to these, but towards the opposite part, will just counteract their effect. Thus the forces at A and B are equivalent to a force at C, equal to their sum and parallel to their direction, and towards the same part; thus we may call the force at C the *resultant* of those at A and B. It must be observed that if the *direction* of the forces at A and B is changed, yet so long as the forces are *parallel* the point C remains the same.



163. Next let there be any number of equal like parallel forces *equidistant from each other;* then their resultant will be equal to their sum, and will be midway between the two extreme forces. For example, let there be four forces, and let them act at points A, B, C, D of a straight rod, such that AB, BC, CD are all equal: let each force be a force of one pound. The resultant of the forces at A and D will be a force of two pounds parallel to them at the point midway between A and D, which we will denote by G; the resultant of the forces at B and C will be a force of two pounds parallel to them at the point midway between B and C, which is the point already denoted by G: hence the resultant of the four forces at A, B, C, D is a force parallel to them of four pounds acting at G. Again, let there be five forces, and let them act at points A, B, C, D, E of a straight rod, such that AB, BC, CD, DE are all equal: let each force be a force of one pound. The resultant of the forces at A and E will be a force of two pounds parallel to them at the point midway between A and E, that is at the point C; similarly the resultant of the forces at B and D will be a force of two pounds parallel to them at C; and by supposition there is also at C a force of one pound: hence the resultant of the five forces at A, B, C, D, E is a force of five pounds acting at C.

164. The two examples of the preceding Article will lead us to a general result of great importance. Take the first example, in which equal forces act at A, B, C, D. Consider the three forces at A, B, C; by the method of the preceding Article these are equivalent to a single force of *three* pounds acting at B. Hence the system of forces is equivalent to a force of *three* pounds at B, and a force of *one* pound at D; and therefore the resultant of the forces of three pounds at B, and of one pound at D, must be a force of four pounds at the point denoted by G. Suppose that AB, BC, CD are each two inches in length; then GB is *one* inch, and GD is *three* inches. Thus the resultant of the forces of three pounds at B and of one pound at D is a force of four pounds acting at a point G such that *GB bears the same proportion to GD as the force at D bears to the force at B.* Again, take the second example in Art. 163, in which equal forces act at

A, B, C, D, E. The three forces at A, B, C are equivalent to a force of *three* pounds at B; the two forces at D and E are equivalent to a force of *two* pounds at a point midway between D and E, which we will denote by H. Thus the resultant of the forces of three pounds at B and of two pounds at H is a force of five pounds at C. Suppose that AB, BC, CD, DE are each two inches in length; then CB is *two* inches and CH is *three* inches. Thus the resultant of the forces of three pounds at B and of two pounds at H is a force of five pounds acting at a point C such that CB bears the same proportion to CH as the force at H bears to the force at B.

165. In the way of the preceding Article we arrive at the following general principle: *let P and Q denote two like parallel forces, which act at the points A and B respectively; then their resultant is the sum of P and Q and it acts parallel to P and Q, at a point G on the straight line AB, such that GA bears the same proportion to GB as the force at B bears to the force at A.* This is a very important proposition which must be carefully remembered by the reader. The *way* in which it is obtained deserves especial notice. The case in which the forces are *equal* may be taken as obvious, or may be founded on experiment; and then the more difficult case in which the forces are of any relative magnitude is deduced by simple *reasoning* alone. The point G is called the *centre* of the parallel forces P and Q acting at A and B respectively.

166. Next suppose we have three like parallel forces of any magnitudes and acting at three points not necessarily in a straight line; we can determine the resultant of the three. For, take any two of them, find their resultant, and put this resultant instead of them; then take this resultant and the third force and find *their* resultant: this will be the resultant of the original *three* parallel forces. The resultant will be equal to the *sum* of the three parallel forces, will be parallel to them, and will act at a point which remains the same however the direction of all the parallel forces may be changed. By proceeding in this way we arrive at the following general result: if any number of like parallel forces act on a body their resultant is a single

force equal to their sum and parallel to them in direction, acting at a definite point which can be found when we know the points at which the forces act; this point remains the same however the directions of the forces may be changed, provided they all remain parallel: the point is called the *centre* of the parallel forces.

167. We shall not have to pay much attention to *unlike* parallel forces, but we will just explain how the resultant of two such forces, supposed unequal, can be found. It appears from Art. 165 that *like* parallel forces P and Q, acting at A and B respectively, can be balanced by a parallel force $P + Q$, acting in the contrary direction at a certain point G. Hence it follows that the *unlike* parallel forces P and $P + Q$ are balanced by a force Q parallel to them and like P; and therefore the resultant of the unlike parallel forces P and $P + Q$ is a force Q parallel to them and like $P + Q$, acting at a certain point. Thus the resultant of two unlike parallel forces is equal to their difference, parallel to them, and like the greater force; its position is such that the greater force is between the less force and the resultant, and its distance from the latter bears the same proportion to its distance from the former as the less force bears to the difference of the two.

168. The result obtained in Art. 166 is immediately applicable to the case of a body under the action of gravity. Every body may be supposed to be made up of a large number of particles, and the force of gravity acts on each particle, producing what we call its weight. Thus the weights of all the particles form a set of parallel forces the resultant of which is equal to their sum, and acts vertically downwards through a definite point of the body, however the body may be turned about. This point is called the *centre of gravity* of the body.

169. The *centre of gravity* of a body then is a fixed point at which the whole weight of the body may be supposed to act. It is very remarkable that there should be such a point, and moreover that for many purposes the effect of the weight of the body is the same as if we supposed all the weight collected at the centre of gravity: this

is the case in all questions relating to the equilibrium of bodies. This affords another illustration of the way in which investigations with respect to the action of forces on *particles* may be applied to *forces* on bodies; see Art. 158.

170. The position of the centre of gravity of a body may be found by experiment. Fasten a string to any point of the body, and hang the other end of the string to a fixed point; let the body be allowed to come to rest, as it will do in a very short time. The forces which act on it are the tension of the string upwards, and its weight downwards. Now this weight may be supposed to be collected at the centre of gravity of the body; and thus it will happen that when the body is at rest the centre of gravity must be on the straight line which we shall get if we suppose the direction of the string produced through the body. Thus if we make a mark on the body directly below the point at which the string is fastened to the body we know that the centre of gravity must lie on the straight line which joins the point with the mark. In this way we determine the position of a straight line in the body which passes through its centre of gravity. Remove the string from the point of the body at which it is fastened, and fasten it to another point, and complete the process as before; thus we determine the position of another straight line in the body which passes through its centre of gravity. Hence the centre of gravity will be at the point of intersection of the straight lines. It may happen that the body is of a certain symmetrical shape so that the situation of *one* straight line passing through the centre of gravity is obvious; thus the centre of gravity of a *poker* must be somewhere on a certain straight line which we may call the *axis* of the poker. In such a case it will be sufficient by one suspension of the body to determine the situation of another straight line passing through its centre of gravity.

171. We will now indicate the position of the centre of gravity for various bodies; these may be obtained by reasoning, but the beginner may take them as verified by experiment. We suppose each body made up entirely of the same substance, or, to use formal language, we suppose each body to be of the *same density* throughout. For a

sphere or globe the centre of gravity is the centre of the sphere. For a cube the centre of gravity is at the point which we may call the centre of the figure, namely the point where straight lines joining opposite corners meet. The same rule gives the centre of gravity of a body shaped like a brick, which in geometry is called a rectangular parallelepiped, or more briefly a right solid. The centre of gravity of a cylinder is midway between the centres of the circular ends.

172. We have hitherto spoken of the centre of gravity of *bodies*, but we may also speak of the centre of gravity of *plane figures*, although strictly these are not *bodies* inasmuch as they have no thickness. Thus we may say that the centre of gravity of a circle is at its centre. This is a short expression of which the full meaning may be easily supplied. Suppose we have a circle cut out of *very thin* metal; then we may fix our attention on either of the two faces, and, speaking roughly, we may say that the centre of gravity of the body is at the centre. The exact truth is that each circular face has its own centre, and that the centre of gravity of the body is midway between the two geometrical centres. This is strictly true whatever be the thickness of the metal; in fact the body is really a cylinder, and the centre of gravity is found by the rule of Art. 171. In like manner we may speak of the centre of gravity of a *triangle*, and the following is the rule for determining its position. Draw a straight line from an angle of the triangle to the middle point of the opposite side; the centre of gravity of the triangle is in this straight line. Draw a straight line from another angle to the middle point of the opposite side; the centre of gravity is also in this straight line. It is therefore at the point of intersection of the two straight lines. It can be proved by geometry, and verified by measurement, that the distance of this point from any angle of the triangle is twice the distance from the middle point of the opposite side. The interpretation of the phrase *centre of gravity of a triangle* is like that we have given respecting a circle. Suppose a triangle to be cut out of metal or wood; if the material is very thin we may take practically for the centre of gravity of the body the point on either face determined by the preceding rule. But if

we wish to be quite exact we may suppose two points found, one on each face, by the preceding rule, and the centre of gravity of the body is midway between the two points. In like manner we may understand what is meant by the centre of gravity of any plane figure. As another example we may say that the centre of gravity of a straight line is at its middle point. We mean that if we take a straight slender rod which is of the same thickness throughout, as for instance a straight piece of wire, then the centre of gravity of the body may be said to be practically at its middle point. If we wish to be quite exact we must observe that the rod is really a cylinder, and the centre of gravity is found by the rule of Art. 171.

173. The centre of gravity of a cone or pyramid is found by the following rule: join the vertex with the centre of gravity of the base, and measure off three quarters of the length of this straight line from the vertex; the point so obtained is the centre of gravity.

174. The centre of gravity of a body may be at a point where no particle of the body is situated. For example, suppose we have a spherical shell, everywhere of the same thickness, which may be called a hollow sphere; then the centre of gravity of it will be at the centre of the sphere. Also the centre of gravity of a ring is at the centre of the ring. Likewise for a wooden bowl, or for a drum, the centre of gravity will fall at some point of a certain straight line which may be called the *axis* of the body, but will not be coincident with any particle of the body. In fact this will be the case for innumerable bodies which we see around us. Take for instance a *chair;* it may by chance happen that the weights of the different parts are so adjusted as to bring the centre of gravity to some point of the seat: but probably this will not be the case, and the centre of gravity may very likely be *below* the seat.

175. The reader must notice that whether the centre of gravity of a body does or does not coincide with some particle of it, what we have stated in Art. 169 holds, namely that we may for most purposes suppose that the

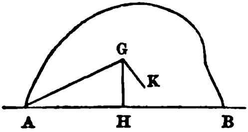
weight of the body is collected at this point. Thus, taking the example of the chair just given, if we hang the chair up by a string attached *anywhere* to it, the line of direction of the string when the chair is at rest will always pass through a certain point which, although not coincident with any particle of the body, has a fixed position with respect to the body: thus in whatever way we hang up the chair the position which it takes is the same as if all the weight were collected at that certain point. Another mode of bringing the nature of the centre of gravity before the mind is sometimes given: suppose this point to be connected with various parts of the body by strong rods without weight, then let the point be supported and the body allowed to turn round the point in any way; it will be found that the body will remain at rest in any position in which it may be left. If the supposition of strong rods without weight appears difficult or extravagant to any reader, we may take another which will answer our purpose as well. Suppose the weights of these strong rods to be so adjusted that the centre of gravity of the whole of them shall just fall at the same point as the centre of gravity of the body: then, as before, the body will remain at rest in any position in which it may be left.

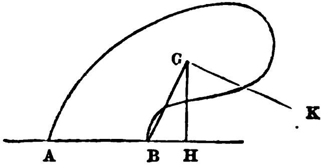
XI. PROPERTIES OF THE CENTRE OF GRAVITY.

176. One of the most important facts relating to the centre of gravity is thus stated: *When a body is placed on a horizontal plane it will stand or fall according as the vertical straight line through its centre of gravity passes within or without the base.*

Let G be the centre of gravity of the body. Let the vertical line through G cut the horizontal plane on which the body stands at H. Let any horizontal straight line be drawn through H, and let AB be that portion of it which is within the base of the body.

First suppose H to be *between* A and B as in the left-hand diagram. No motion can take place round A. For the weight of the body acts vertically downwards at G, and





therefore any motion of turning round A which this weight might produce would tend to make G move in the direction GK; and such motion is prevented by the resistance of the horizontal plane. Similarly no motion can take place round B. Next suppose H to be on AB produced through B as in the right-hand diagram. Then, as before, no motion can take place round A. But motion will take place round B; for the weight of the body would tend to make G move round in the direction GK, and there is nothing to prevent this. The body then would fall over round B.

177. In order to understand the preceding proposition we must pay careful attention to the meaning of the word *base* there used. It may happen that the portions of surface common to the body and the ground on which it is placed form one undivided area, and then the base is this area; for instance, when a brick is placed on the ground the base is the area of the face of the brick which is in contact with the ground. Or it may happen that the portions of surface common to the body and the ground form various separate areas; this is the case with a chair, where there are four separate areas corresponding to the four feet. Here we may suppose a string stretched round the four feet close to the ground, so as to include the four separate areas; then the figure bounded by the string is what we mean by the *base* of the chair.

178. If the vertical straight line drawn through the centre of gravity passes *within* the base the body will stand, but if the vertical passes extremely *near* the boundary of the base the body will not stand very securely; for then a small push or shake may bring the vertical beyond the boundary of the base, and the body will tumble over. Suppose, for example, that one leg of a chair is broken off; then the *base* of the chair is reduced to the figure formed by stretching a string round the other three legs close to the ground. The vertical through the centre of gravity of the chair *may* pass within the base, and so the chair stand on three legs, but the vertical will be extremely near to that portion of the string which passes *diagonally* from front to back, and thus the chair falls over very easily in the direction of the absent leg. An experiment may be easily tried, which is the same in principle, without waiting until accident supplies a damaged chair. Take a common chair and put three pieces of wood of the same thickness under three of the legs; it will most likely be found impossible to keep the fourth leg off the ground, if it be one of the back legs: but if the weight of the back of the chair is considerable the centre of gravity will be decidedly nearer to the two back legs than to the two front legs, and it will be possible by putting the pieces of wood under two back legs and one front leg to keep the fourth leg off the ground.

179. It is easily seen by a little reflection on the diagram of Art. 176 that if the base remains unchanged, the lower the centre of gravity of a body is the more securely the body stands. If the centre of gravity in the left-hand case instead of being at G were between G and H, the body would have to be turned through a large angle about A or B before the vertical through the centre of gravity would pass beyond the base. Thus if a waggon is loaded with stones or coals the centre of gravity of the whole is about half way between the top and the bottom of the load; and if the waggon is by any accident tilted up a little to the right hand or to the left hand, still it does not fall over. But suppose that instead of stones or coals the waggon is loaded with an *equal weight* of hay; then the hay is piled up to a great height, and the centre of gravity

comes to a point much above its former position: thus the same amount of angular tilting as before may bring the vertical through the centre of gravity outside the base, and so lead to an upset. Similarly if persons in a small boat stand up the centre of gravity of the system is raised so high that a very little disturbance of the boat may overturn it.

180. The fact that in some positions a body may stand more securely than in others is well illustrated by the case of a body shaped like a brick. It stands most securely when one of its two largest faces is placed on the ground; the base is then greater, and the centre of gravity is lower, than for any other position. The body stands least securely when one of its two smallest faces is placed on the ground; the base is then smaller, and the centre of gravity is higher, than for any other position in which the body can be made to rest.

181. An example on this subject which is frequently given in popular works requires a little notice. There is a famous tower at Pisa which is called the *Leaning Tower of Pisa* because it is very much out of the perpendicular. It is sometimes said that the tower remains safe in that position because the vertical through the centre of gravity falls within the base; but this is rather a misleading remark. For the tower is not *placed on* the ground but *thrust into* the ground, by reason of the foundation; and the main thing to be regarded is whether the parts of the mass are fastened strongly enough together by mortar and other means. If they are the tower will remain in its position even if the vertical through the centre of gravity falls *without* the base. It is well known that a stick *thrust* a little way into the ground will stop where it is placed whether in an oblique or an upright position. So also trees may be seen so much bent from the perpendicular that the centre of gravity must be beyond the base; but they are held fast by the roots in the ground.

182. A body at rest when acted on by forces is said to be in equilibrium; see Art. 143. Now there are different kinds of equilibrium. Suppose that a body in equilibrium is slightly disturbed by some new force, and then left to

the action of the old forces. The body may move back towards its original position, or it may move farther away from it; in the former case the equilibrium is said to be *stable* and in the latter case *unstable.*

183. A very good example of stable equilibrium is furnished by a weight hanging from a fixed point by a string. If we draw the weight a little aside from its position of rest, and then leave it, the weight moves *backwards* towards its original position. Again, suppose we have a hole bored through a body, so that the body can turn round a rod passed through this hole, and held fast in a horizontal position. As in the former case the body is in stable equilibrium when the centre of gravity is vertically below the rod. The body may however be turned round and placed so as to have its centre of gravity vertically *above* the rod; and then also it will be in equilibrium. But the equilibrium is now *unstable;* for if the body be moved a little way from this position and left to itself, it does not fall back towards its original position, but further away from it. If the hole happens to pass through the centre of gravity of the body, then the body rests in any position in which it may be placed; if it is disturbed and then left to itself it neither goes back to the former position nor further away from it: the equilibrium in this case is said to be *neutral.* This case is comparatively rare in practice; equilibrium is in general either stable or unstable. A common grindstone furnishes an example of neutral equilibrium. A cone standing on its own base is in *stable* equilibrium; it might be balanced on its point and then would be in *unstable* equilibrium; it might rest on its slant side, and then the equilibrium would be *neutral.*

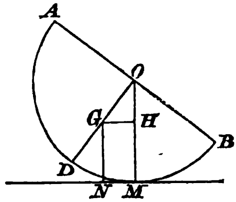
184. We may say then that in general a body is in *stable* equilibrium when it rests with the centre of gravity in its lowest possible position. For the weight of a body is a force tending *downwards;* and if the centre of gravity is originally in its lowest possible position, any slight displacement of it must bring it to a higher point, and then the weight of the body will bring the centre of gravity *down* again, that is turn the body back towards its original position. In like manner when the body rests with the centre of gravity in its highest possible position the equili-

brium is *unstable;* for then any displacement of the centre of gravity must bring it to a lower point, and the weight of the body urges the centre of gravity downwards, that is turns the body further away from its original position.

185. Suppose a sphere of wood placed on a table. The centre of gravity of the sphere is at its centre; and if the sphere is moved about on the table the centre of gravity is always at the *same* height above the table. The sphere rests always where it is placed, and the equilibrium is *neutral.* But if the sphere is *loaded* by having a piece of lead introduced inside it the centre of gravity will in general be no longer at the centre of the sphere. Then the sphere will be in *stable* equilibrium when the loaded part is as low as possible, and in *unstable* equilibrium when it is as high as possible; for in the former case the centre of gravity of the whole is as low as possible, and in the latter case as high as possible, and in both cases the centre of gravity comes vertically over the point of contact of the sphere and the table, that is over the *base.*

186. A common toy furnishes a very good example of *stable* equilibrium.

Take a hemisphere, as for example half a round bullet. Put it on a table with the flat part upwards; displace it slightly, and leave it to itself: then it goes back to its first position. Let ADB represent a section of the hemisphere made by a vertical plane through the centre of the base; let O be the centre of the base, and D the point which is in contact with the table when the flat part is horizontal. Then the centre of gravity will be somewhere on the straight line OD; suppose it to be at G. Let the hemisphere be tilted in the vertical plane ADB, so that M is now the point of contact with the table. From G draw GN perpendicular to the table; then it may be shewn by measurement that GD is less than GN, so the centre of gravity is now higher than it was at first. Hence



when the body is left to itself its weight will bring G down, so that the body will move back towards its first position. The toy is constructed by fastening the figure of a man carved out of very light wood upright on the flat part of the hemisphere; and thus he appears to raise himself again whenever he is depressed. A child's rocking-horse when considered by itself without any rider involves the same principle; when a child is placed on the horse the centre of gravity of the whole shifts about a little as the child bends, so that it does not remain at a point which is fixed with respect to the whole system.

187. We say in the preceding Article that *measurement* will shew GN to be greater than GD; but the same result can be obtained by very simple *reasoning*. Draw OM; then by the nature of the circle this is perpendicular to the table. Draw GH parallel to NM; then the angle OHG is a right angle. Now by the nature of the circle, OG and GD together are equal to OH and HM together; but OG is greater than OH by Art. 33, and therefore HM is greater than GD. But HM is equal to GN by Art. 16; and therefore GN is greater than GD. Simple as this example is it illustrates well the use of Mathematics in questions of Natural Philosophy.

188. When a man stands, the *base*, in the sense of Art. 176, is the figure which would be formed by a string passing round its feet close to the ground. We may see that when he turns out his toes a little he enlarges the breadth of his base without much diminishing the length of it. The attitudes which persons take under various circumstances all depend on the principle that the vertical straight line through the centre of gravity must fall within the base. Thus a porter with a heavy load on his back leans forwards; a nurse carrying a baby in her arms draws her head and shoulders back. A person carrying a pail of water in one hand often holds the other arm and hand outwards, so as to keep the centre of gravity of the system from deviating too much towards the side where the pail is. When two pails are carried, one on each side, by means of a yoke over the shoulders, the centre of gravity of the whole system remains easily in a suitable position. If a

person stands on stilts the base is very small, consisting only of the narrow slip which would be formed on the ground by a string passing round the stilts; hence it is very difficult to remain at rest under such circumstances. But the person on stilts usually carries a pole in his hand which has one end on the ground; and then the base is the figure which would be formed on the ground by a string passing round the *stick and the stilts;* and this is large enough to render it very easy to remain at rest.

189. In walking the centre of gravity of the person is brought alternately over the right and left foot. Like many other bodily acts we learn to perform this in an unconscious manner; but that it is at first somewhat difficult we know by observing the time it takes for an infant to acquire the habit, and the many failures which accompany the early attempt.

190. The diagram of Art. 186 will supply matter for further interesting study. The path which the point G traces out if the body is rocked without sliding is found to be an arc of a curve, resembling an arc of a circle, having its *lowest* point at the place which G occupies when the body is at rest. Again, we have supposed the body to be a hemisphere, but it might be a larger portion of a sphere; still the centre of gravity will be between O and D, and the equilibrium will be stable. Suppose however the body to be a sphere or a portion of a sphere in form, but to be denser in some parts than in others, for instance to be loaded by having a dense piece of metal embodied in it; then the centre of gravity may be *beyond* O, instead of being between D and O. In this case the equilibrium will be *unstable;* the path which the point G traces out if the body is rocked without sliding is then found to be an arc of a curve, having its *highest* point at the place which G occupies when the body is at rest.

XII. THE LEVER.

191. *Machines* are instruments which men use for communicating motion to bodies, for changing the motion of bodies, or for preventing the motion of bodies. They are not origins of force; they merely transmit the force exerted upon them. In all cases our object is to obtain some useful result by the aid of them. Thus a locomotive engine on a railway is a machine for giving motion to loaded carriages, so that we may move people and things from one place to another. A windmill is a machine for turning round some large stones which grind corn into flour.

192. There are certain very simple machines which are called the *Mechanical Powers;* by combinations of these all the more complex machines can be constructed. It is usual to consider these Mechanical Powers to be *seven* in number; namely the Lever, the Wheel and Axle, the Toothed Wheel, the Pully, the Inclined Plane, the Wedge, and the Screw. This division is not a very satisfactory one, as there are not really seven distinct principles involved in these Mechanical Powers, but it has been commonly adopted, and so we will retain it.

193. We shall state what are called the *conditions of equilibrium* for these simple machines, that is we shall suppose them to be used to *prevent* motion; in practice they are more commonly used to *communicate* motion, but they are more intelligible by being considered first in a state of equilibrium. We shall in every case have *two* forces which balance each other by means of the machine, and for distinctness one is called the *Power* and the other the *Weight;* the former being the force we apply, and the latter the resistance we wish to balance. In every case in order that there may be equilibrium, the Power must be in a certain proportion to the Weight; this proportion depending on the nature of the machine. In the present Chapter we shall consider the Lever.

194. The Lever is a rod or bar which can turn in one plane about a point in the rod called the *fulcrum*. The plane in which the Lever can move is called the plane of the Lever, and the forces which act on the Lever are supposed to act in this plane. A force acting at any point of the Lever, provided the direction of the force does not pass through the fulcrum, would turn the Lever round the fulcrum in one direction. If two forces act and tend to turn the Lever round in *contrary* directions the forces may be so adjusted as just to balance each other, and thus keep the Lever at rest: and this is the case we have to consider. The rod or bar may be straight, and then the machine is called a *Straight Lever;* in other cases it is called a *Bent Lever.*

195. One of the most familiar examples of a Lever is supplied by the *common balance;* the fulcrum is at the middle point of the beam; the two forces acting are both weights, namely that of the substance to be weighed in one scale, and that of the counterpoise in the other scale. The forces act on opposite sides of the fulcrum, and tend to turn the beam of the balance in contrary directions; when the forces are equal the balance is in equilibrium. The two parts into which the beam is divided by the fulcrum are called *arms* of the balance; and the term is sometimes used with respect to other examples of the Lever.

196. Levers are sometimes divided into three classes, according to the position of the points of application of the Power and the Weight with respect to the fulcrum. In the first class the Power and the Weight act on opposite sides of the fulcrum. In the second class the Power and the Weight act on the same side of the fulcrum, the Weight being the nearer to the fulcrum. In the third class the Power and the Weight act on the same side of the fulcrum, the Power being nearer to the fulcrum. Hence we may say briefly that the three classes of Levers have respectively the Fulcrum, the Weight, and the Power in the middle position.

197. The balance is an example of a Lever of the first class. Another example is furnished by a crowbar used

for raising a heavy body; one end of the crowbar is placed under the body, and near this end a piece of stone or wood or iron put under the crowbar serves for a fulcrum; the power is exerted by the man who uses the crowbar pressing down the other end of it. A see-saw will supply another example; this resembles a balance, except that if the children who use it are of unequal weights the *arms* will be unequal, the lighter child being at the end of the longer arm. For our present purpose the see-saw should be considered not as in motion, but as remaining in equilibrium. Other examples are a poker used to raise coals in a grate, and the handle of a common pump. A pair of scissors may be regarded as a double Lever of the first class; the Weight here is the substance cut through by the blades of the scissors, the fulcrum is the point where the two blades are connected, and the Power is the pressure exerted by the fingers at the loops. If we have to cut with scissors a very thick and strong piece of paper, we bring the paper as near to the fulcrum as possible; the reason for this will be seen when we explain the proportion which must hold between the Power and the Weight on a Lever.

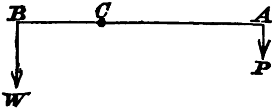
198. The oar of a boat furnishes an example of a Lever of the second class. The fulcrum is at the blade of the oar in the water; the Power is applied by the hand; the Weight is applied at the rowlock. The crowbar may be used in such a manner as to become a Lever of the second class. For suppose that a large round body is to be pushed along the ground; the end of the crowbar rests on the ground under the body, and this forms the fulcrum; the crowbar is pressed against the body, so that the resistance of the body at the point of contact constitutes the Weight; and the Power is applied by the man who uses the crowbar pushing the other end of it. A pair of nutcrackers may be regarded as a double Lever of the second class; the fulcrum is at the hinge, the Weight is the resistance of the nutshell which is to be cracked, and the Power is the pressure applied by the hand.

199. An example of a Lever of the third class is furnished by the familiar process of raising a ladder. The bottom of the ladder rests on the ground as a fulcrum; the

Power is applied by the hands of a man who stands on the ground and pulls at one of the rounds of the ladder; the Weight is the weight of the ladder which may be supposed to act at the centre of gravity of the ladder. The treddle of a turning-lathe or of a knife grinder's wheel is another example; the fulcrum is at the ground, the Power is applied by the pressure of the foot, and the weight is the resistance at the crank of the wheel. A pair of tongs used to hold a coal may be regarded as a double Lever of the third class; the fulcrum is at the hinge, the Weight is the resistance of the coal at the end of the tongs, the Power is the pressure exerted by the hand.

200. We have now to consider the conditions of equilibrium of a Lever. The Power and the Weight must tend to turn the Lever in *contrary directions:* this is so evident that it is usually rather understood than expressed, and the *Principle of the Lever* means the statement of the proportion which the power must bear to the Weight. To this we proceed.

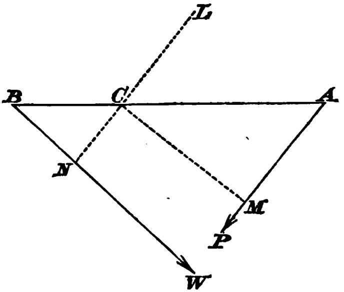
201. Take a straight lever of the first class.



Suppose C the fulcrum, A the point at which the Power acts, and B the point at which the Weight acts. Suppose the plane of the paper to be the plane of the Lever. Denote the Power by P, and the Weight by W; and let them both act in directions at right angles to the Lever. Then in order that there may be equilibrium P *must be to* W *in the same proportion as* BC *is to* AC. For instance if BC is one quarter of AC, then P must be one quarter of W.

202. The preceding statement is merely a fact which we have already noticed presented under a slightly different aspect. For the forces which we denote by P and W are parallel forces, and therefore by Art. 162 they are equivalent to a single force acting at the point C, such that BC is to AC in the same proportion as P is to W. Then since this resultant force acts at the *fulcrum* it does not tend to turn the Lever round, but only presses it against

the fulcrum; so that there is equilibrium. In this way the reader may see that our statement with respect to the Lever really coincides with one which we have already demonstrated with respect to parallel forces; so that we have thus demonstrated the truth of the *Principle of the Lever* in one important case. The truth may be verified by experiment to any required extent.



203. Next suppose that *P* and *W* do not act at right angles to the straight Lever. Draw *CM* perpendicular to the line of action of *P*, and *CN* perpendicular to the line of action of *W*. Then in order that these may be in equilibrium P *must be to* W *in the same proportion as* CN *is to* CM.

204. This case may be deduced from the former by appealing to considerations which seem nearly self-evident. First the effect of the force *P* will not be altered if instead of acting at the point *A* it acts at the point *M* retaining the same direction as before; and similarly the effect of the force *W* will not be altered if instead of acting at the point *B* it acts at the point *N* retaining the same direction as before: see Art. 157. Hence instead of forces acting at *A* and *B* on the *straight* Lever *ACB* we may suppose that we have the same forces respectively acting at *M* and *N* on the *bent* Lever formed of rods *CM* and *CN*. Next produce *NC* to *L* so that *CL* is equal to *CM*, and suppose that *NCL* is a lever. Then we may admit that *P* acting at *M* at right angles to *CM* produces the same effect as if it acted at *L* at right angles to *CL*. Thus we have again a straight Lever, namely *NCL*, and forces acting at right angles to it; so that, by Art. 201, in order that there may be equilibrium *P* must be to *W* in the same proportion as *CN* is to *CL*, that is as *CN* is to *CM*.

205. We may combine the two cases of Art. 201 and Art. 203 into one statement thus: P *must be to* W *in the same proportion as the distance of* W *from the fulcrum is to the distance of* P *from the fulcrum.* Here the *distance of* W *from the fulcrum* means the length of the perpendicular from the fulcrum on the line of action of W; and the *distance of* P *from the fulcrum* is to be understood in a similar way. The same statement will hold with respect to the equilibrium of a Lever, whether it be straight or bent.

206. What we have thus obtained with respect to Levers of the first class holds also for Levers of the second and third classes, straight or bent. Thus, universally, in order that there may be equilibrium on a Lever, P *and* W *must tend to turn the Lever in contrary directions, and* P *must be to* W *in the same proportion as the distance of* W *from the fulcrum is to the distance of* P *from the fulcrum.*

207. There is no difference in *theory* between Levers of the second class and Levers of the third class, but there is considerable difference in *practice.* For it follows from the statement of Art. 206 that in Levers of the second class the Power is *less* than the Weight, and that in Levers of the third class the Power is *greater* than the Weight. Thus we may say that there is a mechanical *gain* by using a Lever of the second class, and a mechanical *loss* by using a Lever of the third class. The *advantage* of a machine may be defined as the proportion of the Weight to the Power when there is equilibrium.

208. It follows from the statement of Art. 206 that a very small Power might be made to balance a very great Weight by using a suitable Lever, that is by making the distance of the Power from the fulcrum very large, and the distance of the Weight from the fulcrum very small. But machines are in general used rather to *produce* motion than to *prevent* it; and this leads to a very important remark. Suppose in the diagram of Art. 201 that BC is one-third of AC, then when there is equilibrium the Power is one-third of the Weight. It is true then that a force just exceeding one-third of the Weight will be sufficient to move the Weight; but on the other hand it will be found

that if the Weight is to be raised through *one* inch the Power end of the Lever must descend through *three* inches. Thus although by the aid of a Lever we can move any Weight by a force much less than that Weight, yet the force must be exerted through a distance which is greater in a corresponding degree. This important principle is found to apply to the other Mechanical Powers, and to combinations of them, and it is usually stated briefly thus: *what is gained in power is lost in speed.*

209. The principle of the Lever which we have explained was first demonstrated by Archimedes, the greatest mathematical philosopher among the ancients, and probably inferior to Newton alone among the moderns. Tradition has recorded in a well-known sentence attributed to him the high opinion which he had formed of the importance of his result: *shew me where I may stand and I will move the world.* A more tolerable form of the boast would be, *I will support the world;* for in order to *move* the world through an appreciable distance, the philosopher by the principle of Art. 208 would have had himself to move through an enormously greater distance. We now know that if a motion of the earth through an *infinitesimal* space is all that is required we may dispense with the Lever which Archimedes proposed to use, for the motion is produced every time a man jumps from the ground: he really pushes the earth from beneath him by his spring, and then draws it towards him by his weight. See Chapter VIII.

210. It is unadvisable to introduce more technical terms than are absolutely necessary into an elementary work of the present kind; but one such term may be noticed by the aid of which the Principle of the Lever can be briefly stated. The *Moment* of a force with respect to a point is the product of the force into the perpendicular from the point on the line of action of the force. This supposes the force to be expressed in pounds, or ounces, or in any other convenient terms; and the perpendicular to be expressed in inches, or feet, or any other convenient terms. But having once chosen the unit of force, and the unit of length, we must keep to these units throughout the investigation on which we may be engaged. With the aid

of the term *moment* we may express the relation which must hold between the Power and the Weight for equilibrium on the Lever thus: *the moments of the Power and the Weight with respect to the fulcrum must be equal.* It is easy to see that this coincides with what is stated in Art. 206.

211. In the present Chapter we have left out of consideration the fact that the rod or bar of the Lever will itself have weight. If the fulcrum be at the centre of gravity of this rod or bar the weight of the rod or bar is entirely supported by the fulcrum, and so need not be regarded; the rod or bar so far as we are concerned with it is practically without weight. But if the fulcrum is not at the centre of gravity of the rod or bar, allowance must be made for the weight of the rod or bar: the account of the Common Steel-yard in the next Chapter will illustrate this point.

XIII. THE BALANCE.

212. The various kinds of Balances form such a very important application of the Principle of the Lever that we shall devote a separate Chapter to them. The use of the Balance, as is well known, is to determine the *weight* of any proposed body, so that in this case we employ the Lever not to *produce* motion, but to *prevent* motion, that is to preserve equilibrium.

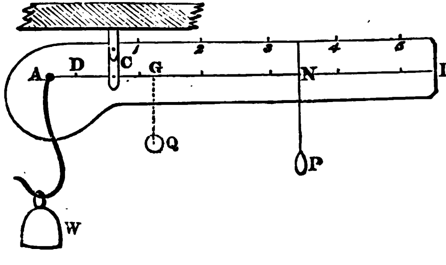
213. *The Common Balance.*

The Common Balance consists of a beam with a scale suspended from each end; the beam can turn about a fulcrum which is above the centre of gravity of the beam, so that if the scales were removed the beam would adjust itself to a position of *stable equilibrium:* see Art. 183. The arms of the beam should be of equal length, and the scales of equal weight, so that the beam may be at rest in a horizontal position when the scales are attached and are empty. If these conditions are satisfied the Balance is said to be *true;* if not it is said to be *false.* The body to be

weighed is placed in one scale and weights in the other until the beam remains at rest in a horizontal position. In this case if the Balance be true the weight of the body is indicated by the weights which have been put in the other scale. We may test whether the Balance is true by observing whether the beam still remains at rest in a horizontal position when the contents of the two scales are interchanged. But even if a Balance be false we may determine by its aid the exact weight of a body, if we employ the process which is called *double weighing*. Put the body which is to be weighed in one scale, and in the other scale put sand or shot so as exactly to counterpoise the body. Remove the body and put in its place weights so as just to restore equilibrium again. Then the sum of these weights indicates the weight of the body. This process of double weighing is very simple in theory and very exact in practice.

214. Another kind of Balance is that in which the arms are unequal, and the same Weight is used to weigh different substances by putting it at different distances from the fulcrum. The Common Steel-yard is of this kind.

215. *The Common Steel-yard.*



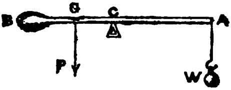
Let AB be the beam of the Steel-yard, C the fulcrum. Let A be the fixed point from which the body to be weighed is suspended. Let Q be the weight of the beam together with the hook or scale-pan at A. Let P be a

weight which may be placed at any distance from the fulcrum. We have now to *graduate* the Steel-yard, that is to put marks on it so that if we observe the position which P has when a body is suspended from A, and the whole is in equilibrium, we may know the weight of that body. Now we might proceed by the aid of theory. For the weights P and Q being parallel forces we can determine their resultant by Art. 165; and then this resultant must balance the weight of the body, according to the Principle of the Lever. But it will be more simple to proceed by the aid of experiment. Take then a weight, say of one pound, and suspend it from A; move P about until such a place is found for it that the beam just remains in equilibrium, and mark the place with the figure 1. Again instead of the weight of one pound at A put a weight of two pounds; move P about as before, and mark with the figure 2 the place which it has when the beam is in equilibrium. Proceeding in this way the beam becomes graduated, and the Steel-yard is fit for use. It will be found by trial that the figures 1, 2, 3, 4,... succeed at *equal distances* on the beam. Thus when we have a body to be weighed we suspend it from A, and then move P about until it comes to such a place that the beam remains at rest in a horizontal position. Let this, for example, be when P is midway between the figures 3 and 4, as in the diagram; then we infer that the body weighs $3\frac{1}{2}$ pounds.

216. Sometimes two different graduations are recorded on the Steel-yard, corresponding to two different moveable Weights. In this way we can extend the range of the machine without making the machine itself inconveniently long; thus one graduation might give us the weights of bodies up to 10 pounds, and then another graduation corresponding to a heavier moveable weight might give us the weights of bodies of 10 pounds and upwards to about 100 pounds. Or the two graduations may correspond to two different positions of the point A from which the body to be weighed is hung, the moveable weight P being the same in both cases.

217. Another kind of Steel-yard is called the Danish Steel-yard.

This consists of a heavy beam which terminates in a knob at one end; and the body to be weighed is placed at the other end, the fulcrum being moveable. Let AB be the beam; let P denote its weight, and G its centre of gravity. The body to be weighed is suspended from A,



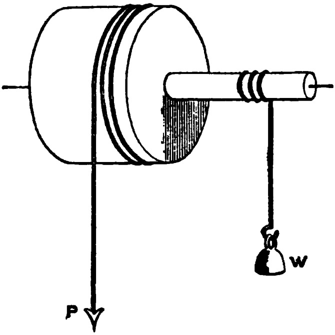
and the fulcrum is moved about until there is equilibrium when the beam is horizontal. The Danish Steel-yard might be *graduated* by the aid of theory; for P at G must balance the body hung from A according to the Principle of the Lever. Or we may proceed by experiment as before. Let a body of one pound weight be suspended at A; move the fulcrum about until there is equilibrium with the beam horizontal, and mark the position of the fulcrum by the figure 1. Again, instead of the weight of one pound at A put a weight of two pounds; move the fulcrum about as before, and mark with the figure 2 the place which it has when the beam is horizontal and in equilibrium. Proceeding in this way the beam becomes graduated and the Steel-yard is fit for use. It will be found in this case that the figures 1, 2, 3, 4,... do not succeed at equal intervals on the beam. Thus in using the Danish Steel-yard to weigh a body if the fulcrum comes precisely under one of the figures marked on the beam we know the weight of the body; but if the fulcrum comes between two of the figures we cannot tell the weight *exactly*, but only two values between which it must lie.

218. There are some weighing machines which do not depend on the Principle of the Lever. They usually consist mainly of a strong spring which is drawn out to a greater extent the heavier the body is which is suspended from it; and a contrivance is furnished by which we can readily observe how far the spring has been drawn out. These machines may be graduated by *experiment*, that is by suspending known weights and recording the corresponding points to which the spring is drawn out.

XIV. THE WHEEL AND AXLE. THE TOOTHED WHEEL.

219. In this Chapter we shall consider two other Mechanical Powers, namely, the Wheel and Axle, and the Toothed Wheel.

220. *The Wheel and Axle.* This machine consists of

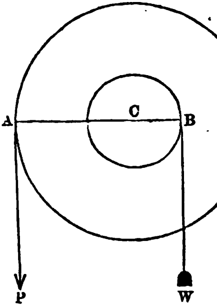


two cylinders which have a common axis; the larger cylinder is called the *Wheel* and the smaller the *Axle.* The two cylinders are rigidly connected with the common axis, which is supported in a horizontal position, so that the machine can turn round it. The Weight acts by a string which is fastened to the Axle and coiled round it; the Power acts by a string which is fastened to the Wheel and coiled round it. The Weight and the Power tend to turn the machine round the axis in opposite directions.

221. When there is equilibrium on the Wheel and Axle the *Power must be to the Weight in the same proportion as the radius of the Axle is to the radius of the Wheel.* For it is easy to see the close resemblance between this machine and a Lever of the first class. It will be obvious that the effect of the Weight must be the same whether it is placed

as in the diagram, or whether it is placed at that part of the Axle which is close to the Wheel; and the effect of the Power must be the same whether it is placed as in the diagram, or whether it is placed at that part of the Wheel which is close to the Axle. Then if we imagine these changes to be made in the position of the Weight and the Power we obtain the following diagram:

Here *CA* is the radius of the Wheel, and *CB* is the radius of the Axle. We may consider *ACB* as a Lever of which *C* is the fulcrum. The Weight *W*, and the Power *P*, act in the manner shewn in the diagram; and in order that there may be equilibrium *P* must be to *W* in the same proportion as *CB* is to *CA*.

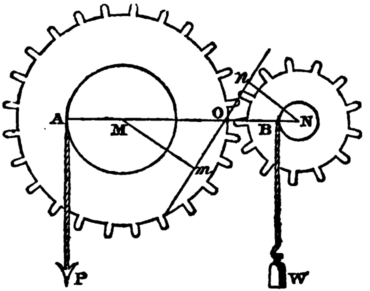


222. We have hitherto supposed that the Power acts by means of a string, but it may act by the direct application of a man's hand, as in the familiar example of the machine used to draw up a bucket of water from a well.

223. The important principle of Art. 208 holds with respect to this machine. Suppose for instance that the radius of the Wheel is four times the radius of the Axle; then a weight of four pounds hanging round the Axle can be supported by a weight of one pound hanging round the Wheel. Thus a power only a very little greater than one pound will be sufficient to move the Weight of four pounds; but still to raise the Weight through any space the Power must descend through four times that space. Thus if the machine turns round just once, so as to raise the Weight through a space equal to the circumference of the Axle, then the Power descends through a space equal to the circumference of the Wheel; and these circumferences are in the same proportion as the radii, so that the circumference of the Wheel is four times that of the Axle.

224. A Windlass and a Capstan may be considered as cases of the Wheel and Axle. The Windlass scarcely differs from the machine used to draw up water from a well; it has however more than one fixed handle for the convenience of working it, or there may be a moveable handle which can be shifted from one place to another. In the Capstan the fixed axis round which the machine turns is vertical; the hand which supplies the Power describes a circle in a horizontal plane, and the Weight is some heavy body which is attached to the Axle by a rope passing in a horizontal direction.

225. *Toothed Wheels.* Let two wheels of wood or



metal have their circumferences cut into equal teeth at equal distances. Let the Wheels be moveable about their centres, and in their own planes, and let them be placed in the same plane so that their edges touch, one tooth of one circumference lying between two teeth of the other circumference. If one of the Wheels of this pair be turned round its centre by any means the other Wheel will also be made to turn round its centre. Or a force which tends to turn one Wheel round may be balanced by a suitable force which tends to turn the other Wheel round in the contrary direction. The two forces may be supposed to act by means of strings on Axles belonging to the Toothed Wheels. Thus the power P may be supposed to act at A, and the Weight W to act at B; also M is the common centre of one

Toothed Wheel and Axle, and N the common centre of the other Toothed Wheel and Axle.

226. The condition of equilibrium is somewhat complex; the reader may take it as verified by experiment: *when there is equilibrium on a pair of Toothed Wheels the moment of the Power round the centre of its Axle must be to the moment of the Weight round the centre of its Axle in the same proportion as the radius of the Power Wheel is to the radius of the Weight Wheel.* The principle of Art. 208 may be shewn to hold with respect to this machine.

227. In practice this machine is used to transmit motion; and then it is necessary to pay great attention to the form of the teeth, in order to secure uniform action in the machine, and to prevent the grinding away of the surfaces. On this subject, however, the student must consult works on mechanism. Toothed wheels are extensively applied in all machinery, as in cranes and steam-engines, and especially in watch-work and clock-work.

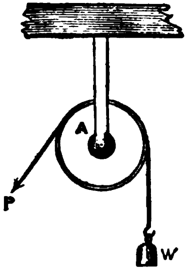
228. Wheels are sometimes turned by means of straps passing over their circumferences: in such cases the minute protuberances of the surfaces prevent the sliding of the straps. The strap passing partly round a Wheel exerts a force on the Wheel at both points where it leaves the Wheel: the effect at each point would be measured by the *moment* of the tension of the strap at that point round the centre of the Wheel. If it were not for the friction, and the weight and stiffness of the strap, the tension would be the same throughout, and so the action at one point of the Wheel would balance the action at the other point.

XV. THE PULLY.

229. The *Pully* consists of a small circular plate or wheel which can turn round an axis passing through the centres of its faces, and having its ends supported by a framework which is called the *block*. The circular plate

has a groove cut in its edge to prevent a string from slipping off when it is put round the Pully.

230. Let *A* denote a Pully, the block of which is fixed; and suppose a Weight attached to the end of a string passing round the Pully. If the string be pulled at the other end by a Power equal to the Weight, there will be equilibrium; if the string be pulled by a Power somewhat greater, the Weight will be raised. Thus a *fixed* Pully is a machine by the aid of which we can change the *direction* of a force without changing its *magnitude*. For example, we might have a Power which could most conveniently act in

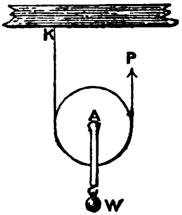


a direction *inclined* at a certain angle to the horizon, and we might wish to use it in supporting a Weight, that is in balancing a *vertical* force; then by transmitting the Power by means of a string, and passing the string round a fixed Pully, as in the diagram, we can support a Weight equal to the Power. A fixed Pully is often used when weights are to be raised, as for example the sails of ships. Thus a *fixed* Pully, though it may be very convenient, does not afford us any *mechanical advantage:* see Art. 207. We shall presently see that by the aid of a *moveable* Pully, or of a system of *moveable* Pullies, we do obtain mechanical advantage.

231. It might at first sight appear that nothing is gained by making the circular plate of the Pully capable of turning round its axis; but practically this is very important. When the circular plate can thus turn round, it is found by trial that in the state of equilibrium the tension of the string is almost exactly the same on both sides of the Pully, so that a Weight can be moved by a Power which is very slightly greater. But when the circular plate cannot turn round it is found that there may be a considerable difference between the tension of the string on the two sides of the Pully; and so a Weight could not be moved unless the Power were considerably greater.

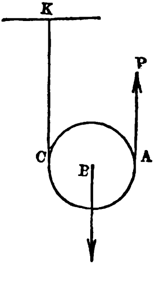
This is owing to *Friction*, which we shall explain hereafter.

232. Now consider the case of a single moveable Pully. Let a string pass round the Pully A, have one end fixed as at K and be pulled vertically upwards by a Power P at the other end. Let a Weight W be attached to the block of the Pully. Then it is found on trial that there is equilibrium if the Power is equal to half the Weight. In fact we may consider the block to be acted on by three parallel forces; namely the Weight downwards, and the two forces upwards arising from the tension of the two parts of the string. Thus the Weight must be equal to the sum of the two upward forces. But the tension of the string is throughout equal to the Power. Therefore twice P is equal to W. The reader will see that there is nothing strange in this result. The Weight W has to be supported in some manner, and on examining we find that the result is the same as if half the Weight were supported by a fixed beam at K, and half by the Power.

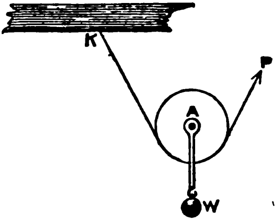


233. If the Power be only a little greater than half the Weight, the Weight will be raised. According to the principle of Art. 208 if the Weight is raised through any space the end of the string at which the Power acts must be raised through *twice* that space. This may be easily shewn.

For suppose the Weight to be raised through any space, say one inch; then the part KC of the string between the fixed end and the Pully must be shortened by one inch, and to keep the string stretched the end at which P acts must be raised through two inches. Thus the Power end of the string moves through twice the space through which the Weight moves.

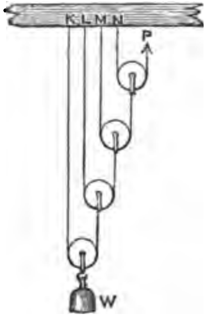


234. Sometimes the two parts of the string are not parallel. But in order that there may be equilibrium the two parts of the string must be inclined to the vertical at the *same angle;* for if they were not the Pully would be drawn towards the side where the string was most inclined to the vertical. Then when there is equilibrium



the Weight is equal to twice that part of the Power which acts in the vertical direction; supposing the Power resolved into two components, one vertical and the other horizontal: see Art. 156. We now pass on to consider various combinations of Pullies.

235. *First System of Pullies.* The diagram represents a system of Pullies in which each Pully hangs by a separate string, and all the strings are parallel; it is usually called the *First System of Pullies.* In this system the string which passes round any Pully, except the highest, has one end attached to a fixed point, and the other end to the block of the next higher Pully; and the string which passes round the highest Pully has one end attached to a fixed point, and the other end supported by the Power. We saw that for equilibrium on the single moveable Pully the Weight must be



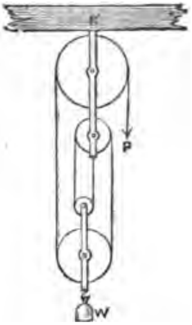
twice the Power; in the *First System of Pullies* it is found that for equilibrium if there are 2 moveable Pullies the Weight must be 4 times the Power, if there are 3 moveable Pullies the Weight must be 8 times the Power, if there are 4 moveable Pullies the Weight must be 16 times the Power, and so on. Thus for every additional moveable Pully the Weight that can be supported by a given Power is doubled.

236. There is no difficulty in the reasoning by which this result is established. Suppose there are 4 moveable Pullies. By the principle of the single moveable Pully the tension of the string which passes under the lowest Pully is $\frac{W}{2}$; thus the next Pully is drawn downwards by a force equal to $\frac{W}{2}$, and consequently the tension of the string which passes round it is $\frac{W}{4}$; in like manner the next Pully is drawn downwards by a force equal to $\frac{W}{4}$, and consequently the tension of the string which passes round it is $\frac{W}{8}$; in like manner the next Pully, which is the highest in this diagram, is drawn downwards by a force equal to $\frac{W}{8}$, and consequently the tension of the string which passes round it is $\frac{W}{16}$. This last tension must be equal to the power which acts at the end of the string; so that P is equal to $\frac{W}{16}$.

237. In the System of Pullies considered in the preceding Article let K, L, M, N denote the points at which the ends of the strings are fastened. The part of the Weight W supported at K is $\frac{W}{2}$, the part supported at L is $\frac{W}{4}$, the part supported at M is $\frac{W}{8}$, and the part supported at N is $\frac{W}{16}$; also the power supports $\frac{W}{16}$ as it acts in the contrary direction to W. It will be easily found that the sum of these parts, that is of $\frac{W}{2}$, $\frac{W}{4}$, $\frac{W}{8}$, $\frac{W}{16}$ and $\frac{W}{16}$ is equal to W, as we might have expected.

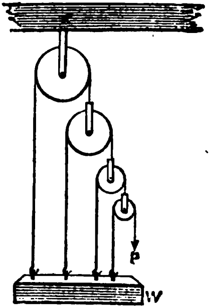
238. As in former cases, if the Power be a little greater than is necessary for equilibrium, the Weight will be moved; and the principle of Art. 208 will be found to hold.

239. *Second System of Pullies.* The diagram represents a system of which the same string passes round all the Pullies, and the parts of it between the Pullies are parallel; it is usually called the *Second System of Pullies.* In this diagram there are *four* strings at the lower block, and when there is equilibrium the Weight is *four* times the Power. In like manner, if there are *six* strings at the lower block, then when there is equilibrium the Weight is *six* times the Power. In this diagram one end of the string is represented as fastened to the *upper* block, and the number of strings at the lower block is an *even* number. But the end of the string might be fastened to the *lower* block, and then the number of strings at the lower block would be an *odd* number.



240. There is no difficulty in the reasoning by which the condition for equilibrium in the Second System of Pullies is established. The tension of the string is the same throughout, and is equal to the Power; so that if there are four strings at the lower block we may regard that block as drawn upwards by four parallel forces each equal to the Power, and drawn downwards by the Weight. Therefore the Weight must be equal to four times the Power. It will be seen that there are five strings at the upper block, so that the fixed point K must support altogether five times P, that is the sum of W and P, as they both act in the same direction, that is downwards. A remark similar to that of Art. 238 may be repeated here.

241. *Third System of Pullies.* The diagram represents a series of Pullies in which each string is attached to the Weight, and all the strings are parallel; it is usually called the *Third System of Pullies.* In this system the string which passes round any Pully, except the lowest, has one end attached to the block of the next lower Pully; the string which passes round the lowest Pully has one end attached to the Weight, and the other end supported by the Power. The highest Pully is fixed, and the others are moveable. The condition of equilibrium for the *Third System of Pullies* can be expressed most easily by stating what proportion the *sum* of the Weight and the Power must bear to the Power. If there is one Pully the sum is twice the Power, if there are 2 Pullies the sum is 4 times the Power, if there are 3 Pullies the sum is 8 times the Power, if there are 4 Pullies the sum is 16 times the Power, and so on. Thus for every additional Pully the proportion which the sum bears to the Power is doubled. If the sum is twice the Power the Weight is equal to the Power; if the sum is 4 times the Power the Weight is 3 times the Power; if the sum is 8 times the Power the Weight is 7 times the Power; if the sum is 16 times the Power the Weight is 15 times the Power, and so on.



242. There is no difficulty in the reasoning by which the condition of equilibrium for the *Third System of Pullies* is established. Suppose that there are four Pullies. Let W denote the Weight to which all the strings are fastened, and P the Power which acts vertically downwards at the end of the string which passes over the lowest Pully. The tension of the string which passes over the lowest Pully is P, hence this Pully is drawn downwards by a force equal to $2P$, and consequently this must be the tension of the string which is fastened to it, and draws it upwards, passing over the second Pully. Hence the second

Pully is drawn downwards by a force equal to $4P$, and consequently this must be the tension of the string which is fastened to it, and draws it upwards, passing over the third Pully. In like manner $8P$ is the tension of the string which passes over the fourth Pully, that is the highest in our diagram. Now all the strings are fastened to the Weight, and so help to support it; thus W must be equal to the sum of P, $2P$, $4P$, and $8P$; that is, W must be equal to $15P$. Or we might shorten the process a little thus. The tension of the string which goes over the highest Pully is $8P$, so that this Pully is drawn downwards by a force equal to $16P$; but the whole Weight supported at K must be equal to the sum of W and P, as they both act in the same direction, that is downwards; therefore the sum of W and P is equal to $16P$, and consequently W is equal to $15P$. A remark similar to that of Art. 238 may be repeated here.

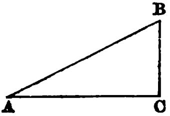
243. We have hitherto supposed that the weights of the Pullies themselves are neglected, but in practice it may be necessary to take these weights into account; it will be sufficient to treat one case as an example. Consider the *Third System of Pullies*, and suppose, as in Art. 241, that there are four Pullies. The weight of the lowest Pully here assists the Power, and acts just like the Power, except that it has a system of *three* Pullies above it instead of *four*; thus it will support 7 times its own weight. Similarly the weight of the next Pully will support 3 times its own weight, and the weight of the next to that will support just its own weight. The weight of the highest Pully will not give any aid. Thus finally we have the following result: the Weight W is equal to the sum of $15P$ together with 7 times the weight of the lowest Pully, 3 times the weight of the next, and the weight of the next to that.

244. The Pully is one of the most useful of the simple machines, on account of its portability, the cheapness of its construction, and the ease with which it may be applied in almost any situation. It is much used in building when weights are to be raised to great heights. But its chief employment is in connexion with the rigging of ships, where almost every arrangement is accomplished by its aid. In practice however it is found that the mechanical advan-

tage is far less than that which theory assigns: this arises from the stiffness of the string or rope and the friction between the wheels and the blocks: it appears that in most cases, owing to these causes, the Power produces only one-third of its theoretical effect.

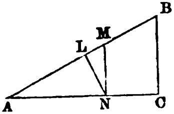
XVI. THE INCLINED PLANE, THE WEDGE, AND THE SCREW.

245. An *Inclined Plane* in Mechanics is a smooth plane supposed to be made of wood or metal or some other rigid material, and fixed in a position inclined to the horizon. It is assumed to be capable of resisting in a direction perpendicular to its surface, to any required amount. When an Inclined Plane is used as a Mechanical Power the straight lines indicating the directions in which the Power and the Weight act are supposed to be both in one vertical plane, namely in the plane perpendicular to the straight line in which the Inclined Plane meets the horizon. Thus the Inclined Plane is represented by a right-angled triangle such as *ABC*; the horizontal side *AC* is called the *base*, the vertical side *BC* is called the *height*, and the hypotenuse *AB* is called the *length*. The angle *BAC* is the inclination of the Inclined Plane to the horizon.



246. Let a heavy body be placed on an Inclined Plane. The Weight of the body tends vertically downwards, but owing to the resistance of the Plane the body cannot move in that direction; it will however slide down the Plane unless prevented by a suitable force, and the amount of the force which we must use will depend on the direction in which it acts. We will suppose that the force acts *along* the Plane, or *parallel* to it; the proposition which applies to this case is the following: *When a Weight is put on an Inclined Plane, and kept in equilibrium by a Power acting parallel to the Plane, the Power is to the Weight in the same proportion as the height of the Plane is to its length.*

247. The preceding statement may be taken as an experimental truth, or it may be established by reasoning, as we will now shew. Let W denote the Weight, and P the Power. From any point L in the Inclined Plane draw LN at right angles to the Plane, meeting the base at N; and draw NM vertical, meeting the Plane at M. The body on the Inclined Plane is kept in equilibrium by three forces, the Power which is supposed to act along the Plane, the Weight of the body which acts vertically downwards, and the Resistance of the Plane which acts at right angles to the Plane. Now the sides of the triangle LMN are parallel to the directions of these three forces, namely LM to that of the Power, MN to that of the Weight, and NL to that of the Resistance. Hence, by Art. 155, the sides of this triangle are in the proportion of the forces, so that the Power is to the Weight in the same proportion as LM is to MN, and the Resistance is to the Weight in the same proportion as LN is to MN. But by measurement, or by theory, it may be shewn that the triangles LMN and CBA are *similar;* so that LM is to MN in the same proportion as CB is to BA, and NL is to MN in the same proportion as AC is to BA. Hence finally the Power is to the Weight in the same proportion as CB is to BA, and the Resistance is to the Weight in the same proportion as AC is to BA. Strictly speaking we required only the proportion of the Power to the Weight; but the proportion of the Resistance to the Weight will be useful hereafter.

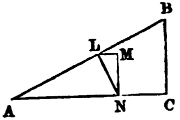


248. If we suppose the Power to be a little greater than is necessary for equilibrium the Weight will be moved along the Plane. Suppose the Weight to be drawn along the Plane from A to B, so that the Power has passed over the *length* of the Plane; then the Weight has passed over as much space as the Power, but the *vertical* height through which the Weight has passed is BC. Thus we have here a fresh illustration of the important principle of Art. 208, and at the same time an indication of the way in which the principle is to be understood: the motion of the Weight *estimated in the direction of the Weight* bears the

same proportion to the motion of the Power *estimated in the direction of the Power*, as the Power bears to the Weight in equilibrium.

249. There is another case with regard to the *Inclined Plane* which it is usual to notice, namely that in which the Power acts *horizontally;* the proposition which applies to this case is the following: *When a Weight is put on an Inclined Plane and kept in equilibrium by a Power acting horizontally, the Power is to the Weight in the same proportion as the height of the Plane is to its base.*

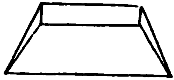
250. The preceding statement may be taken as an experimental truth; or it may be established by reasoning, as we will now shew. Let W denote the Weight, and P the Power. From any point L in the Inclined Plane draw LN at right angles to the Plane, meeting the base at N; and draw NM vertical, meeting at M the horizontal straight line drawn through L. The body on the Inclined Plane is kept in equilibrium by three forces, the Power which acts horizontally, the Weight of the body which acts vertically downwards, and the Resistance of the Plane which acts at right angles to the Plane. Now the sides of the triangle LMN are parallel to the directions of these three forces, namely LM to that of the Power, MN to that of the Weight, and NL to that of the Resistance. Hence, by Art. 155, the sides of this triangle are in the proportion of the forces, so that the Power is to the Weight in the same proportion as LM is to MN. But by measurement, or by theory, it may be shewn that the triangles LMN and BCA are similar, so that LM is to MN in the same proportion as BC is to CA. Hence finally the Power is to the Weight in the same proportion as BC is to CA.



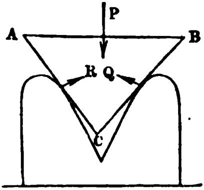
251. If we suppose the Power to be a little greater than is necessary for equilibrium the Weight will be moved along the Plane. Suppose the Weight to be drawn along the Plane from A to B, so that the Power has passed

horizontally over the space AC; then the Weight has passed *vertically* over the space BC. Hence the space passed over by the Weight *estimated in the direction of the Weight* is to the space passed over by the Power *estimated in the direction of the Power* as the Power is to the Weight in the state of equilibrium. Thus we have here a fresh illustration of the important principle of Art. 208, and an indication, as in Art. 248, of the way in which it is to be understood.

252. The *Wedge* is a hard solid body bounded by five plane figures, two of which are triangles and the others are four-sided figures. The four sided-figures are often rectangles, and then the triangles are in parallel planes.



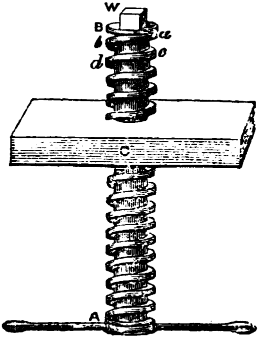
253. The Wedge may be employed to separate bodies. We may suppose the Wedge urged forwards by a force P acting on one of the four-sided faces, and urged backwards by two resistances Q and R arising from the bodies which the Wedge is employed to separate, and acting on the other four-sided faces. These forces will be supposed all to act in one plane which is perpendicular to the edge of the Wedge; and we shall assume that the Wedge is smooth, so that the force on each face is at right angles to the face. Let the triangle ABC represent a section of the Wedge made by a plane perpendicular to its edge; and suppose the Wedge kept in equilibrium by the forces P, Q, R at right angles to AB, BC, CA respectively: then by reasoning which we do not give here it is shewn that P, Q, R are in the same proportion to each other as AB, BC, CA respectively. If AC and BC are equal the Wedge is called an *Isosceles Wedge;* in this case Q and R must be equal, and each of them be in the same proportion to P that AC is to AB.



254. There is very little value or interest in the preceding Article, because the circumstances there supposed

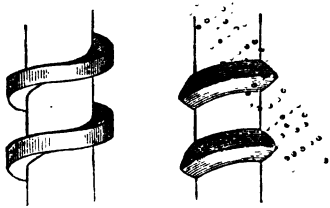
scarcely ever occur in practice. A nail is sometimes given as an example of the Wedge; but when the nail is at rest the resistances on its sides are balanced by friction, and not by a Power at the head. The nail is indeed driven into its place by blows on the head; but the discussion of the *motion* produced by such blows in conjunction with the resistances and the friction is too difficult for a work like the present.

255. *The Screw.* Everybody is familiar with the use of a Screw for fastening pieces of wood together, and this will

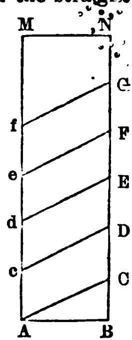


supply great help towards understanding the action of a Screw as a Mechanical Power. The Screw consists of a right circular cylinder AB, with a uniform projecting thread *abcd*...traced round its surface, making always the same angle with straight lines parallel to the axis of the cylinder. This cylinder fits into a block C pierced with an equal cylindrical aperture, on the inner surface of which is cut a groove, the exact counterpart of the projecting thread *abcd*...Thus when the block is fixed and the cylinder is introduced into it, the only manner in which the cylinder can move is backwards or forwards by turning round its axis.

256. In practice the forms of the *threads* of screws may vary, as we see exemplified in the annexed diagrams.



257. We may obtain in the following way some notion of the most essential characteristic of the Screw, namely its making at every point the same angle with the straight lines parallel to the axis of the cylinder. Let $ABNM$ be any rectangle. Take any point C in BN, and make CD, DE, EF,... all equal to BC. Join CA, and through D, E, F,...draw straight lines parallel to CA, meeting AM at the points c, d, e,...respectively. Then if we conceive $ABNM$ to be wound round a pencil or ruler of such a size that the two edges Am, Bn, meet without overlapping, the straight lines AC, cD, dE, eF,...will compose a connected curve which takes the shape of a thread of a Screw, supposing the thread to be excessively fine. In this diagram BC represents what is called the *distance between two consecutive threads of the Screw.*



258. Suppose the axis of the Screw to be vertical, and let a Weight W be placed on the Screw. Then the Weight, by its tendency to descend, would cause the Screw to turn round in the block unless this motion were prevented by some Power. We will suppose this Power P to act at the *end* of a horizontal arm perpendicular to the lever, and horizontally; the arm is firmly attached to the cylinder, as is shewn in the diagram of Art. 255, in which the arm is represented as attached to the cylinder at A. The distance between the axis of the cylinder and the point

of application of the Power we shall call the Power-arm. It is found that when there is equilibrium P is to W in the same proportion as the distance between two consecutive threads of the Screw is to the circumference of the circle having the Power-arm for radius. The reasoning on which this depends is not simple enough to find a place here, so that this may be taken as an experimental fact. It is interesting however to observe that it agrees with the principle of Art. 208, as illustrated in Arts. 248 and 251. For suppose the Power to be a little greater than is necessary for equilibrium, then the Weight will be moved; by turning the Screw once round the Weight will be raised through a space equal to the distance between two consecutive threads. Also the whole space passed over by the end of the Power-arm, estimated in the direction of the Power, consists of a multitude of small spaces which are together equivalent to the circumference of the circle having the Power-arm for radius.

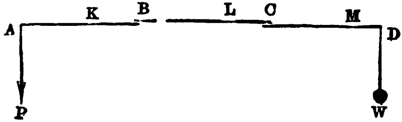
259. The most common use of a screw is not to support a Weight, but to exert a Pressure. Thus suppose a fixed horizontal board above the body denoted by W in the diagram of Art. 255; then by turning the Screw the body will be compressed between the head of the Screw and the fixed board. A bookbinder's press is an example of this mode of using the Screw. The proportion between P and W will be that stated in Art. 258, where W now denotes the whole force exerted parallel to the axis of the Screw by the body which is compressed; a force arising partly from the weight of the body, but mainly from the resistance which it offers to compression. The Screw-pile is another exemplification of the same thing; it is used for foundations which are to be laid under water. The thread of a Screw is cut out in the lower part of a wooden or metal pile; and by means of a capstan the pile is gradually screwed down to the depth which it is required to take: this process is found to succeed where it would be practically impossible to drive a pile down by blows.

260. In practice there will be much friction in the use of a screw; in the familiar case to which we allude at the beginning of Art. 255 this friction is in fact the Weight which the Power has to overcome.

XVII. COMPOUND MACHINES.

261. We have already spoken of the *mechanical advantage* of a machine, and have defined it to be the proportion of the Weight to the Power when the machine is in equilibrium: see Art. 207. Now we might theoretically obtain any amount of mechanical advantage by the use of any of the Mechanical Powers. For example, in the Wheel and Axle the advantage is the proportion of the radius of the Wheel to the radius of the Axle, and this proportion can be made *theoretically* as great as we please; but *practically* if the radius of the Axle is very small the machine is not strong enough for use, and if the radius of the Wheel is very great the machine becomes of an inconvenient size. Hence it is found advisable to employ various compound machines, by which great mechanical advantage may be obtained, combined with due strength and convenient size. We will now consider a few of these compound machines.

262. *Combination of Levers.* Let AB, BC, CD be three Levers, having fulcrums at K, L, M respectively.

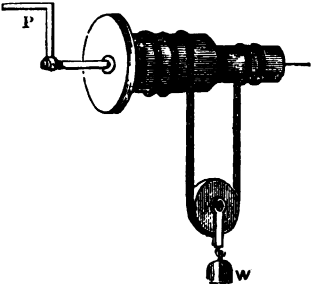


Suppose all the Levers to be horizontal, and let the middle Lever have each end in contact with an end of one of the other Levers. Suppose the system in equilibrium with a Power P acting downwards at A, and a weight W acting downwards at D. It is easy to see that equilibrium can be secured by a proper adjustment of P and W; for P tends to raise the right-hand end of the Lever which has K for its fulcrum; thus the left-hand end of the Lever which has L for its fulcrum is pressed upwards, and therefore the right-hand end of the same Lever is pressed downwards: then the left-hand end of the Lever having M for its fulcrum is pressed downwards, and therefore the right-hand end of the same Lever is pressed

upwards, and if this upward pressure is sufficient W will be supported. It is found by theory and by trial that the advantage of this combination of Levers is expressed by the *product* of the numbers which express the advantages of the separate Levers. For example, suppose that AK is 3 times KB, that BL is 4 times LC, and that CM is 5 times MD; then the advantages of the Levers separately are expressed by 3, 4, and 5 respectively, and the advantage of the combination is expressed by $3 \times 4 \times 5$, that is by 60. Hence any Power at A will support a Weight of 60 times that amount at D. If we suppose the Power to be a little greater than is necessary for equilibrium the Weight will be moved, but in order to raise the Weight through any space the Power must descend through 60 times that space.

263. Combinations of Wheels and Axles are often used. The Wheel of each of the pieces which form the combination is made to act on the Axle of the next by means of teeth or of a strap. It is found by theory and by trial that the advantage of this combination is expressed by the *product* of the numbers which express the advantages of the separate pieces.

264. *The Differential Axle, or Chinese Wheel.*

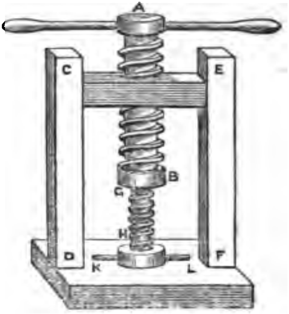


This machine may be considered as a combination of the Wheel and Axle with a single moveable Pully. Two

cylinders of different radii have a common axis with which they are firmly connected; the axis is supported in a horizontal position so that the two cylinders can turn as one body round the axis. A string has one end fastened to the larger cylinder, is coiled several times round the cylinder, then leaves it, passes under a moveable Pully and is coiled round the smaller cylinder to which the other end is fastened. The string is coiled in opposite ways round the two cylinders, so that as it winds off one it winds on the other. A Weight W is hung from the moveable Pully; and the equilibrium is maintained by a Power P applied at the end of a handle attached to the axis. It is found by theory and by trial that there is equilibrium on this machine when the Power is to the Weight in the same proportion as half the difference of the radii of the two cylinders is to the length of the arm at which the Power acts. Thus by making the difference of the radii of the two cylinders sufficiently small we can secure any amount of mechanical advantage.

265. It is not difficult to shew that the preceding statement is consistent with the principle of Art. 208. For suppose the Power to be a little greater than is necessary for equilibrium, then the Weight will be raised. Let the Power describe the circumference of the circle of which the Power-arm is the radius. Then from the smaller cylinder a piece of the string is unwound equal in length to the circumference of the cylinder; and on the larger cylinder a piece of the string is wound equal in length to the circumference of the cylinder. Thus the excess of the circumference of the larger cylinder over the circumference of the smaller is equal to the whole length of string which is removed from the hanging position; so that each of the two vertical portions is shortened by half this length, which is therefore the space through which the Weight is raised. Thus the same number which expresses the proportion of the Weight to the Power when there is equilibrium, expresses also the proportion of the space passed over by the Power to the space passed over by the Weight when there is motion.

266. *Hunter's Screw, or the Differential Screw.*



AB is a right circular cylinder, having a Screw traced on its surface; this fits into a corresponding groove cut in the block *CE*, which forms part of the rigid framework *CDFE*. The cylinder *AB* is hollow, and has a thread cut in its inner surface, so that a second Screw *GH* can work in it. The second Screw does not turn round, for it has a crossbar *KL* the ends of which are constrained by smooth grooves, so that the piece *GHLK* can only move up and down. The machine is used to produce a great pressure on any substance placed between *KL* and the fixed base on which the framework *CDFE* stands; this pressure we will call the Weight, and denote by *W*: the Power *P* is applied by a handle at the top of the outer screw. It is found by theory and by trial that there is equilibrium in this machine when the Power is to the Weight in the same proportion as the difference of the distances between two consecutive threads in the two Screws is to the circumference of the circle having the Power-arm for radius.

267. It is not difficult to shew that the preceding statement is consistent with the principle of Art. 208. For suppose the Power to be a little greater than is necessary for equilibrium, then the Weight will be moved. Let the outer Screw be turned round once. The whole space passed over by the end of the Power-arm, estimated in the direction of the Power-arm, is equal to the circumference of the circle having the Power-arm for radius, as in Art. 258. By turning round the outer Screw the piece *KL* descends through a space equal to the distance between two consecutive threads; at the same time some of the lower Screw enters into the other, namely a length equal to the distance

between two consecutive threads. Therefore, on the whole, the piece KL descends through a space equal to the difference of the distances between two consecutive threads in the two Screws. Thus the same number which expresses the proportion of the Weight to the Power, when there is equilibrium, expresses also the proportion of the space passed over by the Power estimated in the direction of the Power to the space passed over by the Weight, when there is motion.

XVIII. COLLISION OF BODIES.

268. In the last nine Chapters we have been concerned mainly with questions relating to *equilibrium;* we now return to some which relate to *motion.*

When the application of force results in motion we measure the force by the *momentum* which is produced in a definite time, as for instance, one second; and as long as we keep to the action of force on the *same* body we may measure the force by the *velocity* which is produced. One of the forces with which we are familiar is *gravity*, which takes an appreciable time to produce a moderate velocity. There are however other forces which seem to produce a large velocity almost instantaneously. For example, when a cricket-ball is driven back by a blow from a bat the original velocity of the ball is taken away and a new one is given to it in a contrary direction; the velocity taken away, and also that given, are very large, while the whole operation takes place in an extremely brief time. Similarly, when a bullet is discharged from a gun a very large velocity is given to the bullet in an extremely brief time. Forces which produce such effects as these are called *impulsive forces;* and the following is the usual definition: *An impulsive force is a force which produces a large change of motion in an extremely brief time.*

269. Thus impulsive forces do not differ in *kind* from other forces but only in *degree;* and an impulsive force is merely a force which acts with very great intensity during a very brief time. As the laws of motion may be taken to

be true whatever be the intensity of the forces which produce the motion, we can apply these laws to the action of impulsive forces. But since the duration of the action of an impulsive force is too brief to be appreciated, we cannot measure such a force by the momentum produced in a definite time; it is usual to measure an impulsive force by the whole momentum which it produces.

270. We shall not have to consider the result of the simultaneous action of impulsive forces and ordinary forces for the following reason: the impulsive forces are so much more intense than the ordinary forces that during the brief period of simultaneous action the latter do not produce an effect of any importance in comparison with that produced by the former. Thus, to make a supposition which is not extravagant, an impulsive force might produce a velocity of 1000 feet per second in less than one tenth of a second, while the earth's attraction in one tenth of a second would produce a velocity of about 3 feet per second.

The words *impact* and *impulse* are often used as abbreviations for the *action of an impulsive force*, or for *impulsive action*.

271. We are now about to consider some questions relating to the collision of two bodies; the bodies may be considered to be small spheres of uniform substance. We shall not take account of any possible *rotation* of these spheres; that is to say, the motion we are about to consider is that which all the particles of the body have in common, leaving out such as may be different for different particles. The collision of spheres is called *direct* when at the instant of contact the centres of the spheres are moving in the straight line in which the impulse takes place, that is, in the straight line which joins the centres of the two spheres; the collision is called *oblique* when this condition is not fulfilled.

272. When one body impinges directly on another, the following is considered to be the nature of the mutual action. The whole duration of the impact is divided into two parts. During the first part a certain impulsive force acts in opposite directions on the two bodies, of such an amount as to render their velocities equal. During the

second part another impulsive force acts on each body in the same direction as before, and the magnitude of this second impulsive force bears to the magnitude of the former a proportion which is constant for a given pair of bodies. This proportion lies between the values 0 and 1, both inclusive. When the proportion is 0 the bodies are termed *inelastic;* when it is greater than 0 and less than 1 the bodies are called *imperfectly* elastic; and when it is 1 the bodies are called *perfectly* elastic. This proportion is called the *coefficient of elasticity*, or the *index of elasticity.*

273. There are three assumptions involved in the preceding Article.

We assume that there is an epoch at which the velocities of the two bodies are equal: this will probably be admitted as nearly self-evident.

We assume that during each of the two parts into which the whole duration of the impact is divided by this epoch, the action on the two bodies is equal and opposite: this is justified by the Third Law of Motion.

We assume that the action on each body after the epoch is in the same direction as before, and bears a constant ratio to it: this assumption may be taken on trial as a matter to be tested by observation.

274. The theory of the collision of bodies appears to be chiefly due to Newton, who made some experiments on the subject, and recorded the results in his *Principia.* In his experiments the two balls used together seem always to have been formed of the same substance. He found that the value of the *index of elasticity* was for balls of worsted about $\frac{5}{9}$, for balls of steel about the same, for balls of cork a little less, for balls of ivory $\frac{8}{9}$, for balls of glass $\frac{15}{16}$.

275. We have still to explain why the words *elastic* and *inelastic* are used in Art. 272. It appears from experiment that bodies are compressible in various degrees,

and recover more or less their original forms after the compression has been withdrawn; so likewise they may be bent or twisted to some extent, and will recover their original forms when the forces which bent or twisted them cease to act: this property is called *elasticity*. When one body impinges on another we may naturally suppose that the surfaces near the point of contact are compressed during the first part of the impact, and that they recover more or less their original forms during the second part of the impact.

276. By the aid of the principles which we have now explained the change of motion in bodies produced by collision may be calculated; but it is not suitable to our plan to enter on this calculation, and so we will merely state the result for some special cases. We now consider only *direct* collision.

277. Suppose that there is a collision between two balls *equal* in mass and *perfectly elastic;* then the two balls *interchange* velocities. There are various particular examples included in this single statement. Thus let a ball A impinge on a ball B at rest; then A is brought to rest, and B moves on with the velocity which A originally had. Again, let A and B be both moving in the same direction, and let A *overtake* B; then after the collision they both move on in the same direction as before, A with the velocity which B originally had, and B with the velocity which A originally had. Finally let A and B be moving in opposite directions and meet; then after the collision A moves backwards with the velocity which B originally had, and B moves backwards with the velocity which A originally had. Thus in fact if we suppose the two balls exactly alike, so that one cannot be distinguished from the other, the result is the same in all these examples as if one body had *gone through* the other, or as if one had passed *close* by the side of the other.

278. It is easy to verify the statement made with respect to the three preceding examples by experiments. The first example especially is interesting. We may take a row of equal elastic balls, say B, C, D at rest in a

straight line, either close together or separated. Then let an equal ball A impinge on B in the direction of the straight line. By this collision A is brought to rest, and B proceeds, with the velocity which A had, to strike C; then B is brought to rest and C proceeds to strike D; finally C is brought to rest, and D proceeds with the velocity which A originally had. If the balls B, C, D were at first close together it is curious to see D fly off apparently immediately A strikes B. Ivory balls, though not perfectly elastic, are sufficiently elastic to exhibit the experiment well.

279. Next suppose that there is a collision between two balls which are *inelastic.* Then after collision the balls do not separate, but move on together with the *same* velocity. This velocity can be determined when we know the velocities of the balls before the collision by the aid of the principle that *the momentum of the system is the same after collision as before.* The principle can be expressed briefly in the foregoing words; but a little explanation is necessary in order to fix the meaning of the term *momentum of the system.* If the balls are moving in the *same* direction the momentum of the system is the *sum* obtained by adding the momentum of one ball to the momentum of the other; if the balls are moving in *opposite* directions the momentum of the system is the difference obtained by subtracting the momentum of one ball from the momentum of the other. If one body is at rest before the collision then the momentum of the system is the momentum of the other body. This principle of the identity of the *momentum of the system* before and after collision is shewn by theory to be an obvious consequence of the Third Law of Motion.

280. Suppose for example that a ball of mass 5 moving with a velocity 6 strikes a ball of mass 4 moving with a velocity 3; the velocities may be understood to be expressed throughout in feet per second. Then if the balls are moving in the *same* direction the momentum of the system before impact is 30 + 12, that is 42. After the collision the balls move on together with the velocity

$\frac{42}{9}$; for with this velocity, since the whole mass is 9, the momentum of the system after the collision will be 42, the same as before. But if the balls are originally moving in contrary directions the momentum of the system before the collision is $30 - 12$, that is 18; and so after collision the balls move on with the velocity $\frac{18}{9}$, that is 2: the direction of this velocity is the same as that of the ball which had the greater momentum before the collision.

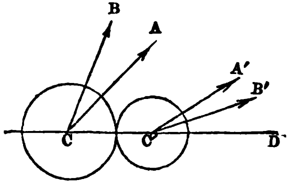
281. The general problems of the direct collision of elastic balls which are not equal in mass, and of imperfectly elastic balls, do not yield results which we can express simply and easily in words; except that the principle just stated with respect to momentum always holds, namely that the *momentum of the system* is the same after the collision as before. But numerical results are easily obtained by following the steps of Art. 272. Suppose for example that a ball of mass 5 moving with a velocity 6 overtakes a ball of mass 4 moving with a velocity 3. If the balls are inelastic we found in Art. 280 that they would move on together with the velocity $\frac{42}{9}$ that is $\frac{14}{3}$. Suppose however that instead of being inelastic the balls have $\frac{3}{4}$ for their index of elasticity; then $\frac{14}{3}$ will still be the common velocity at the end of the *first part* of the impact. Thus the ball which had originally the velocity 6 has lost $6 - \frac{14}{3}$, that is $\frac{4}{3}$; and therefore during the second part of the impact it will lose $\frac{3}{4}$ of $\frac{4}{3}$, that is 1: so that its final velocity will be $\frac{14}{3} - 1$, that is $\frac{11}{3}$. The ball which had originally the velocity 3 gained during the first part of the shock $\frac{14}{3} - 3$, that is $\frac{5}{3}$; and therefore during the second

part of the shock it will gain $\frac{3}{4}$ of $\frac{5}{3}$, that is $\frac{5}{4}$: so that its final velocity will be $\frac{14}{3} + \frac{5}{4}$, that is $\frac{71}{12}$.

282. We have hitherto supposed that both balls are in motion; or at least if one ball is at rest before collision we have supposed that it is *moveable*. But a particular case may be noticed of another kind, namely that in which one body moves and strikes another which is *fixed;* we may for simplicity take this fixed body to be a *plane*. We suppose the collision to be *direct*. Then it is found that if the moving ball and the fixed plane are *inelastic* the moving ball remains close to the fixed plane after collision; and if the moving ball and the fixed plane are *perfectly* elastic the moving ball recoils after collision in the same straight line and with the same velocity as before. If the moving ball and the fixed plane are imperfectly elastic the moving ball recoils after collision in the same straight line as before, with a velocity which is equal to the product of the former velocity into the index of elasticity.

283. The next subject which naturally occurs for consideration is the *oblique* collision of bodies.

In the diagram let C represent the centre of one ball, and CA the direction in which it is moving at the instant

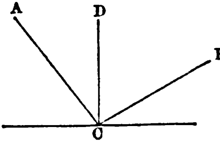


of collision; let C' represent the centre of the other ball, and $C'A'$ the direction in which it is moving at the instant of collision. Then it is found by theory that we may treat the problem thus. Resolve the velocity of the ball whose centre is C into two components, one along CC', and the other at

right angles to CC'; also resolve the velocity of the ball whose centre is C' into two components, one along CC', and the other at right angles to CC'. Then the velocities *along* CC' are changed in precisely the same way as if the balls moving with these alone came into direct collision; and the velocities at right angles to CC' are not affected at all; that is they remain the same for each ball after collision as before. Since we thus know the two component velocities of the ball whose centre is C, we can find the resultant velocity after collision, and the direction, CB, of this velocity. Similarly we can find the resultant velocity after collision of the other ball, and its direction $C'B'$.

It is often convenient to *resolve* velocities into components in the manner just exemplified; the method is the same as for resolving forces: see Art. 156.

284. An important case of oblique collision is that in which a moving ball strikes a fixed plane.



Let AC represent the direction in which the ball moves before it strikes the fixed plane at C; let CD be at right angles to the plane. After striking the plane the ball will go off in some direction which we denote by CB. The angle ACD is called the *angle of incidence*, and the angle BCD the *angle of reflection.* If the ball and the fixed plane are *perfectly* elastic these angles are equal, and the velocity of the ball after collision is equal to the velocity before. If the ball and the fixed plane are *imperfectly* elastic the angle of reflection is greater than the angle of incidence, the relation between the two depending on the index of elasticity. In the case in which the ball and the fixed plane are *inelastic* the angle of reflection is a right angle, so that the ball after collision moves close to the plane. The velocity after collision is always less than the velocity before collision, except when the ball and the fixed plane are perfectly elastic.

285. Many remarkable results are obtained by the collision of balls on a billiard-table, which the principles we have stated would not be sufficient to explain. These results depend on two circumstances which we have not considered, namely the *rotation* of the balls, and the *friction* between the balls, and between the balls and the table: the theory of such results would be altogether beyond the present work.

XIX. MOTION DOWN AN INCLINED PLANE.

286. We have already spoken about the motion of a body falling freely, but we will now make a few additional remarks on the subject. The motion in this case is said to be *uniformly accelerated:* this means that in successive equal intervals of time the velocity of the falling body receives equal additions. The laws of the motion involve, as we saw, two numbers, namely 16 which expresses the number of feet fallen through in the first second of time, and 32 which expresses in feet per second the velocity at the end of the first second. The first number is *half* the second, and the reason for this may be seen without difficulty. The velocity increases in the same proportion as the time, and in the first second the velocity begins with the value 0 and ends with the value 32. Hence 16 may be called the *average* velocity; for instance at the end of the first tenth of a second the body is falling with the velocity $\frac{1}{10}$ of 32, and at the end of nine-tenths of the second it is falling with the velocity $\frac{9}{10}$ of 32: the sum of these two velocities is 32, so that the half sum is 16. It is easy to admit that when the velocity increases or decreases uniformly as the time increases, then the space described in a given time is just the same as would be described by a body moving during that time uniformly with the *average* velocity, that is with a velocity equal to half the sum of the velocities at the beginning and the end of the given time.

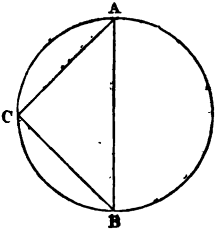
287. Let us apply the principle just stated to find the space through which the body will fall in the *fourth* second of its descent. The velocity at the beginning of the *fourth* second, that is at the end of the *third* second, is 3×32, that is 96. The velocity at the end of the fourth second is 4×32, that is 128. The half sum of 96 and 128 is 112, so that a body moving uniformly with the average velocity would describe 112 feet in a second. This, as we saw in Art. 91, is exactly the space through which a body falls in the fourth second of its descent, as it should be according to our principle.

288. Of the two numbers which thus present themselves in the laws of falling bodies, namely 16 and 32, we might take either as the representative of the force of gravity; but it is found most convenient to take 32 which denotes the *velocity gained in the first second* by a body falling freely. This number is very important in Mechanics; it is usually denoted by the letter g in books which discuss the mathematical theory of the subject. The strength of any other *constant* force, may be compared with that of gravity, by observing the appropriate number which now takes the place of 32. Thus at the surface of the sun for a falling body the number would be 27 times 32; the attraction of the sun at its surface being about 27 times the attraction of the earth at its surface. At the surface of the moon for a falling body the number would be about $\frac{1}{6}$ of 32, that is rather more than 5.

289. We have already mentioned a contrivance, called *Atwood's machine*, by which we can exhibit a motion of the same *kind* as that of a body falling freely, but much slower, and so better adapted for observation: see Art. 140. Another case of such motion is that furnished by a body sliding down a smooth inclined plane. We have seen in Art. 246 that when a body is placed on an Inclined Plane it may be supported by a force acting along the Plane less than the weight of the body, namely by a Power having the same proportion to the Weight of the body as the height of the Plane bears to its length. This leads to the conclusion that a body will slide down the inclined

plane in the same manner as a body falls freely, but at a slower rate. Instead of the number 32 we must now take a smaller number, namely a number in the same proportion to 32 as the height of the plane is to its length. For example, if the height of the plane is $\frac{1}{8}$ of its length the standard number with which we shall be concerned will be $\frac{1}{8}$ of 32, that is 4. A body sliding down such a plane would gain in the first second a velocity of 4 feet per second, and an equal additional velocity in every other second; and it would slide down through 2 feet in the first second. An important fact connected with this case of motion is that the velocity gained by a body in sliding down the inclined plane is precisely the same as would be gained by the body if it fell freely through the height of the plane.

290. Various interesting results are obtained by theory and may be verified by experiment respecting the motion of bodies down smooth inclined planes.



Thus, for example, let A be the highest point of a circle in a vertical plane, AB a diameter, AC any chord. Then the time of sliding down AC is equal to the time of falling freely down AB; so also the time of sliding down CB is equal to the same time.

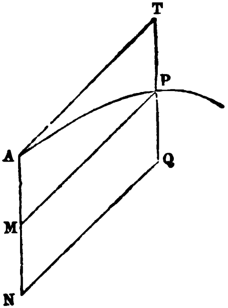
291. Another example of motion of the same *kind* as that of a falling body is furnished by placing one body on a smooth horizontal table and allowing it to be drawn along the table by another body which descends vertically, the two bodies being connected by a string which passes over a pully at the edge of the table. Suppose for instance that the weight of the body on the table is 5 pounds, and the weight of the descending body 3 pounds. Then the mass to be moved is the sum of the two masses, and the corresponding weight is 8 pounds. But the weight of the body on the table is resisted by the table, and so it does

not produce any motion; and thus the weight of 3 pounds has to move all the mass instead of just moving itself. Therefore the effect produced is $\frac{3}{8}$ of what would be produced if the descending body were free; and the motion is like that of a falling body, only instead of the standard number 32 we must use $\frac{3}{8}$ of 32, that is 12.

XX. PROJECTILES.

292. In Art. 124 we have considered the motion of a body projected vertically *upwards*, and have shewn that the body will reach the height from which it would have had to fall in order to gain the velocity with which it was projected upwards. A few words may be given to the case of a body projected vertically *downwards*. A person might stand on a high tower and send a body vertically downwards, starting it say with a velocity of 64 feet per second. In this case the body starts with the velocity which would be gained in falling for two seconds, and the subsequent motion is precisely the same as that of a body which falls freely, but which began its descent just two seconds before we turned our attention to it. As in Art. 126 we must notice that *during* the motion, that is *after* the body has been projected, the only force acting is the force of gravity.

293. We have hitherto considered only motion in a *straight line*, but daily observation presents us with examples of other kinds of motion. The most familiar case is that in which a body is started in some direction neither vertically upwards nor vertically downwards, and is left to move under the action of gravity. As examples we may take a cricket-ball thrown by the hand, an arrow shot from a bow, and a ball shot from a cannon. A body thus projected and left to the action of gravity is called a *Projectile*.



294. Let a body be projected from the point A, in any direction which is not vertical; let AT be the space which would be described by the body in any assigned time if gravity did not act, so that AT is the direction of projection. Draw AM vertically downwards, equal to the space through which a body would fall from rest in the assigned time under the action of gravity. Complete the parallelogram $ATPM$; then P, the corner opposite to A, will be the place of the body at the end of the assigned time. For by the Second Law of Motion gravity will communicate the same vertical velocity to the body as it would if the body had not received any other velocity. Thus at any instant there will be the same vertical velocity as if there had been no velocity parallel to AT, and the same velocity parallel to AT as if there had been no vertical velocity. Therefore the spaces described parallel to AT and AM respectively will be the same as if each alone had been described. Thus P will be the place of the body at the end of the assigned time.

295. It is obvious that by the method just given we may determine the position which the projectile has at any assigned instant; and if we go through the process for a great number of different instants, marking on a piece of paper the places obtained, we shall obtain a good representation of the path of the projectile. It is found to be a curve which mathematicians call a *parabola*, and of which they have discovered many interesting properties. But we do not assume that the reader has at present studied the nature of this curve. Some idea of the form of the curve may be gained by watching the flight of an arrow. Or suppose we make a small hole in the lower part of the side of a barrel full of water; the drops of water are forced out and become projectiles, and as one follows another we have a continued stream which takes the form of a parabola.

296. The parabola is not a *closed* curve like a circle, but stretches on without end; and in this respect it resembles a straight line. Suppose one arrow to be shot nearly vertically upwards, and another to be shot very obliquely; then at first sight the two paths may seem to be not much alike. The reason for the apparent diversity is that the two paths are not of corresponding extent; but in reality all parabolas are *similar*, that is, a portion of one parabola is an exact copy of the corresponding portion of any other, though it may be on a larger or a smaller scale.

297. Return to the diagram of Art. 294. Produce AM to N, so that MN is equal to AM, and complete the parallelogram $ATQN$; then when the body is at P it will be moving for an instant in the direction parallel to the diagonal AQ. Suppose, for example, that 3 seconds elapse, after starting, before the body is at P; then AT represents the space through which the body would move in 3 seconds if gravity did not act: thus if the original velocity is 120 feet per second AT represents 3×120 feet. Also AM represents the space fallen through under the action of gravity in 3 seconds, so that $AM = 9 \times 16$; and therefore $AN = 9 \times 32$. Hence AT bears the same proportion to AN as 3×120 bears to 9×32, that is as 120 bears to 3×32. But 120 represents the original velocity along AT, and 3×32 represents the vertical velocity given by gravity in 3 seconds. Hence AT and AN are proportional to the component velocities of the body at the end of 3 seconds, and are in the directions of these velocities respectively. Hence when the body is at P the *direction* of the resultant velocity is parallel to AQ; and the magnitude of the resultant velocity bears the same proportion to that of the original velocity as AQ bears to AT. See Art. 108.

298. While the projectile is in motion the only force acting on it is that of gravity; if at the instant the projectile is at P this force were to cease acting, the body would thenceforward move on uniformly in the direction, and with the velocity, just determined.

299. Another method of treating the problem of projectiles may be briefly noticed, as it is found very advantageous in mathematical calculations. This consists in

resolving the velocity with which the body is projected into *two components*, one vertical and one horizontal.

For example, suppose that the velocity with which the body is projected is 100 feet per second, and that the direction of projection is such that this velocity can be resolved into components of 80 feet and 60 feet respectively, the former vertical and the latter horizontal. Then theory shews that the *height* to which the projectile rises in any time is just that which a body would have reached if sent *vertically upwards* with a velocity of 80 feet per second. And the distance measured *horizontally* from the starting point is just that which a body would have reached if it had moved uniformly at the rate of 60 feet per second. Hence the position of the projectile at any assigned instant can be readily determined.

300. In all that we have said in this Chapter we have left out of consideration the resistance of the air; but in practice this is a very important matter, and in consequence of it the path of the projectile is not what mathematicians call a *parabola*. The resistance of the air becomes more considerable as the velocity of projection is increased; and if this velocity is very great the motion is quite different from that which we have theoretically determined. For example, a cannon-ball of certain weight and size being projected at a certain inclination to the horizon with a velocity of 1000 feet per second was found to strike the ground again at a distance of about 5000 feet from the starting point, whereas according to the theory which we have given the ball ought to have gone more than six times as far. But the consideration of the motion of projectiles, taking into account the resistance of the air, is much too difficult for an elementary book like the present.

301. We have supposed in this Chapter as in other places that the projectile never goes very far from the surface of the earth: see Art. 100. The earth's radius is about 4000 miles, and at the surface of the earth the force of gravity is measured by the number 32. If we could ascend to a height of 4000 miles from the surface, so as to be at *double* the original distance from the centre, the earth's attraction

would be only a *fourth* of what it was before, and therefore a body instead of falling through 16 feet in the first second would fall through 4 feet, and instead of gaining a velocity of 32 feet per second would gain a velocity of 8 feet per second. In like manner at a triple distance from the earth's centre the attraction would be only a *ninth* of what it was at first; and so on. This is technically expressed by saying that the *attraction varies inversely as the square of the distance:* see Art. 77. This would be strictly true if the earth were a sphere of the same substance throughout; and in the actual state of things it is *very nearly* true. Hence if a body were to fall from a very great height it would not gain in every second precisely the same velocity; in fact in every second it would gain a trifle *more* velocity than it gained in the second immediately preceding. This would be practically of no moment in the case of any motion, such as we could observe, continued for only a few seconds; but it would become important in the case of a fall which lasted for a long time.

XXI. MOTION IN A CIRCLE.

302. The force of gravity is *constant* in magnitude and direction; that is so long as we keep near the same place on the earth's surface the force undergoes no change in these respects. More complex problems of motion arise when we consider the operation of forces which may change in magnitude, or in direction, or in both.

303. Suppose that a body is observed to describe a circle with uniform velocity; then theory demonstrates the following results. *The body is acted on by a force passing through its centre of gravity, constant in magnitude, and always directed to the centre of the circle; and in order to compare this force with the weight of the body we must take the square of the velocity, divide it by the radius, and find the proportion of this quotient to* 32.

304. No single statement in connexion with motion is more important than that just given, and the student should regard it with earnest attention. To prevent mistake we add that in estimating the velocity and the radius we must take a *foot* as the unit of length, and a *second* as the unit of time. Also it may happen that the moving body is under the operation of *two or more* forces; then our statement means that these forces must be equivalent to a *single resultant* having the value assigned. This case of motion gives us an example in which the force though *constant* in magnitude, is *variable* in direction; for at any instant the direction of the force is the straight line which at that instant can be drawn from the centre of gravity of the body to the centre of the circle.

305. We proceed to illustrate the statement of Art. 303. Suppose, for example, that the velocity in the circle is 20 feet per second, and the radius of the circle 5 feet. The square of 20 is 400; and if 400 be divided by 5 the quotient is 80. Thus the force which acts on the body, and makes it describe the circle, is $\frac{80}{32}$ of the weight of the body, that is $\frac{5}{2}$ of the weight of the body. Again, suppose that the radius of the circle is 7 feet, and that the body describes the circumference of the circle in 2 seconds. By Art. 28 the circumference of the circle is about $\frac{22}{7} \times 14$ feet, that is about 44 feet. Hence the velocity is about 22 feet per second. The square of 22 is 484; and dividing this by the radius we obtain $\frac{484}{7}$. Thus the force which acts on the body, and makes it describe the circle, is $\frac{484}{7 \times 32}$ of the weight of the body, that is $\frac{121}{56}$ of the weight, that is rather more than twice the weight.

306. Thus we may hope that the important statement of Art. 303 is intelligible, and we have next to consider

how it is known to be true. It is in fact capable of demonstration by strict mathematical reasoning, but it may also be verified by trial. There is however no very obvious example which immediately presents itself; a boy's sling is sometimes mentioned, but this does not strictly fulfil all the conditions. For the loaded end of the sling in general does not describe an exact circle; the hand at the other end shifts its position perpetually, and keeps urging the loaded end to increased speed. If the hand remains quite still, so that the loaded end describes an exact circle, still this circle is not described with uniform velocity, when, as is usually the case, the sling moves in a vertical plane; the velocity is greater at the lower than at the upper points of the circle.

307. In the absence of appropriate spontaneous examples we must contrive experiments. Put a ball on a smooth table, fasten it by a string to a fixed point in the table, and start the ball so as to describe a circle round the fixed point as centre. If the table is smooth the ball will move for some time pretty uniformly, and the velocity with which it moves can be observed. It is easy to devise means for measuring the force which acts on the ball and tends towards the centre of the circle; this is in fact the tension of the string. We may have the string formed of some material which will stretch, and observe carefully the length of the string when the ball describes a circle uniformly. Then stop the motion, take away the ball from the string, fix one end of the string and hang a weight at the other end just heavy enough to stretch the string to the length it had in the case of motion: then the tension of the string in both cases is equal to this weight. Thus we know the velocity with which the ball moved, the radius of the circle, and the force acting towards the centre of the circle; and accordingly we can test the truth of the statement in Art. 303 as to the relation between these quantities. Or instead of fastening one end of the string to a fixed point in the table, when the ball at the other end is describing a circle, we may pass the string through a small hole in the table and hang a weight at the other end. When this weight remains at rest it measures the tension of the string, and

therefore the force which is directed towards the small hole as centre, and acts on the body describing a circle round that centre.

308. In this experiment of the ball on the smooth table the weight of the ball vertically downwards is just balanced by the resistance of the table upwards, and thus these two forces do not affect the motion of the ball on the table. Setting aside the weight and the resistance of the table the only force which acts on the ball *during the motion* is the force *towards the centre* of the circle. Suppose we stop the action of this force at any instant, which we may do by cutting the string, then the ball will continue to move uniformly in the direction in which it is moving at that instant; this direction is at right angles to the straight line drawn from the ball to the centre of the circle at that instant, or in the language of Geometry it is the *tangent* to the circle at the point which the ball occupies at the instant. Thus it must be remembered that *while* a body describes a circle with uniform velocity the resultant of all the forces which act on it is a single force *towards* the centre of the circle. We say the *resultant* of all the forces, because there may be forces which just balance each other as in the case of the ball on the smooth table, where the weight and the resistance balance each other.

309. There is still another mode of making the experiment which may be noticed. Let one end of a string be fastened to a weight and the other end to a fixed point. Let the string be drawn aside from the vertical direction, and let a velocity be given to the weight in a horizontal direction. By trial it will be found possible to get the weight to move for some time in a horizontal plane and to describe a circle. The tension of the string may be supposed to be resolved into two components, one vertical and the other horizontal: see Art. 156. The vertical component will balance the weight of the body, so that the body goes neither up nor down. The horizontal component constitutes the force towards the centre of the circle which makes the weight describe the circle. It is easy to determine the value of the tension of the string, by

such methods as those of Art. 307; and then the components into which it is resolved can be found: thus the truth of the statement in Art. 303 can be tested.

310. Suppose a man to run round a circle of which the radius is 20 feet, at the rate of 8 feet in a second. Then the resultant force which acts on him is directed towards the centre of the circle, and is equal to $\frac{64}{32 \times 20}$ of his weight, that is to $\frac{1}{10}$ of his weight. This resultant force must be produced by a combination of the man's weight, and the action of the ground. Hence the action of the ground must not be entirely vertical but oblique; the vertical component of it must just balance the man's weight, and the horizontal component of it must be equal to $\frac{1}{10}$ of the weight. In order that the action of the ground may pass through the man's centre of gravity, which is necessary in order that it may combine with the weight to form the horizontal force, the man must lean inwards towards the centre of the circle: the amount of this leaning must be at the rate of 1 inch horizontal to 10 inches vertical.

311. We find in Astronomy some of the best illustrations of the motion of a body under the influence of a force which has its direction always changing but always passing through a fixed point. For instance the Earth moves round the Sun under the action of the Sun's attraction. The Earth does not describe a circle, and so does not furnish exactly a case of the motion considered in the present Chapter; but still the path in which the Earth moves is very nearly a circle, and the amount of the Sun's force is not much different from that assigned by Art. 303. So also the Moon relatively to the Earth describes a path which is very nearly a circle. The distance of the Moon from the Earth is about 240,000 miles; thus the circumference of the circle which the Moon describes round the Earth, expressed in feet, is about $\frac{22}{7} \times 480000 \times 5280$. The time in which this

circle is described is about $27\frac{1}{4}$ days, that is about $27\frac{1}{4} \times 24 \times 60 \times 60$ seconds. Hence we obtain the velocity by dividing the former number by the latter. Then by the statement of Art. 303 we can compare the force which the Earth exerts on a body moving like the Moon moves, with the weight of the body; that is in fact, we compare the force which the Earth exerts on a body moving like the Moon moves, with the force which the Earth would exert on the body if it were close to the Earth's surface. This comparison was the foundation of Newton's system of Astronomy; the result is that the force on a body in the situation of the Moon is about $\frac{1}{3600}$ of the force on the same body if it were at the Earth's surface: see Art. 301.

XXII. SIMPLE PENDULUM.

312. Let one end of a fine string be fastened to a fixed point, and the other end to a small heavy particle. In the position of equilibrium the string will be vertical. Let the particle be displaced from its position of equilibrium, the string being kept stretched, and then allowed to move. The particle will go backwards and forwards; this is called *oscillating*. The particle thus describes arcs of a circle; owing to friction and the resistance of the air the arcs described become gradually less and less, until at last the particle comes to rest. The string and particle together constitute what is called a *simple pendulum*.

313. The forces which act on the heavy particle are its own weight and the tension of the string. The former force acts always vertically downwards, and is always of the same amount; so that it is *constant* in direction and magnitude. The latter force perpetually changes its direction, though the direction always passes through a fixed point. The weight acting vertically may be supposed at any instant to be resolved into two components, one along the string at that instant, and the other at right angles to it. The former produces no motion, being resisted by the string; the latter urges the heavy particle along the circular arc

towards the lowest point. The motion is found to be of the following kind: the particle being at one of the extreme points of an arc starts, as if from rest, and the velocity continually increases until the particle reaches the lowest point of the arc; then as it goes up through the rest of the arc the velocity diminishes until the particle reaches its highest point at the other end of the arc. The time of moving from the starting point to the lowest point is the same as that of moving from the lowest point to the other end of the arc; and when the arc is very small it is found that this time does not sensibly change as the arc becomes smaller and smallor. The time of passing from one end of an arc to the other is called the *time of oscillation;* it may be found according to theory, by the following rule: *Take the length of the string in feet, divide by 32, and extract the square root of the result; then multiply by* $\frac{22}{7}$ *and the product will be the time in seconds.*

314. The important point to notice with respect to the preceding rule is that it supposes the arc through which the particle moves to be very small; but then it is true without taking into account the greater or less extent of this small arc. The rule may be made more accurate by using instead of $\frac{22}{7}$ the number 3·1416: see Art. 28. Also for the sake of extreme precision we should have instead of 32 to put a slightly different number, different for different places: see Art. 98. The length of a simple pendulum which oscillates in a second at the latitude of London is 39·1393 inches. This is about ·994 of the *metre*, the French standard of length.

315. We have said that the rule in Art. 313 supposes the arc of oscillation to be *very small;* and therefore it will be proper to give some notion of the correction which must be made when the arc is not very small. The time found by the rule must then be increased by a small fraction of itself, and this fraction may be found with sufficient accuracy in the following manner: the numerator is the square of the number of degrees in the angle between the extreme position of the pendulum and the position of

equilibrium, and the denominator is 50000. Thus, for example, suppose the pendulum oscillates through an angle of 10 degrees altogether, then there are 5 degrees in the angle with which we are concerned; the square of 5 is 25, and $\frac{25}{50000} = \frac{1}{2000}$. Therefore the time found by the rule of Art. 313 must be increased by $\frac{1}{2000}$ of itself; so that if that rule gives one second for the time when the arc is very small the correct time when the arc corresponds to 10 degrees will be $1\frac{1}{2000}$ seconds.

316. The time of oscillation does not depend on the nature of the substance of which the heavy particle is composed; this corresponds with the fact that, setting aside the resistance of the air, all bodies fall to the ground from the same height in the same time. The word *oscillation* is used by some writers to denote the time taken by the heavy particle in passing from one end of an arc to the same point again; this amounts to *twice* the time which we assign to an oscillation. Also the word *vibration* is sometimes used instead of oscillation.

317. Instead of compelling a particle to describe an arc of a circle by means of a string we might have a fine tube made in the form of an arc of a circle, and fixed in a vertical plane; and then the particle might be placed within the tube so as to slide up and down. Theory shews that the motion is of the same kind as the other, provided the tube is smooth internally. The resistance of the tube in this case takes the place of the tension of the string in the other.

318. We may also have other cases of motion by supposing a fine smooth tube, as in the preceding Article, not in the form of an arc of a circle but in that of an arc of any other curve. One interesting result obtained by theory then is that whatever be the form of this curve the velocity of the heavy particle at any point is just the same as if it had fallen freely through a vertical space equal to the depth of this point vertically below the starting point. We have already remarked in Art. 289 that this is the case when the tube is in the form of a *straight line*.

319. Two very curious results in connexion with this subject may be noticed. Suppose the fine smooth tube made in the form of half a particular curve which mathematicians call a *cycloid*, and let it be placed as they would say with its base horizontal and its vertex downwards; denote the highest point of the curve by A, and the lowest point by B. Then the heavy particle would slide from A to B down this curve in *less time* than down any other curve from A to B. And if C denote any point of the curve between A and B the particle would slide down the portion of the curve from C to B, starting at C, in the same time as down the whole curve from A to B. The two statements can be well demonstrated experimentally by constructing tubes or troughs on a large scale. In particular the truth of the second statement can be very effectively shewn; a man takes a ball in each hand, and by stretching out his arms he can put one ball at a point of the trough far above the point at which he puts the other, and let both start at the same instant; then the upper ball just overtakes the lower ball at the bottom of the curve.

320. It is easy to give a notion of the curve which we call a *cycloid*. It is the curve which a point in the circumference of a carriage wheel would trace out as the wheel turns once round in rolling along the ground; the point being supposed the lowest point of the wheel at the beginning and at the end of the turning. The curve thus formed will bear some resemblance to the outline of a very flat arch of a bridge. The curve must be supposed turned upside down and half of it taken when used in the manner of Art. 319.

XXIII. FRICTION.

321. We have hitherto supposed that all bodies are *smooth*, but practically this is not the case, and we must now examine the results which follow from the roughness of bodies.

322. The ordinary meaning of the words *smooth* and *rough* is well known, and a little explanation will settle the sense in which these words are used in Natural Philosophy. Let there be a fixed plane horizontal surface formed of polished marble; place on this a piece of marble having a plane polished surface for its base. If we attempt to move this piece of marble by a horizontal force we find that there is *some* resistance to be overcome; the resistance may be very small, but it always exists. The same thing will appear if we change the material with which we make the experiment, as for instance if we use wood instead of marble, or if we have the fixed plane of one material and the moveable body of another. We say then that the surfaces are not perfectly *smooth*, or we say that they are to some extent *rough*. Thus surfaces are called *smooth* when no resistance is caused by them to the motion of one over the other, and they are called *rough* when such a resistance is caused by them; this resistance is called *friction*, and it always acts in the contrary direction to that in which motion takes place or is about to take place. Although we may imagine *smooth* bodies to exist, yet strictly speaking there is always some degree of roughness in practice.

323. The following is another method of explaining the meaning of the words *smooth* and *rough* in our subject. When bodies are such that if they are pressed together the force which each exerts on the other must be *at right angles* to the two surfaces the bodies are called *smooth;* when this is not the case they are called *rough*. If the two surfaces which are pressed together are both *plane* surfaces this definition is immediately applicable, but if one or each of the surfaces is a *curved* surface some explanation is required. Suppose that one surface is curved and the other plane, as for example when a sphere is pressed against a plane; then a straight line at right angles to the plane at the point of contact is to be considered as also at right angles to the curved surface. Next suppose that each surface is curved, as for example when one sphere is pressed against another sphere; then a plane must be supposed to touch each surface at the point of contact, and a straight line at right angles to this plane is to be considered as also at right angles to the curved surfaces.

324. Suppose we want to *support* a Weight by the aid of a machine; then friction may be said to *help* the Power, for the Weight may be increased beyond the value which according to theory the Power would support, and yet motion may be prevented by the friction. But suppose we want to give motion; then friction may be said to *oppose* the Power, for the Power must be increased beyond the value which according to theory would move the Weight in order to overcome the friction. Suppose we increase the Power sufficiently then we actually overcome the friction and produce the motion which we desire. Thus there is in every case a limit to the friction, and experiments have been made in order to obtain information with respect to the extreme amount of friction which can be brought into action between two surfaces when they are pressed together. The following *Laws* have been thus obtained.

(1) The friction varies in the same proportion as the force with which the bodies are pressed at right angles to the surfaces in contact, so long as the materials of the bodies in contact remain the same.

(2) The friction remains the same whatever may be the extent of the surfaces in contact so long as the force pressing the bodies at right angles to the surfaces is the same.

These two Laws are true not only when motion is just about to take place, but when there is sliding motion. But in sliding motion the friction is not always the same as in the state bordering on motion; when there is a difference the friction is greater in the state bordering on motion than in actual motion.

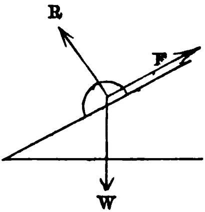
(3) The friction is the same whatever may be the velocity when there is sliding motion.

325. *Coefficient of Friction.* Let two bodies be pressed together by any force at right angles to the surfaces in contact, and let us try to make one body slide on the other by a force parallel to the surfaces, increasing the force we apply until it is *just sufficient* for the purpose; then the proportion of this transverse force to the force at right angles to the surfaces is called the *Coefficient* of *Friction.* For example suppose that two bodies are pressed together

by a force of 10 pounds, and that we can just make one body slide on the other by a force of 3 pounds; then the coefficient of friction is $\frac{3}{10}$.

326. The following results have been obtained by experiment; they apply to the case of actual motion. The coefficient of friction for iron on stone is between $\frac{3}{10}$ and $\frac{7}{10}$; for timber on timber between $\frac{1}{5}$ and $\frac{3}{5}$; for metals on metals between $\frac{3}{20}$ and $\frac{1}{4}$. Thus, for example, if two metallic bodies are pressed together with a force of 100 pounds, then in order to keep one in motion over the other we must exert a force between $\frac{3}{20}$ of 100 pounds and $\frac{1}{4}$ of 100 pounds, that is between 15 pounds and 25 pounds. The precise amount of force will depend on the nature of the metals and the degree of smoothness of their surfaces.

327. *Angle of friction.* Let a body be placed on an Inclined Plane; if the plane were perfectly smooth the body would not remain in equilibrium. Let W denote the weight of the body, which acts vertically downwards; let R denote the Resistance of the Plane, which acts at right angles to the Plane; let F denote the Friction, which acts along the Plane. Now, by Art. 246, we know that so long as the body remains in equilibrium, the Weight, the Resistance, and the Friction are in the proportion of the length, the base, and the height of the Plane respectively. Thus the Friction is to the Resistance in the same proportion as the height of the Plane is to its base. Let the Plane be gradually tilted until the body just begins



to slide down the Plane; then in this case the proportion of the Friction to the Resistance is what we call the *coefficient of friction* for the state bordering on motion. Thus we have an experimental method of finding the value of this coefficient. The angle which the Inclined Plane makes with the horizon in the state of the body bordering on motion is called the *angle of friction.*

328. We have hitherto supposed that one body *slides* over another; the friction then may be called for distinction *sliding* friction. There is however another case in which the friction may be called *rolling* friction. Thus a solid cylinder may roll on a fixed plane, or within a fixed hollow cylinder; or a hollow cylinder may roll round a fixed cylindrical axis. It is found by experiment that in this case the friction is very nearly proportional to the pressure, but is much less than for the case of sliding surfaces kept in contact by the same pressure.

329. Friction may be diminished in various ways. Thus we may make the surfaces in contact very smooth; or we may interpose some lubricating material, such as oil or grease, between the surfaces in contact. It is found advantageous to have the bodies in contact of different substances; thus axles may be made of steel, and the parts on which they turn of gun metal or brass; in time-pieces the steel axles often turn on agate or on diamond. We endeavour also as much as possible to avoid *sliding* friction and to introduce *rolling* friction; thus small wheels called *castors* are placed at the feet of tables and chairs for this purpose. Large masses of stone are often moved by the aid of many castors in the form of cannon balls placed under them. The following example has been given to illustrate the diminution of friction by various contrivances. A roughly hewn block of stone weighing 1080 pounds was drawn from the quarry on the surface of the rock by a force of 758 pounds. The stone was placed on a wooden sledge and then a force of 606 pounds was sufficient to draw it over a wooden floor. When the wooden surfaces in contact were smeared with tallow the force necessary to draw the stone was reduced to 182 pounds. Finally when the load was placed on wooden

rollers three feet in diameter the force was reduced to 28 pounds.

330. A very important contrivance is used for diminishing friction in the case of a body which turns round an axis in the way a grindstone does. The axis instead of resting on an immoveable support at each end rests on *friction wheels* as they are called. Two equal wheels are placed parallel and very near to each other; the distance between their centres is less than a diameter of the wheels. Thus at the upper part a kind of an angle is made on which rests one end of the axis of the body which is to turn. The other end rests on another similar pair of wheels. The friction wheels turn with the body, and the friction is found to be much less than it would be if the body turned on immoveable supports.

331. There are cases in which we find the assistance of friction very useful. Thus in frosty weather the iron rails become so slippery that the wheels of a locomotive engine turn round without *biting* the rails, and it is necessary to scatter a little sand on them to obtain the necessary roughness and consequent friction. When first railways were proposed it was maintained by some persons that the friction would always be inadequate to make the wheels bite, and that it would be necessary to cut *teeth* on the wheels and on the rails. Sometimes to procure enough friction we change rolling motion into sliding motion; thus the wheel of a carriage is *locked* when descending a hill in order to moderate the velocity by increasing the friction.

332. A remarkable case of friction is that which occurs when a rope is coiled round a solid body. Thus one end of a rope may be fastened to a barge, and if the rope is coiled two or three times round a strong post the barge will be easily held fast by a very small force at the other end of the rope, in spite of the current of the river in which it may be floating. The whole friction in this case increases very rapidly with the number of coils. Thus for example suppose that when the rope is coiled *once* round a force of one hundred weight supports eight hundred

weight by the aid of the friction; then the same force will support 8 times 8 hundred weight when the rope is coiled *twice* round, and 8 times this when the rope is coiled *thrice* round, and so on.

333. Friction may naturally present itself to the reader at first in the light of an imperfection or obstacle in nature. By reason of friction the simplicity which we should otherwise often see in virtue of the First Law of Motion disappears. By reason of friction our machines never produce so much effect in moving bodies as they would otherwise. Nevertheless it is not difficult to shew that friction promotes in many respects the comfort of man, and a very interesting Chapter is devoted to the subject in Dr Whewell's *Bridgewater Treatise;* from this work the next two Articles are mainly derived.

334. The simple operations of standing and walking would scarcely be possible without the aid of friction; every person knows how difficult and how dangerous they are when performed on ice. Now there is really considerable friction in the case of ice, as we may see by the fact that a stone sliding on ice is brought to rest after it has gone but a slight distance. But the friction on ice is much less than on ordinary ground, and from our experience in moving on ice we may learn how embarrassing would be our condition on a perfectly smooth plane. At every step we take it is the friction of the ground which prevents the foot from sliding back, and thus allows us to push the other foot and the body forwards. And in the more violent motions of running and jumping it is easily seen that we depend entirely on friction for the possibility of the feat. Likewise when we wish to hold things in our hand it is friction which enables us to succeed; and on the contrary it was formerly the custom for wrestlers to rub their bodies with oil that they might be less easily grasped by their adversaries. Again the objects which surround us in our rooms, as chairs, tables, and books, would yield to the slightest push or current of air, and be in a state of perpetual motion if it were not for friction. The stability of our buildings is largely due to friction. It is true that mortar is used to assist in binding the bricks

and stones together, but were it not for friction the strength of the mortar would be always on trial as it were, at every shock and every breeze; and would give way under the long continued strain. But owing to friction the stability would subsist in many cases even without the mortar, and thus the tenacity of the mortar is reserved as it were for extreme occasions.

Were it not for friction rivers that now flow gently would be converted into rapid torrents. By the aid of friction we can form long threads and sheets out of the short fibres of cotton, flax or hemp; for it is friction consequent upon the mutual pressure of the fibres which are twisted together that keeps the material of these fibres together.

335. It is remarkable that friction which is so important in the concerns of the world disappears almost entirely when we turn to the larger motions of the heavenly bodies. All motions on the earth soon stop, but the moon and the planets continue in their courses for ages. So great is the apparent difference that the ancients were quite misled, and divided motion into two kinds, *natural* like that of the heavenly bodies, continually preserved, and *violent* like that of earthly objects, soon extinguished. Modern philosophers maintain that the nature of motion is the same, and the laws the same, for celestial and terrestrial bodies; that all motions are natural, but that in terrestrial motions friction comes into play and alters their character. Moreover there is strong reason for believing that all space is occupied by a medium, which though excessively rare does impede the motions of the heavenly bodies.

XXIV. GENERAL MOTION.

336. We have more than once drawn attention to the circumstance that the *motion* with which we have been concerned is of a simple and restricted kind. We have spoken of it as the motion of a *particle*, and as the motion of a body where all the points move in the same manner, and as excluding all motion of rotation; see Arts. 123 and 285. The motion of bodies considered without this

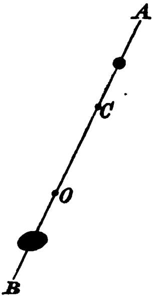
restriction is beyond an elementary work like the present, and we must confine ourselves to a very few remarks respecting it. One of the most simple cases is that of motion round a fixed axis. Take, for example, the diagram of Art. 220, and suppose that P and W are not in the proportion necessary for equilibrium. Then motion ensues; one of the two, P and W, descends and the other ascends, while the piece consisting of the Wheel and Axle turns round a fixed horizontal axis. Suppose that W is larger than it ought to be for equilibrium; then W descends, and it is found by theory that W moves down with a velocity which increases in the same proportion as the time, that is W moves in the same manner as a body falling freely; but the motion is less rapid than that of a free body. Instead of the number 32 of Art. 92 we have now a smaller number, the value of which depends on P and W and on the weight and size of the machine. Also P *ascends* according to the same law, but with another number instead of 32. If the machine is very small and light compared with P and W its own motion will be unimportant, and we have very nearly the same case as that in Art. 142.

337. In the preceding case we have a body which can turn round a fixed axis, and which is kept in motion by the action of *constant* forces, namely P and W. But such motion might be produced by the action of forces which are not constant. For example, in raising water from a well the hand which turns the machine might exert force irregularly, sometimes more and sometimes less; and then the ascending body would no longer move like a body under the influence of gravity only.

338. We have spoken in Art. 312 of a *simple pendulum*, and have defined it as a heavy particle at one end of a fine string, the other end being fixed. But this is rather an *ideal* pendulum than a really existing object. A real pendulum may be defined to be a body of any form which can turn round a fixed horizontal axis.

Let AB be a body of any form, as for instance a rod with two fixed balls, one near each end. Suppose the plane of the paper to be vertical, and to contain the rod; and let C denote the point at which a horizontal axis at

right angles to the plane of the paper passes through the body. The ends of this axis are supported; and the body being drawn away from its position of equilibrium and then left free will move to and fro. Now it is found by theory that the motion of this real pendulum is exactly the same as that of a simple pendulum of some definite length which can be calculated when the form and the substance of the body are known; this length is called the length of the *equivalent simple pendulum.* If we measure along the line through C and the centre of gravity of the body, a distance CO equal in length to the length of the equivalent simple pendulum, then O is called the *centre of oscillation,* while C is called the *centre of suspension.* The centre of gravity of the body will be at some point between C and O.



339. It is remarkable that the *centres of oscillation and suspension are convertible;* this means that if the body instead of turning round the horizontal axis at C turns round a parallel axis at O, then C becomes the new centre of oscillation.

340. The position of the centre of oscillation can be determined as we have said by theory; but it may also be found by experiment. For example if a slender rod oscillate about an axis through one end at right angles to the rod it is found that it oscillates in the same time as a simple pendulum *two thirds* of the length of the rod. Thus the centre of oscillation is distant two thirds of the length of the rod from the fixed rod. The statement can be verified by making the rod oscillate about an axis through the point thus assigned; then by Art. 339 the time of oscillation will be the same as before.

341. The rule found by theory for the length of the equivalent simple pendulum in the case of any body is the

following. Suppose the body to consist of any number of equal small particles, then the required length is a fraction to be calculated thus: The numerator is the sum of the squares of the distances of the particles from the horizontal axis; the denominator is the sum of the distances of the particles *below* the horizontal *plane* through the axis from that plane, diminished by the sum of the distances of those *above*, when the body is in its lowest position.

342. The centre of oscillation does not necessarily fall within the body. It is obvious from the diagram of Art. 338 that the parts of the body above and below C respectively are always tending by their weights to move the pendulum in *contrary* directions, so that if these two parts are so adjusted as to produce nearly equal effects the motion may be very slow indeed, and thus CO may be very long and consequently the point O quite beyond the body. Musicians use a small pendulum called a *metronome* for the sake of marking time; though very short it can be made to oscillate in a second or even in a longer time. It is of the form represented in the diagram, namely a rod with balls at the ends. The upper ball can be moved to any position which may be desired, and held fixed in that position by a screw; thus the metronome can be made to oscillate at a quicker or slower rate as may be required.

343. It will be observed that a heavy body oscillates in the same manner as if the whole mass were collected at the *centre of oscillation*, not as if it were collected at the *centre of gravity*. It is true that the *weight* of a body may always be supposed to be collected at the centre of gravity; for this amounts merely to the substitution of a *resultant* force instead of the *component* forces to which it is equivalent. Beginners sometimes incautiously imagine that the *mass* of a body may always be supposed to be collected at the centre of gravity; but the present case shews that such a statement is too wide: see Art. 169.

344. The following very important result is demonstrated by theory with respect to the motion of any body. *The motion of the centre of gravity of a body is exactly the same as the motion of a particle having a mass equal*

to the mass of the body and acted on by forces equal and parallel to those which act on the body. The reader will scarcely be prepared to understand completely this very remarkable statement, but even an imperfect notion of it will be of service. Take, as a simple example, a top spinning and moving on the ground. There are various forces acting, the weight of the top, the resistance and the friction from the ground, and the resistance of the air. Suppose all these forces moved up to the centre of gravity of the body, each force remaining parallel to its original direction, and let their resultant be found; then if this resultant act on a particle of the same mass as the whole top the motion will be the same as the actual motion of the centre of gravity of the top.

345. Another result of the same kind is the following. *The motion of a body round its centre of gravity is the same as the body would take if its centre of gravity were fixed and the body were left to turn round under the influence of the forces really acting.* But this, like the former proposition, is beyond the range of an elementary work.

XXV. FLUIDS.

346. Two opposing principles are found to operate extensively throughout the material world; one is the principle of *cohesion* which tends to bind the component particles of bodies together, and the other is the principle of *repulsion* which tends to separate the particles. The principle of cohesion is perhaps connected with that of attraction between bodies at a distance; the principle of repulsion is perhaps identical with heat, or at least intimately connected with it. Now the three forms under which matter presents itself depend upon the relative influence of these two principles. In solid bodies cohesion prevails over repulsion, so that the particles form one connected mass, not to be separated without the application of force. In air and gases the principle of repulsion predominates, and the particles require the application of force in order to keep them in contact or near each other. The third form of matter, namely that of water and other liquids, is one in which neither of the two principles is predominant; the particles can be separated by the application of forces so slight as to be practically insensible, but they do not require to be confined in *every* direction, like those of air and gases to prevent them from escaping.

347. The term *fluid* includes two classes of objects, namely *liquids* like water, and *gaseous* bodies like air; the two classes have some properties in common, and each class has also some of a special kind. We shall treat first of liquids and then of gases. The most common liquid is *water*, and this may be taken as the type of all. Hence the science which we are about to consider has received names derived from the Greek word for water. The term *Hydrostatics* has been applied to all that concerns the mechanical properties of liquids in equilibrium, and *Hydrodynamics* to the subject of liquids in motion: the term *Hydraulics* is sometimes applied to the theory of machines which depend on the action of liquids.

348. The general properties which we are about to consider are those which belong to what are called *perfect* liquids. By perfect liquids we mean such as offer no resistance whatever to the separation of their parts, and on this account adapt themselves to the shape of the vessels containing them. Strictly speaking no liquids are *perfect;* but still for water and other liquids there will be no error of practical importance introduced in the statements we shall make. There are however substances which though liquids are far from being *perfect* in the sense we have explained, and to which therefore our subsequent statements will not apply; for instance tar or melted glue. Water approaches more nearly than oil to the idea of a perfect liquid, and alcohol more nearly than water.

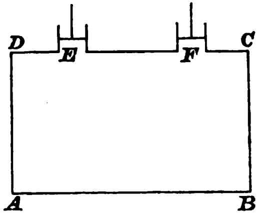
349. It was formerly supposed that liquids were *incompressible;* that is to say it was held that a liquid could not have its bulk diminished by any pressure however great. An experiment was made at Florence, and thence known as the *Florentine experiment,* which seemed to confirm this notion. Water was enclosed in a hollow globe of silver; the globe was squeezed in such a manner as to alter its form, and therefore by the conclusions of Geometry to diminish its size, and it was found that the water was forced through the pores of the silver. But it is now well ascertained that water is compressible, though the compressing force must be very great in order to produce a sensible effect. The standard fact may be put in the following form, which will be fully comprehended as the reader proceeds with the subject: *water when pressed by a column of water* 33 *feet high has its density increased by* ·000046 *of its original density.* Also the increase of density will be in proportion to the pressure; so that under the pressure of a column of water 3300 feet high the density would be increased by ·0046 of the original density, and under the pressure of a column of water 7000 feet high the density would be increased by about ·01 of the original density. Or we may put the last fact in this form: at the depth of 7000 feet in the sea a mass of water will lose ·01 of the bulk it would have at the surface of the sea. When the force which compresses a

liquid is removed the liquid regains its original bulk and density.

350. We may if we please imagine that a liquid is composed of very small smooth spherical particles, and thus connect the properties of a liquid with those of an assemblage of particles; but such a supposition is not necessary for our purpose.

XXVI. PRESSURE TRANSMITTED IN ALL DIRECTIONS.

351. The foundation of all we have to teach about liquids is a principle which seems to have been first enunciated by Pascal; it is called the *transmissibility of pressure in every direction.*

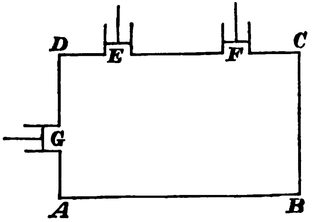


Let $ABCD$ be a vertical section of a closed vessel full of liquid. At two places in the upper surface, E and F, let there be *equal* holes in which are placed tubes of equal bore; the holes and the tubes may for simplicity be supposed circular. In these tubes let there be pistons which can work easily up and down, remaining water tight, like the moveable part of a boy's squirt. Push one of these pistons down with a certain force; say that the piston at E is pushed down with a force of one pound. Then it will be found on trial that the piston at F is thrust up, and if we wish it to stop in its place we must push it down also with a force of one pound. In other words, if we apply any force on a part of the upper surface of the liquid in the closed vessel, that force is as it were transmitted in equal amount to any other equal part of the same upper surface.

352. Next let the tubes at E and F be of *unequal* bore; suppose the area of F to be double the area of E. Then it will be found on trial that if the piston at E is pushed down with a force of one pound, and we wish to keep the piston at F in its place, we must push it down with a force of *two* pounds. This is an immediate result from the principle of Art. 351; for according to the principle a pressure equal to that exerted on the piston at E is transmitted to each of the portions of the same area of the piston at F. In like manner if the area of the tube at F is ten times the area of the tube at E, then when the piston at E is pushed down with a force of one pound the piston at F must be pushed down with a force of ten pounds if we wish to keep it in its place.

353. The preceding two Articles supply rather an illustration of the meaning of the principle of the transmissibility of pressure than a mode of establishing it very strictly. For in practice there would be friction which would impede the motion of the pistons, and prevent the accurate accordance of the facts with the theory. But the truth of the principle may be readily admitted, as it will be confirmed by numerous results which can be deduced from it and verified by trial.

354. We have hitherto supposed the two pistons to be placed in the upper surface of the vessel. But suppose we have a piston at G, a place in the side of the vessel; let this be of equal area with the pistons at E and F. It will



be found on trial that even if we exert no force at E and F the piston at G will be thrust out; this arises from the weight of the liquid, as we shall see in the next Chapter.

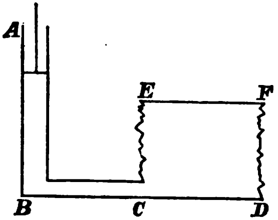
Suppose that a force is applied to the piston at G, *just sufficient* to keep it in its place, so that the liquid remains in equilibrium. Let now the piston at E be pushed down with any force, say a force of one pound; it will be found, as we said before, that to preserve equilibrium the piston at F must be pushed down with a force of one pound: and moreover we must push in the piston at G with a force of one pound in *addition* to the force already exerted on it. Thus the force applied at E is transmitted to the equal area at G. Also if the area of the piston at G is ten times the area of the piston at E, then when a force of one pound is applied to the piston at E we must in order to preserve equilibrium apply a force of ten pounds to the piston at G, in *addition* to the force which it was necessary to exert to keep this piston from being thrust out before any force was applied to the piston at E.

355. In our illustration we have supposed the tubes at E and F to be *vertical*, and that at G to be *horizontal;* but the principle is not to be restricted to these cases. The side of the vessel in which the tube is supposed to be inserted need not be necessarily either horizontal or vertical, but may be inclined at any angle to the horizon. Still the result will hold, namely, that when equilibrium has been obtained by applying proper forces to the pistons, then if any additional force be applied to one piston we must apply an equal additional force to every portion of the same area in all the other pistons, in order to maintain equilibrium.

356. The principle of the transmissibility of pressure through a fluid explains the action of a little contrivance which is called the *hydrostatic paradox;* the name is given because at first sight the effects seem out of proportion to the causes in action.

CD and EF are flat boards, which are connected by flexible leather, or cloth, so as to form a water-tight vessel. AB is a vertical tube which communicates with the vessel. Let the vessel and a part of the tube be filled with water, and suppose a piston to work in the tube and to be retained in its place by a suitable force. Suppose, for an example, that the area of the bore of the tube is one square inch, and that the area of the upper board EF is

a thousand square inches. Then if the piston is pushed down with an additional force of one pound the board EF



will be thrust upwards with a force of a thousand pounds; so that in fact the board would support the weight of a thousand pounds placed on it without sinking down. It will be seen after reading the next Chapter that instead of using a piston in the tube AB the required force may be obtained by making the column of water in the tube of sufficient height. We shall see hereafter that the principle of the hydrostatic paradox is the essential part of a valuable machine called *Bramah's Press.*

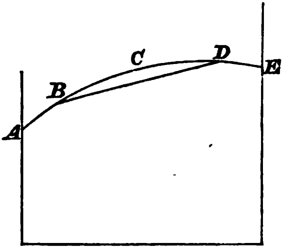
The important principle of Art. 208 applies here, namely that what is *gained in power is lost in speed:* for if we were to force the piston in the tube down through one inch the board EF would ascend through only one thousandth of an inch.

XXVII. PRESSURE FROM THE WEIGHT OF LIQUIDS.

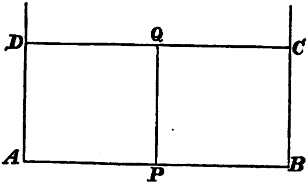
357. We have hitherto considered liquids as contained in closed vessels and transmitting to all points any pressure which may be applied at their surfaces; but we have now to treat of the pressure produced by the *weight* of liquids.

358. Suppose liquid put into a vessel open at the top; then the upper surface will be a *horizontal plane.* That it is a *plane* surface is obvious from common observation. To

say that it is a *horizontal* plane means that it is at right angles to the direction of gravity, and this may be established by an easy experiment. If a plumb line be hung over the surface of a liquid at rest the eye can discern that the direction of the plumb line and the direction of its image reflected in the liquid seem to fall in the same straight line; and when the student is acquainted with the elements of Optics he will know that this shews the surface of the liquid to be at right angles to the direction of the plumb line. The result may also be established by reasoning. Suppose the surface of a liquid to be curved, as denoted by *ABCDE*. Consider a portion of the fluid *BCD* such as would be cut off by a plane *BD* inclined to the horizon. Then this portion would be like a body placed on a smooth Inclined Plane, acted on only by its own weight; and so it would not be in equilibrium but would run down the Plane.



359. Let there be an open vessel with vertical sides containing liquid. Consider any portion of the area of the



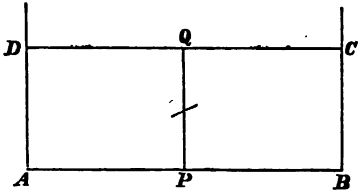
base, as for instance, one square inch near the point *P*; then the liquid itself produces on this square inch a pressure equal to *the weight of a column of the liquid, of which the area of the base is one square inch, and the height is the vertical depth of* P *below the surface of the liquid.* Thus if the depth *PQ* is 28 inches, and the liquid is water, the pressure on a square inch of the base at *P* would be

equal to the weight of a column of water of which the base is one square inch and the height is 28 inches. Such a column would contain 28 cubic inches of water. Now a cubic foot of water weighs about 1000 ounces Avoirdupois, so that a cubic inch weighs $\frac{1000}{1728}$ ounces, and 28 cubic inches weigh $\frac{28000}{1728}$ ounces, that is about a pound. It will be convenient to remember that a column of water of which the area of the base is a square inch and the height is 28 inches weighs about a pound Avoirdupois.

360. But let us advert to the evidence for the truth of the preceding statement. We might contrive some experimental test. For instance the vessel might be placed high on supports at its corners, so as to allow of easy access to the base; then a tube might be inserted at P in which a piston should work; and the force necessary to sustain this piston in its place could be found by trial. Or we might adopt some methods of reasoning. For instance the sides of the vessel being vertical it seems obvious first that the *whole pressure* on the base must be equal to the *whole weight* of the liquid, and next that the pressure on any assigned part of the base will be proportional to the area of the part; and from these two natural suppositions the result will follow. There is also a method of reasoning which may appear somewhat artificial to the reader at first, but which well deserves attention as it is very useful in the theoretical investigations of the subject. Consider a vertical column of the liquid which has for its base an area of a square inch at P, and reaches up to the surface of the liquid. *Conceive this to become solid;* then we may take it as obvious that the pressure on the square inch is not altered. The weight of this solid column must be supported by the resistance of the base, which is equal and opposite to the pressure the liquid exerted on the base. For the liquid around the column will exert pressures on it only in *horizontal* directions, and so will in no degree counteract the weight of the column. Thus finally the pressure on the square inch of area at P is equal to the weight of the column of liquid standing on this square inch of area as base.

361. Next suppose a plane area of one square inch to be placed at any point between P and Q, in a *horizontal* position. The pressure on one side of it, say the upper side, will be equal to the weight of the column of liquid above it. This will appear obvious on reflection. We might suppose all the liquid *below* the plane area to become solid, and allow that the pressure on the plane area would remain unchanged: then this case reduces to the former.

362. Next suppose that at any point of the liquid we put an area of one square inch *inclined to the horizon*. The



pressure on one side will be the same in amount as if the area were horizontal and *at the same depth*. The words *at the same depth* are used for brevity; they require a little explanation in order to bring out their strict sense. The depth of the inclined plane must be understood to mean the *average* depth, that is the depth of the *centre of gravity*. It would not be easy to obtain a very simple direct verification of this statement; but we may give an experimental illustration which will serve to render the meaning clear. Let there be a flat piston moving in a tube closed at the bottom and quite water-tight; and in the tube let there be a spring which resists the motion of the piston, so that a certain pressure must be exerted on the piston to maintain it at a certain position in the tube. Put the whole under the surface of the liquid; then the pressure exerted by the liquid pushes the piston in until there is equilibrium between the pressure and the resistance of the spring. Then for all positions of the piston so long as the *centre of gravity* of its area remains at the *same depth* the piston will remain in equilibrium.

363. All the results we have given in this Chapter are obtained on the supposition that the upper surface

of the liquid is left free. If a lid is put on the upper surface, and pushed down, this gives rise to an *additional* pressure which is transmitted to every point of the liquid. It will appear hereafter that the atmosphere produces a pressure of about fifteen pounds on every square inch of surface exposed to it; and this pressure is transmitted through the liquid to a square inch of area placed in any position within the liquid.

364. But at present we leave out of consideration the action of any other force except the weight of the liquid itself; and the results at which we have arrived may be summed up briefly thus: *the pressure at any point of a liquid is proportional to the depth of the point below the surface, and is the same in every direction.* Like many other brief statements this would be scarcely intelligible without previous explanations. We measure *pressure at any point* by the pressure *on* a certain small area, say a square inch, so placed as to have its centre of gravity at the point; and when we say that the pressure is the same in *every direction* we mean that this area may be placed at any inclination to the horizon.

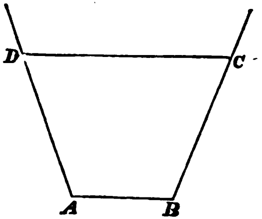
365. The fact that the pressure is the *same in all directions* round any assigned point, to which we have just drawn attention, is quite distinct from the fact that liquids *transmit* pressure from one point to another: both are very important properties of liquids.

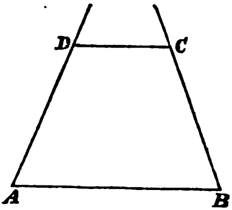
366. The reader will observe that we speak of the pressure of a liquid *at* a point and not of the pressure *on* a point; in order to form a notion of the pressure of a liquid we must suppose that it is exerted on some definite area; this area may be very small, but it is not what is called a *point* in geometry.

XXVIII. VESSELS OF ANY FORM.

367. In the preceding Chapter we supposed liquid to be contained in a vessel with *vertical* sides; but we must now proceed to some other cases.

Let us suppose liquid to be contained in vessels which have sides that are not vertical; these sides may slope *outwards* as in the left-hand side diagram, or *inwards*



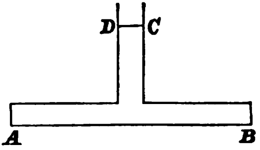


as in the right-hand side diagram. The two results which were obtained in the preceding Chapter, and summed up in Art. 364, are still true, and thus we shall be led to some curious and important consequences.

368. Consider the case represented by the left-hand side diagram. The pressure on the *base* of the vessel is equal to the weight of such a column of the liquid as would stand vertically over the base; thus it is *less* than the weight of all the liquid contained in the vessel. The weight of the liquid contained in the vessel is equal to the vertical component of the pressure on the vessel; but this does not fall entirely on the base; part falls on the inclined sides. Next consider the case represented by the right-hand side diagram. The pressure on the *base* of the vessel is equal to the weight of such a column of the liquid as would stand vertically over the base; thus it is *greater* than the weight of all the liquid contained in the vessel. In this case, as in the former, there is pressure by the liquid on all the vessel in contact with it, and therefore resistance from the vessel on the liquid. But in this case the vertical component of the resistance from the inclined sides tends *downwards;* and the difference between this and the resistance of the base *upwards* is equal to the weight of the liquid in the vessel.

369. The following is the general result. Let there be a series of vessels all having flat bases of the *same*

area, all *open* at the top, and filled with the same liquid up to the same *height;* then the pressure on the *base* of *any* vessel will be the same, namely the weight of such a column of the liquid as would stand vertically over the base. The vessels may have any shape whatever; they may be like cups, or jugs, or decanters, or pails; and the opening at the top may be as small as we please. It is plain that we have thus a fact of the same nature as that involved in the *Hydrostatic Paradox.* Suppose that the vessel is in the form suggested by the diagram, large and shallow, with a tall slender neck. Pour in liquid until it fills all the shallow part and the neck up to *CD*. Then the pressure on the base of the vessel, represented by *AB*, is equal to the weight of such a column of the liquid as would stand on this base and reach up to *CD*: and it is obvious that this may be many times as large as the weight of all the liquid contained in the vessel.

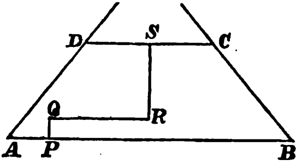


370. The reader will observe that the pressure about which we are speaking is the pressure of the liquid on that side of the base with which it is in contact, and not the pressure between the other side of the base and the table or ground on which the vessel may be supposed to stand. The latter is equal to the sum of the weights of the vessel and the liquid which it contains, by the ordinary principles of mechanics.

371. Experimental evidence can be furnished of the truth of the general statement of Art. 369. Vessels are constructed of various shapes, as suggested in that Article, and having bases of the same area. These bases are not *fixed* to the sides of the vessels, but are kept in contact with them by forces which can be exerted by means of a lever. Then it is found that when the vessels are filled up to the same height the same force must be exerted in every case in order to keep the moveable base in its place.

372. The theoretical demonstration of the statement is so simple that it well deserves the little attention which is necessary in order to understand it.

Let P be any point in the base of a vessel containing liquid: we wish to shew that the pressure on an assigned area at P is equal to the weight of a column of the liquid which would stand on that area, and reach up to the surface of the liquid CD. If a vertical straight line can



be drawn *in the liquid* from P to the open surface the proposition has been already established, namely, in Art. 359; the case which we have to examine is that in which this vertical straight line cannot be drawn in the liquid owing to the inclined sides of the vessel. In this case however it will be possible to pass from the point P to the open surface by a zigzag composed of vertical and horizontal straight lines; thus in the diagram we have PQ and RS vertical, and QR horizontal.

Now in the first place the pressure at R is known by Art. 359; it is proportional to the depth RS.

Next we shall shew that the pressure at Q is the same as the pressure at R. For suppose the liquid in the form of a slender horizontal rod along QR, with parallel vertical ends, to become solid; the pressures on its ends are the only *horizontal* forces acting along the rod, and these must therefore be equal for equilibrium.

Finally the pressure at Q being equal to the pressure at R the column of liquid PQ is in precisely the same circumstances as it would be if it were placed vertically under RS instead of in the position it occupies. Hence the pressure on the assigned area at P is precisely the same as it would be if a vertical straight line could be drawn *in the liquid* from P to the open surface.

373. Thus the pressure of the liquid on the base of any vessel, which is open at the top, is equal to the weight

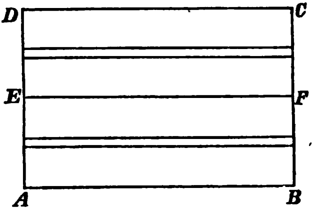
of such a column of the liquid as would stand vertically over the base, and reach up to the open surface. The pressure may be supposed to act at the centre of gravity of the base : see Art. 172.

XXIX. PRESSURE ON THE SIDES OF VESSELS.

374. We have now sufficiently considered the pressure on the *base* of a vessel containing liquid; we proceed to the pressure on the *sides*. The fact that the pressure increases as the depth increases suggests an obvious practical remark with respect to constructions which are intended to resist the pressure of liquids. Suppose we have to carry a canal across a low valley, so that it is necessary to make embankments to serve as artificial sides for the canal. Since the pressure of the liquid increases in the same proportion as the depth, the strength of the embankment ought also to increase with the depth: thus the embankment should be wide at the bottom, and may become gradually thinner towards the top.

375. Again, the pressure in a liquid depends on the *depth* but not at all on the *length* of the vessel in which it is contained. Hence if the water of a pond or canal is to be restrained at one end by a flood-gate or dam, it will not matter whether the channel of water is a few yards or a mile long, so far as the flood-gate or dam is concerned; the pressure is the same on it in the two cases. This is a fact which often seems very puzzling to persons who have not attended to natural philosophy; they do not consider that when the channel is lengthened so as to involve more water the sides are also lengthened which confine it, so that there is no necessary increase of pressure on the end. It must be remembered however that the statement assumes the water to be *at rest:* if the water is liable to be thrown into commotion by the wind or other causes it is plain that a large mass of water will in general produce more impression on the restraints than a small mass.

376. Let $ABCD$ represent a *vertical* side of a vessel, which is in the form of a rectangle; AB is supposed at the bottom, and CD at the surface of the liquid. Let

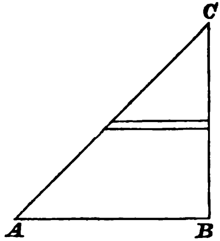


EF be parallel to the top and bottom, and midway between them. Take two equal and parallel strips of the side, one as much below EF as the other is above it; the pressure on the former is *greater* than it would be for an equal strip close to EF, and the pressure on the latter is just as much *less*. Hence the *sum* of the pressures on the two strips is the same as if they were both placed close to EF. Proceeding in this way we see that the pressure on points in EF may be called the *average* pressure all over the side; and the whole pressure is the same as if the whole side were at the depth of EF. Thus the whole pressure on the side is equal to the weight of a column of the liquid having the vertical side for base, and half the depth of the side for height. For instance, if the vessel is a cube open at the top and full of liquid, the whole pressure on one side is just half the pressure on the base.

377. The pressures on all parts of the plane side are *parallel*, being all at right angles to the plane; hence in this case the *whole pressure* is the same thing as the *resultant pressure:* see Art. 166.

378. We know that for every system of parallel forces there is a *centre* at which the resultant of the whole system may be supposed to act: see Art. 166. When the parallel forces are the pressures of a liquid on a plane this point is called the *centre of pressure.* In the case in which the plane is the rectangular side of a vessel full of liquid the position of the centre of pressure can be easily determined.

For join the middle point of CD to the middle point of AB; then the centre of pressure must be at some point of this straight line, because the pressure on each horizontal strip may be supposed to act at the middle point of the strip. Thus the only question is how far down this straight line the *centre of pressure* will be; and the answer is *two thirds of the way down*, so that its distance from the top will be twice the distance from the bottom. In fact the problem of finding the *centre* of these *pressures* is the same as that of finding the *centre of gravity of a triangle*. For suppose a triangle ABC, such that AB and BC are the same as in Art. 376. Divide this triangle into narrow strips parallel to the base, all of the same width. Then the size of these strips will increase in just the same proportion as their distance from C, that is in just the same proportion as the pressures of the liquid on the successive strips into which we may suppose the side $ABCD$ in Art. 376 to be divided. Hence the *weights* of the successive strips of the triangle will represent the *pressures* on the successive strips of the side of the vessel; and thus the centre of pressure will be as far down in the side of the vessel as the centre of gravity is in the triangle; that is two thirds of the way down: see Art. 172.



379. We have hitherto supposed the rectangular side of the vessel to be *vertical;* but similar considerations apply to the case in which the rectangular side is inclined to the horizon. As in Art. 376 we shall find that the *average* pressure is that along the middle horizontal line of the rectangle, and is measured by the vertical depth of this straight line below the surface of the liquid. Thus the whole pressure on the side is equal to the weight of a column of the liquid having the side for base, and half the vertical depth of the side for height. The position of the centre of pressure is the same as if the side were vertical.

380. We need not pursue the subject further, but we may state a general result that is obtained by theory. If a *plane area* of any form is immersed in a liquid the pressure is the same at all points if the area is in a horizontal position; but if the area is not in a horizontal position the pressure is greater as the vertical depth becomes greater. The *average* pressure is that at the *centre of gravity* of the plane area. The whole or resultant pressure is equal to the weight of a column of the liquid having the plane area for base, and the vertical depth of the centre of gravity of the plane area for height. No simple rule can be given for determining the position of the *centre of pressure.*

381. The preceding result admits of a certain extension, which, though of no practical importance, requires notice, for it is sometimes given in books in such a manner as might mislead an incautious reader. Suppose a body having a *curved* surface, for example a sphere, to be immersed in a liquid. Or suppose a vessel in the form of a curved surface, for example a common bowl, to contain liquid. It is still true that the *sum* of the pressures of the liquid on the curved surface is equal to the weight of a column of the liquid having this surface for base, and the vertical depth of the centre of gravity of the surface for height. But we must remember that it is the *sum* of the pressures and not the *resultant* of them which has this value; the pressures not being all parallel, their sum and their resultant are altogether different. Now there is no special mechanical importance belonging to the *sum* of a set of forces, though there often is to their *resultant:* hence this proposition relative to the *whole* pressure on any *curved* surface is really of no practical value.

382. One remark may be placed here, which will be of use as we proceed. Suppose a mass of liquid at rest in a vessel, and fix the attention on any definite portion of this mass; the portion may be in the form of a cube or of a sphere, or of any body whatever, regular or irregular. The liquid surrounding it will exert pressures all over it, but as the definite portion remains in equilibrium the resultant of all these forces must be a *vertical force,*

equal to the weight of the definite portion, and passing through its centre of gravity. For if these conditions are not satisfied, the definite portion of the liquid cannot be at rest; it would not be at rest even if it were solid, but would go up or down or turn round; and so it will not be at rest when it is liquid.

XXX. LIQUIDS STAND AT A LEVEL.

383. We have shewn that the surface of a liquid in equilibrium in a vessel is a horizontal plane. Now suppose we put a liquid into a vessel composed of two vertical tubes connected by a horizontal tube. The surface of the liquid in each tube will be a horizontal plane as we have

already stated; and moreover the two surfaces will be in the *same horizontal* plane; thus if AB and CD denote the surfaces of the liquid in the tubes then AB and CD are in the *same horizontal plane.* This last fact we have not hitherto explicitly stated, though it is intimately connected with some of our previous results; the fact in its various forms is expressed by saying that liquids *seek their level.* It amounts to this: if liquid can pass from one vessel to another by means of a connecting channel it will do so, until the upper surface of the fluid is throughout in the *same* horizontal plane.

384. The preceding statement admits of easy experimental illustrations. It will be found, for instance, to hold with respect to a common tea-pot and its spout. If only a very small quantity of water is put into the tea-pot it may remain below the point of communication with the spout; but when more water is added it will pass into the spout, and then it will stand at the same level in the two parts of the vessel.

385. The fact is closely connected in theory with two others which have come before us. We have shewn

in Art. 362 that the pressure at any point inside a liquid is in proportion to the depth of the point below the open surface; and we have shewn in Art. 372 that the pressure is equal at any two points in the same horizontal plane. Now these two statements would not be consistent with each other unless the liquid in communicating vessels stood at the same level. All the facts too are connected with the principle of Art. 184 that for stable equilibrium the centre of gravity should be as low as possible; for instance if the liquid in different communicating vessels did not stand at the same level, we could bring the centre of gravity of the whole to a lower position by taking liquid from the place where it stood highest and putting it into another vessel in a lower position.

386. So long as we keep within a few yards of the same spot on the earth liquid in a vessel or in a small pond has its surface practically a *plane*. But this is not true with regard to large expanses of water; we know for instance that the Pacific Ocean must be curved into a hemispherical form, and even for lakes of moderate size the deviation from a plane may be recognized. Thus, suppose a circular lake of four miles in diameter; if an accurately straight line could be made to pass from a point just in the circumference of the boundary to a point on the circumference diametrically opposite, it would dip under the surface of the water, and at the middle of the lake would be about 32 inches below the surface.

387. Thus for a large expanse of water the surface is not plane but curved. This leads us to give a strict definition of a *level surface;* it is such that at all points of it the pressure has the same value, and the direction of the force of gravity is at right angles to the surface. The level surfaces are very nearly spherical in form round a common centre; the pressure is less at any point of the outer of two such surfaces than at any point of the inner.

388. The properties of liquids which we have considered produce various phenomena that are exhibited on the surface of the globe. Water confined in a pond or lake maintains itself at rest, and takes a level surface, practically plane if the confined space is small, but other-

wise curved into a nearly spherical form. If however there be an outlet for the water, as the particles have little cohesion they yield to the force of gravity and descend. Thus rain falling on the tops of mountains, if the soil is not soft and easily penetrable, collects in rills which unite and form larger streams. These descend along the sides of mountains and mix with others so as to produce rivers. The course is determined by the nature of the ground, and the general tendency of the water to descend. It is found that if the descent be about a foot in four miles the stream in a straight channel would flow at the rate of about three miles in an hour: the average slope of the large rivers of the world is greater than this. It belongs to Physical Geography to describe the various peculiarities which rivers present in their courses from the mountains in which they rise to the seas into which they fall, such as the cataracts which they form when they change their level suddenly and violently, and their occasional disappearance and reappearance after flowing for some time underground.

389. A canal is an artificial channel of water made to connect two places. If the two ends are not in the same level surface the entire course cannot be in one level surface; and even if the two ends are in the same level surface it may be difficult or impossible to construct the canal entirely on one level owing to the presence of mountains. If the canal were one unbroken channel the water would descend from the higher parts leaving them dry, and would overflow the banks at the lower parts. To obviate this the canal in part of its course consists of separate portions called *locks* which stand at different levels, and which are separated from each other by flood-gates. When a boat is taken through this part of the canal a communication is opened between the compartment in which the boat is and that into which it is to pass: the water in the two compartments is thus brought to the same level, the gates between them are opened and the boat is drawn onwards. Thus every time a boat passes up or down through the locks some water is lost from the highest part of the canal; and the supply must therefore be perpetually renewed by natural or artificial means.

390. The mode in which water is conveyed through our large towns offers an interesting exemplification of the principle that *liquids stand at a level.* A reservoir is formed at as high an elevation as the water is desired to reach; this is kept full by means of water falling into it or being pumped up into it from lower levels. Pipes proceed from the reservoir through the town which is to be supplied, and in any of these the water will rise theoretically to the height which it has in the reservoir; so that it can be brought to the upper rooms of tall houses. Practically however, owing to the fact that water is drawn off for use at various points, the pressure becomes diminished, and so the water will not rise so high as it should according to the theory: this is especially the case during the day-time. The ancient Romans were in the habit of bringing water to their towns from a distance, by means of *aqueducts*, that is by artificial channels constructed on a level surface, or on a gentle descent. Hence it has been supposed that they were not acquainted with the principle that *liquids stand at a level;* but it seems to be now made out that it was not ignorance of this principle but a want of the necessary pipes which kept them from using the modern system. Even in recent times the ancient system has been adopted as possessing some special advantages; an example is furnished by an aqueduct for supplying water to New York.

XXXI. VOLUMES OF SOLIDS IMMERSED IN LIQUIDS.

391. If we wish to determine the volume of a solid body of known regular form, as a cube or a sphere, we have only to use the rules for the process which are given in books on Mensuration. Thus, for instance, the solid body may be in the form of a brick 9 inches long, 3 inches broad, and 2 inches deep; and then we know that the volume is expressed in cubic inches by the product of the numbers 9, 3, and 2; that is the volume is 54 cubic inches. But rules cannot be given for finding the volume of any irregular body, as a stone or a coal. It is a natural consequence

that we usually estimate the quantity of solids by *weight* rather than by *volume*, that is by pounds and ounces rather than by cubic feet and inches. On the other hand liquids, by their property of yielding and filling all the corners of a vessel in which they may be placed, allow us to determine their volumes easily; and accordingly we usually estimate the quantity of liquids by *volume*.

392. But we may also use the fundamental property of liquids, namely their extreme mobility, to determine the volume of a solid. We suppose that the solid will sink in a certain liquid if left to itself; then if the solid be put into a vessel of the liquid it will *displace liquid equal in bulk to its own*. There are various forms in which this fact may be presented. Thus suppose a vessel of sufficient size just full of water; let a solid be carefully dropped in and the water which runs out accurately collected: then this water is obviously just equal in bulk to the solid. The volume of the water collected may be ascertained by pouring it into a vessel which has already been measured and has lines marked on its surface indicating how much it holds when filled up to an assigned level. Or again, take a vessel containing some water, though not full, and observe the level at which the water stands; then put in the solid, which we suppose to go to the bottom and to be perfectly covered by the water. The water now rises to a higher level than before, and the bulk of the solid is exactly equal to that of the water which would be comprised between the two levels. This quantity can be easily calculated if the vessel be of suitable shape; for instance, if the vessel have a rectangular base and its four sides vertical, the volume is found by the rule which we have already exemplified in Art. 391.

393. We have supposed the solid to *sink* in the water, but we know that many substances, as wood for example, will not sink in water. In this case we must press the solid into the water by a slender wire, or by other means. Or we may attach the solid to another of such a nature that both together will sink in water, and thus we can find the volume of both together; then we can find separately the volume of the *sinker*, and finally, by subtraction, the volume of the solid with which we are concerned.

394. In the same manner as we propose to find the *whole* volume of a solid we may also find the volume of any *part* of it we please, provided the part is such as could be cut off by a *plane*. We have only to keep that part with which we are concerned just below the surface of the water, and observe how much of the water runs over if the vessel were originally full, or through what space the level rises if the vessel were originally only partly filled.

395. There are however practical difficulties which may obstruct the process in the case of some bodies. Thus the solid may be soluble in water; then perhaps some other liquid may be found in which the solid is not soluble. Or the water may make its way into the pores of the substance, as it would within a sponge; then perhaps a thin coat of varnish can be applied sufficiently durable to keep out the wet during the short time occupied by the process.

XXXII. WEIGHTS OF SOLIDS IMMERSED IN LIQUIDS.

396. In the preceding Chapter we have treated of the immersion of solids in liquids as affording a method of determining the volumes of solids. In that Chapter there is no *mechanical* principle involved; the whole is a matter of mensuration, that is of elementary Geometry: but we are now about to introduce the reader to some very important mechanical facts. Let us suppose that a person takes a stone weighing about 5 pounds, fastens a string to it, and holds the other end of the string; then he supports the stone, that is he exerts a force sufficient to balance the weight of 5 pounds. Let him now hold the string so that the stone may be immersed in a bucket of water; if the stone rests on the bottom of the bucket it is supported without the exertion of any force by the person. But let us suppose the stone not to touch the bottom of the bucket; in this case the weight is apparently much less than before the stone was immersed, and will seem to the person holding the string to be about 3 pounds. The fact is one which can be easily verified to any extent, and it is universally found that when a heavy body is thus suspended in a liquid in which it would sink if left alone its weight seems *dimin-*

ished; the heavy body is as it were to some degree supported by the liquid. It is customary to say that the solid *loses* a portion of its weight.

397. The next point to settle is the *amount* of this diminution of weight. The following is found to be the law: *when a solid is suspended in a liquid the weight is diminished by the weight of an equal bulk of the liquid.* Or instead of saying *by the weight of an equal bulk of the liquid* we may say *by the weight of the liquid displaced.* This law can be easily verified. In the case of the stone which we considered in the preceding Article the weight can be accurately determined before immersion. Again, when the stone is immersed let the end of the string instead of being held by the hand be fastened to the end of the arm of a balance, or to a spring which serves as a weighing machine; thus the apparent weight can be accurately determined. Therefore, by subtraction, the diminution of weight becomes known. And, as in the preceding Chapter, we can find the bulk of the liquid which is equal to the bulk of the solid; and consequently the weight of so much liquid becomes known. From these results we can make the requisite comparison, and thus the truth of the law which we have stated is established.

398. Besides the direct comparison of weights by which, as we have shewn in Art. 397, the truth of the law is established, there are indirect methods by which we obtain the same result. Before the stone is immersed its whole weight is supported by the hand. Suppose the sides of the vessel are vertical. When the stone is immersed the weight of which the hand is relieved must be thrown on the vessel in some way, and we may naturally infer that in consequence there must be an increase of pressure on the base just equal to this, and therefore the same increase in the pressure of the vessel on the ground or on any supports on which it rests. Take a vessel *full* of water and attach it to some weighing machine; suspend the stone in the vessel; then water runs out equal in bulk to the stone, but the spring weighing-machine does not alter its reading. The relief afforded to the weight of the stone immersed is thus inferred to be exactly equal to the weight of an equal bulk of the water.

399. Or we may establish the truth of the law by reasoning. Suppose a solid immersed in a liquid. The resultant force of the liquid on the solid may be naturally taken to be exactly equal to what it would be on any body of precisely the same size and shape as the solid whatever might be the material of which it was composed. Hence it would be exactly equal to the resultant action on so much of the liquid itself as would occupy just the same space as the solid; and therefore, by Art. 382, this resultant action is a force *upwards* equal to the *weight of the liquid displaced.* Thus the diminution of the weight of the suspended solid is equal to the *weight of the liquid displaced.*

400. We have hitherto supposed the solid *entirely* immersed in the liquid; but similar considerations apply to the case of a solid *partially* immersed. The diminution of weight will be equal to the weight of so much liquid as agrees in bulk with the immersed portion of the solid; or we may say briefly that the diminution of weight is equal to the *weight of the displaced liquid.*

401. Next suppose we have a solid that does not sink but *floats* on the liquid. In this case the *whole weight* is lost, that is the whole weight is supported by the liquid. Hence by Art. 400 we see that when a solid floats on a liquid the weight of the solid is *equal to the weight of the liquid which it displaces.*

402. It will be convenient to state some facts relating to the weight and volume of water which are wanted for numerical applications.

The *grain* is thus determined: a cubic inch of pure water weighs 252·458 grains.

A pound Avoirdupois contains 7000 grains.

A cubic foot of water weighs $1728 \times 252·458$ grains, that is $\frac{16 \times 1728 \times 252·458}{7000}$ ounces Avoirdupois: it will be found that this number to three decimal places is 997·137. Thus it is usually sufficient in practice to take 1000 ounces Avoirdupois as the weight of a cubic foot of pure water.

A gallon is a measure which will hold 10 pounds Avoirdupois of pure water, that is 70000 grains. Hence the

number of cubic inches in a gallon is $\frac{70000}{252 \cdot 458}$; it will be found that this number to three decimal places is 277·274. Thus it is usually sufficient in practice to take 277¼ as the number of cubic inches in a gallon.

XXXIII. APPLICATIONS.

403. The principles of the preceding Chapter lead to various interesting applications and illustrations. One of the most important is the comparison of the weights of equal bulks of various substances. The *specific gravity* of a body is the proportion which the weight of the body bears to that of an equal bulk of some standard substance; and the standard substance is usually pure water at the temperature of 62 degrees of Fahrenheit's thermometer. The mode of determining the specific gravity of solids is in principle that of Art. 397, with due precaution to ensure accuracy. The solid is weighed, the weight of an equal bulk of the water is found; and the former result divided by the latter gives the specific gravity. We shall recur to the process hereafter, and shall consider also the specific gravity of liquids and gases; we give here a few of the results which have been obtained with respect to solids; the figures are to be found to *three* decimal places in various works, but with much diversity; so that it will be sufficient here to go to one decimal place.

Platina	21·5.	Copper	8·9.
Gold	19·4.	Iron	7·8.
Mercury	13·6.	Tin	7·3.
Lead	11·4.	Marble	2·7.
Silver	10·5.	Ivory	1·9.

Thus platina is 21·5 times heavier than water, bulk for bulk; gold is 19·4 times heavier; and so on. Some of these results are liable to a little modification under circumstances; thus for *hammered* gold the specific gravity is nearly 19·4, and for *cast* gold it is 19·26. Since a cubic foot of water weighs very nearly 1000 ounces Avoirdupois we can immediately determine the weights of known volumes of other substances, with sufficient practical accuracy, by means of a *Table of Specific Gravities*. For

instance a cubic foot of iron will weigh 7·8 times as much as a cubic foot of water, that is it will weigh 7·8 times 1000 ounces, that is 7800 ounces. Hence a cubic inch of iron will weigh $\frac{7800}{1728}$ ounces.

404. Platina and gold are comparatively scarce substances, so that we are limited in the use of very *dense* materials, that is of materials which are extremely heavy for a given bulk. The power of man over matter is in many ways great; in particular we shall see when we describe the *air pump*, and some other machines, that we can obtain matter in a state of extreme tenuity; but as yet no means have been found for obtaining matter in a state of extreme density. It is obvious that there are various useful applications which might be made of a very dense substance if such could be readily procured. Thus in forming sea walls, or the foundations of the arches of bridges and other constructions under water, a stone as dense as gold if it could be easily found or composed artificially would be of great service. So, too, a strip of extremely dense material would be very advantageous for the keel of a ship.

405. The support, whether partial or total, which a solid body in a liquid receives from the liquid is a very familiar fact, and is often expressed by the term *buoyancy*. It is obvious that the degree of support depends on the nature of the liquid. Thus the Table in Art. 403 shews that silver, lead, copper, iron and tin will all sink in water, and all float in mercury. So also as oil is a little lighter than water, bulk for bulk, a body might sink in oil which would float on water.

406. The human body when the chest is expanded by drawing in air is rather lighter than an equal bulk of water, so that it would float on water with some portion not immersed. A person who floats has to exercise care to preserve the mouth and the nose above the surface of the water, so as to secure the power of breathing. When the air is expelled from the chest the bulk of the body is sensibly diminished while the weight suffers no appreciable change; and then the body sinks a little in the water.

Thus the body is in a perpetual state of oscillation up and down; the body in sinking falls a little below its equilibrium position, and in rising ascends a little above it. When a man uses his hands to assist himself in swimming or in floating, his proceeding resembles that of a bird when flying; the reaction of the water for the man, and of the air for the bird, is a force which is perpetually urging the body upwards.

407. The buoyancy afforded by water is obvious to persons wading or bathing. In deep water so much is the weight diminished that the feet are scarcely sensible of any pressure. Stones and rocks may be trodden on without inconvenience, which by their sharpness or roughness would cause great pain to a person walking barefoot on them without the support of the water. In trying to ford a river where there is a current men and animals have sometimes been thrown down by a comparatively slight stream, on account of their small pressure on the ground, and consequently the small sustaining friction. Salt water is denser than fresh water, and therefore it is easier to swim and float in seas than in rivers. A cubic foot of salt water weighs about 1027 ounces. An experiment is sometimes performed which shews the difference in buoyancy between salt water and fresh water. An egg will sink to the bottom in a vessel of fresh water, and will float in a vessel of salt water. Let fresh water be poured gently on some salt water in a vessel; then a mixture of the two takes place where they are in contact. Put an egg carefully into the upper part; then it will descend, and after a little oscillation it will rest in a position where it displaces liquid in weight equal to its own. This position of the egg is one of *stable* equilibrium; for if the egg be depressed a little it comes into the place of somewhat denser liquid, and is urged up again; and if the egg is elevated a little it comes into the place of somewhat rarer liquid, and is urged down again.

408. It is possible to give to a body composed of any material such a form that it will float upon water. Thus a basin composed of metal or china will float with the convex side downwards. In this case the body dis-

places as much water as a body of the same shape and weight would do, if instead of being hollow it were completely closed up; at least such is the case if we neglect the insignificant weight of the air which the hollow body holds. A tea-cup may be put into water with its convex part downwards, and it will be observed to float, and it will continue to float if some water be poured into it; by gradually increasing the water we find that before the cup is full it will sink, namely when the weight of the cup and of the contained water is greater than the weight of the water which the whole cup would displace.

409. A ship which is formed of wood might float even if filled with water; for wood in general is lighter, bulk for bulk, than water. But some wood is heavier, bulk for bulk, than water; and in all cases, taking into account the metal used in the construction, the whole weight of a ship may be greater than the weight of an equal bulk of water, so that the ship would sink if filled with water. But as long as the water is kept out the ship floats with a part above the surface of the water. Ships can be made of iron, as we know by constant observation; for increased safety they should be divided into water-tight compartments, so that if the water enters one compartment by a leak or any accident, the water displaced by the other compartments may still keep the vessel afloat. The buoyancy of fluids is seen in a remarkable degree in the case of the great ironclads which are now constructed. Notwithstanding the weight of the armour, several inches thick, which covers the sides, and the additional burden of the enormous guns, the vessels float. The fact is that they are of vast size, and so the weight of the displaced water is very considerable.

410. We may *weigh* a ship by means of the principle that the weight of the ship is equal to the weight of the displaced water. For the form of the ship is in general sufficiently regular to enable us to determine the volume of the displaced water by the aid of rules given for the purpose: see *Mensuration*, Chapter XXXI. And when we know the volume of the displaced water we can immediately find its weight. A simpler calculation of the same

kind will tell us how much *more* freight can be put on a ship, when it already floats in a certain position. The *area of the plane of floatation* means the area of a horizontal section of the ship made at the surface of the water. Now suppose this area is 1000 square feet in the case of a certain ship, and that it is safe to sink the ship an additional foot in the water; then the additional water displaced will be about 1000 cubic feet, and consequently the ship will bear an additional weight equal to that of 1000 cubic feet of water. If this is salt water the weight is about 1027000 ounces, that is 64137 pounds.

411. The use of bladders and corks to enable persons to float is well known. The bladder filled with air is practically of no weight, so that when it is attached to the person and kept below the surface an upward force is obtained equal to the weight of the water which the inflated bladder displaces. In the case of the cork the upward force is equal to the weight of the water displaced diminished by the weight of the cork itself; and the weight of the cork is appreciable, though small.

412. The *pontoons* used in military operations may be noticed. These are simply water-tight casks which are usually made of metal for greater strength. They are put into a river in sufficient number and connected together; they float, and will continue to do so even when laden with heavy weights, so that they can be used as a temporary bridge for the passage of an army and its artillery. The same principle is applied to life-boats; they have round them a hollow metallic tube which by itself would float, and so when fastened to the boat below the surface of the water it gives buoyancy to the boat.

413. A contrivance called a *camel* has been used in Holland for enabling ships to pass over a spot in the water which would otherwise be too shallow. Large chests full of water are fastened to the sides of the ships; the water is then removed and the increased buoyancy enables the ship to float over the shallow spot. The water may be removed from the vessels in various ways. Pumps may be employed; or if the vessels have no tops and their upper parts are above the surface, the water may be drawn out in buckets.

414. The principle of floating bodies is used to regulate the supply of liquids to reservoirs or other vessels. For instance, water is admitted to a tank, and it is required to keep the water always at or below a certain level in the tank. For this a *ball-tap* is the usual contrivance. A hollow ball of metal floats on the surface, and therefore rises as the level of the water rises. This ball can be connected by a wire or a lever with a tap or valve placed at the pipe through which the water enters the tank; and the wire or lever is so adjusted as to close the valve when the water has risen to the prescribed level. A *valve* is a contrivance much used in machines which are connected with fluids; it is a little door which can open or shut so as to allow passage to a fluid through a pipe in *one* direction, but *not* in the contrary direction.

415. The *Diving Bell* is an instrument which we shall describe hereafter; but the reader perhaps already knows that by means of it work may be done under water. For instance, the contents of a sunken ship may be examined and recovered, and the foundations of buildings may be laid in the sea. The workmen find that their power of moving objects seems to be vastly increased under the water; the weight of most stones is little more than half the weight on dry land, so that a man can move a stone nearly twice as great as the largest he could move on dry land.

XXXIV. DIFFERENT LIQUIDS.

416. We have hitherto considered only one kind of liquid at a time, but there are various phenomena connected with the presence of two or more liquids in communication.

417. Suppose that oil and water are mixed together in a vessel; it will be found that after a little time has elapsed the water which is the heavier liquid will occupy the lower part, and the oil which is the lighter liquid will occupy the upper part, and that the boundary between the two liquids will be a *horizontal plane*. It might be possible with great care to get the oil into the lower part of the vessel and the water over it, but the

equilibrium would be *unstable;* any accidental blow would derange the system, and the water would finally get to the bottom. In a similar manner if water be mixed with mercury the mercury will go to the bottom, and the water to the top. If oil, water, and mercury be mixed together the mercury goes to the bottom, the water takes the middle position, and the oil goes to the top; and the boundary between two different liquids is a horizontal plane.

418. It is easy to see that when we have thus two or more liquids in a vessel some modification must be made in the verbal statement of results obtained in the case of a single liquid. We must not say universally, as in Art. 364, that the pressure is proportional to the depth; though this will be true so long as we take points within the highest layer of liquid. The pressure at any point will be measured by the weight of a column consisting of portions of different liquids, namely, of the liquids which occur between the level of the point and the level of the topmost surface. It will still be true that the pressure is the same at all points in the same horizontal plane; and from this we deduce by reasoning that the boundary between two different liquids is a horizontal plane.

419. We suppose that when different liquids are put together they form what is called a *mechanical mixture*, and not a *chemical combination*. The reader may probably know that when two liquids are put together they sometimes form a compound possessing distinct properties of its own, and which cannot be easily separated again into the two liquids from which it arose. An example, though not a very striking one, may be seen in the mixture of wine and water; when such a mixture is made it will not very readily separate itself again like the mixture of oil and water considered in Art. 417.

420. The principle that liquids *stand at a level*, which was explained in Chapter XXX, must now receive a little limitation when different liquids communicate.

Suppose we have oil and water in different vessels, but still in communication. For example, let there be a bent tube; let water occupy the lower part, and suppose

it to rise on the left hand side to the level AB. Let GH be the common boundary of the oil and the water, and suppose the oil to extend from GH up to the level CD. Then AB and CD will *not* be in the same horizontal plane; CD will be higher than AB. We may easily state the relation between the two levels. Let EF be in the same horizontal plane as GH; thus CG represents the height of the oil, and AE the height of the water, above the level of their common boundary GH. It is found that CG is in the same proportion to AE as the specific gravity of water is to the specific gravity of oil. The specific gravity of olive oil is about ·9, so that in this case AE is $\frac{9}{10}$ of CG. This important result can be fully verified by experiment, but the verification is almost unnecessary because the result is an obvious consequence of principles already established. For the pressure at E is measured by the weight of a column of water of the height EA; and the pressure at G is measured by the weight of a column of oil of the height GC; see Art. 359. And the pressure at G is equal to the pressure at E, by Art. 418. Thus finally the weight of a column of water of the height EA must be equal to the weight of a column of oil on a base of the same size and of the height GC. Then since the weights are equal, the height GC must be to the height EA in the same proportion as the specific gravity of water is to the specific gravity of oil.

421. Various illustrations of the principle involved in Art. 417 present themselves. A simple case is the way in which cream is formed by the lighter particles of milk rising to the upper part of the vessel containing it. Again by the application of *heat* a substance is in general *expanded*, so that it becomes lighter, bulk for bulk, than it was originally. Let us suppose that heat is applied at the bottom of a vessel of water; then as the lower layer of water gains heat it expands, and so becomes lighter and rises to the surface. The heavier and colder water on the

other hand descends, and thus in time the heat is communicated to the whole mass of water. The motion may be easily watched, if the vessel be made of glass, by throwing in some coloured particles of about the same specific gravity as the water: for these are carried up and down by the moving fluid. If, however, the heat is applied at the top of the vessel the water at the top is rendered lighter than the rest and so does not descend; in this case although the heat is ultimately communicated to the whole mass of water, yet it is a much slower process than in the former case. On the contrary if we wish to *cool* a liquid the lowering of the temperature should be effected at the top; for then the cooler liquid, being heavier than the rest, descends, and other liquid comes to the top to be exposed to the same cooling influence.

422. When heat is continually applied to water it is found that if the water is in an open vessel its heat cannot be raised beyond a certain point. At this point the water becomes changed into vapour called *steam*. If the heat is applied at the bottom of the vessel the steam is formed there first in the shape of bubbles. Steam is several hundred times lighter than water, bulk for bulk, so that the bubbles rise rapidly to the surface and escape; this is the well-known process called *boiling*.

XXXV. EQUILIBRIUM OF FLOATING BODIES.

423. We have already paid some attention to the equilibrium of floating bodies, but we must now consider the subject more fully. We have shewn that when a solid floats in equilibrium on a liquid the weight of the solid is always equal to the weight of the liquid which it displaces; but as we shall now see something more is requisite to ensure the equilibrium of the solid.

424. Take in the first place a *sphere* of wood, and depress it very gently in water until it has reached a suitable depth; then it will remain at rest. Next take a solid in the shape of a brick, made of wood, and depress it very gently, keeping the *upper face always horizontal;* the same result will happen. But take this brick-shaped solid, and put it into the water *obliquely*, so that it has no

face parallel to the horizon; let it be depressed very gently until the weight of the displaced water is equal to the weight of the solid, and then be left to itself. The solid most probably will not remain in equilibrium, but *will turn over.*

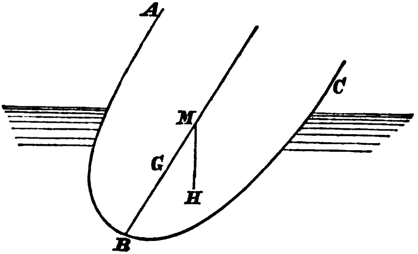
425. In order that a solid may be in equilibrium when floating on a liquid two conditions must be satisfied. (1) The weight of the solid must be equal to the weight of the liquid displaced. (2) The centre of gravity of the solid and the centre of gravity of the liquid displaced must be in the same vertical straight line. The first of these two conditions has been already explained. If a solid be wholly or partially immersed in a liquid it is acted on by two forces, its own weight vertically downwards, which may be supposed to act at its centre of gravity, and a force equal to the weight of the displaced liquid vertically upwards, which may be supposed to act at the centre of gravity of the displaced liquid. If these two forces are not equal the solid will move downwards or upwards according as the former or the latter force preponderates. But suppose that the two centres of gravity are not in the same vertical straight line, then even if the two forces are equal they do not keep the solid in equilibrium because they are not *directly opposed* to each other; they will turn the body round.

426. If a solid composed of materials lighter than water, bulk for bulk, is put into still water we know as an experimental fact that it will at last come to a position of equilibrium. There may be for a time a movement up and down, or a rocking to and fro; but the friction at last stops the motion, and the solid remains at rest. Also even if a body is composed of material which is heavier than water, bulk for bulk, yet by giving to it a hollow form we can in general secure for it a position of equilibrium when put on water. The subject is very important, and is connected with that of the *stability* and *instability* of equilibrium noticed in Art. 182.

427. If we suppose the floating solid to be *symmetrical* in shape, like a sphere, then it is easy to see that the centre of gravity of the floating solid and the centre of gravity of the water displaced do lie in the same vertical

straight line whatever may be the depth of immersion; and thus if this depth be suitably taken the solid will remain in equilibrium. The same remark applies to the brick-shaped solid when one face is kept horizontal. In such cases the equilibrium is *stable* so far as regards any movement up and down. For if the solid is pushed down a little the weight of the water displaced is *greater* than in the position of equilibrium; and so the upward force preponderates, and the solid rises when left to itself. In like manner if the solid be drawn up a little the weight displaced is *less* than in the position of equilibrium; and so the downward force predominates, and the solid descends when left to itself.

428. Let us suppose a ship or such like body floating in equilibrium on water. Let it be tilted by the wind or some other force sideways. Let ABC represent a vertical



section of the ship, taken at right angles to the length, passing through G, the centre of gravity of the ship, and cutting the keel at B. Let H be the centre of gravity of the water displaced by the ship in its tilted position. Then the ship is acted on by its own weight downwards at G, and by a force vertically upwards at H equal to the weight of the water displaced. If these forces are not equal there will be motion upwards or downwards; but this is of small consequence, because by such motion there is a tendency to promote the required adjustment for equilibrium, as explained at the end of the preceding Article. The important question is as to the direction in

which the ship will turn round. Draw a vertical straight line through H, and let it meet BG produced, if necessary, at M. This point M is called the *metacentre*, and in books which discuss the theory of the subject it is shewn how the position of this point may be determined when the amount of tilting is very slight; but the process is not sufficiently elementary for our purpose. We may however easily see the importance which attaches to the position of the point M. Suppose M to be, as in the diagram, *above* G. Then it may be taken as tolerably obvious that the joint effect of the upward force at M and the downward force at G is to turn the ship back again so as to bring BG to be vertical as at first. Thus the original position of the ship is one of *stable* equilibrium with respect to this tilting. Suppose however that M falls *below* G; then in the same way we see that the joint effect of the upward force at M and the downward force at G is to turn the ship further away from the position in which BG is vertical. Thus the original position of the ship is one of *unstable* equilibrium with respect to this tilting. See Art. 345.

429. Hence we see that it is essential for the safety of a ship that the centre of gravity should not be too high up. The proper situation is secured by putting the heavy goods which the ship carries as low in the hold as possible. After a ship has discharged the cargo it is found necessary to put into the hold sand or stones or such things for the sake of bringing down the centre of gravity of the whole as low as possible; these things are called *ballast*. So also if people go on the water in a small boat they must be careful to remain sitting down so as to keep the centre of gravity low; and especially they should avoid any sudden rising, which may elevate the centre of gravity, and tilt the boat at the same time.

430. We may observe that we have not taken the most general form of the investigation. We have assumed that the body is of the nature of a ship so as to have its two sides symmetrical, and we have supposed that the tilting is from side to side. Under these circumstances G and H remain always in the same vertical plane in which the tilting takes place; otherwise the matter becomes too

complicated for an elementary book. As a simple example let us suppose a sphere of wood floating on water. The centre of gravity of the solid is the centre of the sphere; and it is a result of geometry that the metacentre is also at the centre of the sphere. Thus the equilibrium is of the kind which we have called *neutral* in Art. 183. If instead of a whole sphere the floating body is a portion of a sphere cut off by a plane, then whether this portion is greater or less than a hemisphere, the centre of gravity will be below the centre of the sphere, while the *metacentre* is at the centre of the sphere; hence the body will float in stable equilibrium when the flat part is horizontal and outside the water.

XXXVI. SPECIFIC GRAVITY OF SOLIDS.

431. We have often in the preceding Chapters spoken of one body as heavier than another, *bulk for bulk;* thus gold is more than nineteen times as heavy as water *bulk for bulk.* In other words a cubic inch of gold is more than nineteen times as heavy as a cubic inch of water; and so for a cubic foot. When we speak of one body as heavier than another we may mean heavier *bulk for bulk;* in this sense gold is heavier than iron. Or we may mean that one assigned body is heavier than another, as that a certain iron bar is heavier than a certain gold coin. It is always plain from the circumstances in which of these senses we use the word *heavier;* the former is usually the sense required in the present work. We sometimes use the words *heavy* and *light* as if there were no comparison intended between the body with which we are concerned and other bodies. Thus we may say that lead is heavy and that cork is light. But some comparison is really intended; we mean that lead is heavier, bulk for bulk, than most objects with which we are familiar; and that cork is lighter, bulk for bulk, than most objects, or at least than most kinds of wood, with which we are familiar.

432. We have already in Art. 403 defined *specific gravity* as the proportion of the weight of any substance to the weight of an equal volume of the standard substance; and we have stated that the standard substance is usually

water. But we must now be a little more precise with respect to this standard substance. Water as obtained from springs or rivers is not always the same thing; it contains various substances mixed with it in greater or less degree, and hence the condition is added that the water must be *pure*. Water is made pure by distillation, that is, the water must be boiled and the vapour collected and condensed by cooling: in this way it is found that the substances which common water holds in solution are left behind, and pure water obtained. Moreover the bulk of water changes as the temperature changes, other things being the same. It is found that pure water diminishes in bulk as the temperature diminishes until the temperature is about 40 degrees of Fahrenheit's thermometer, and after that if the temperature is still lowered the bulk increases. Hence the temperature of 40 degrees of Fahrenheit's thermometer is that which it is found convenient to take for the standard. Thus finally we may say that the specific gravity of any substance is the proportion of the weight of the substance to the weight of an equal volume of pure water at the temperature of 40 degrees of Fahrenheit's thermometer.

433. The words *dense* and *density* are often used in books on Natural Philosophy, and we may here exemplify the meaning of them. We say that water has its greatest *density* at the temperature of 40 degrees of Fahrenheit's thermometer, or that water is more *dense* at this temperature than at any other. The simple fact which we have to express is that a cubic foot of water at this temperature weighs more than a cubic foot of water at any other temperature. As a convenient mode of representing this to our imagination we may suppose that the particles of water are closer together at the standard temperature than at any other. The *density* of a given body then is greater the smaller the volume of that body is; thus if a body is brought by cold or by pressure to occupy half its original space we say that the density is doubled. It would not be easy to double the density of a solid or of a liquid; but the density of a gaseous body can be easily doubled or even still more increased. We sometimes extend the range of the words *dense* and *density*, and use them in the com-

parison of two bodies of different material; thus we may say that gold is more dense than silver, or has greater density; but in such a case we mean simply that gold is heavier than silver, bulk for bulk.

434. In order to find the specific gravity of any solid which will sink in water we proceed thus: weigh the solid in water and out of it, the difference is the weight of an equal bulk of the water; divide the weight of the solid out of the water by this and the quotient is the specific gravity of the solid. For example, a piece of gold is found to weigh 97 grains, and on being immersed in water to weigh only 92 grains; thus the weight of an equal bulk of water is 5 grains, and therefore the specific gravity of the gold is $\frac{97}{5}$, that is $19\frac{2}{5}$. If the body is in the form of small fragments it may be put into a cup, and the whole immersed in a vessel of water, and the weight in water determined. Then the weight of the cup when immersed alone in the water must be determined, and by subtraction we have the weight in water of the collection of small fragments. Their weight out of water can also be found, and then the specific gravity becomes known.

435. If we know the specific gravities of two metals we can determine the specific gravity of a compound formed by melting together known quantities of these metals, assuming that the volume of the compound is equal to the sum of the volumes of the two metals, and also that in the compound the two metals are thoroughly mixed so as to form a compound of the same density throughout. For example, suppose we take 5 cubic inches of gold of which the specific gravity is 19·4, and combine them with 20 cubic inches of copper of which the specific gravity is 8·9, and want to know the specific gravity of the compound. We may if we please work with cubic feet instead of cubic inches, and our language will then become more simple.

A cubic foot of water weighs	1000 ounces;
a cubic foot of gold weighs	19400 ounces;
a cubic foot of copper weighs	8900 ounces;
thus five cubic feet of gold weigh	97000 ounces,
and twenty cubic feet of copper weigh	178000 ounces.

Therefore the twenty-five cubic feet of the compound weigh 275000 ounces, and therefore one cubic foot weighs 11000 ounces; and the specific gravity of the compound is $\frac{11000}{1000}$, that is 11.

436. Various other questions may be proposed with respect to compound bodies. Thus we may suppose that we know we have 5 cubic inches of gold in the compound, and 20 cubic inches of some other metal; also we may have found by experiment the specific gravity of the compound, and may wish to know the specific gravity of the other metal in the compound. For example, suppose that the specific gravity of the compound is found to be 11. Then we know that a cubic foot of the compound will weigh 11000 ounces, and therefore 25 cubic feet of it will weigh 275000 ounces. But 5 cubic feet of gold weigh 97000 ounces, and therefore 20 cubic feet of the other metal weigh 178000 ounces; thus one cubic foot of it weighs 8900 ounces, and the specific gravity of the metal is $\frac{8900}{1000}$, that is 8·9. The Tables of Specific Gravity shew us then that this metal has just the same specific gravity as copper, so that if we know it to be a simple metal we infer that it is copper.

437. Or again we may have a compound body which we know is made of gold and copper, but how much of each we are not told. Then if we know the specific gravities of gold, of copper, and of the compound, we can find the quantities of gold and of copper in any assigned quantity of the compound. The following is the rule: as the difference of the specific gravities of gold and of the compound is to the difference of the specific gravities of gold and of copper, so is the bulk of copper to the whole bulk. In any case the reader might find the quantities of gold and of copper by this rule and then verify the result by the method of Art. 435. The demonstration of the rule itself is not sufficiently elementary for the present book.

438. A famous story relating to the philosopher Archimedes is always told in books which treat on the subject now before us. Hiero King of Syracuse gave to an artist

a certain weight of gold to be made into a crown. The crown was furnished, and of course of the proper weight, but the king suspected that some of the gold had been replaced by silver, and he wished to settle this point without doing any injury to the crown. He consulted Archimedes, and it is said that the mode in which the problem might be solved flashed across the mind of the philosopher as he was in his bath; and that in a transport of joy he rushed from his chamber exclaiming in Greek, *I have found it, I have found it.* But the story, as repeated in modern times, seems to ascribe much more to Archimedes than he really then discovered. What he did was probably this: he used the principle that if a solid sinks in a vessel full of water the *volume* of the water ejected is exactly equal to the *volume* of the solid. He found in this way the volume of the crown, the volume of an equal weight of gold, and the volume of an equal weight of silver. This would be sufficient to enable him to determine how much gold and how much silver there was in the crown. He used in fact the *geometrical* principle involved in Chapter XXXI. but not the *mechanical* principle involved in Chapter XXXII.

439. We assume in the last four Articles, as stated in the beginning of Art. 435, that the volume of the compound is equal to the sum of the volumes of the metals which form it. But in practice this is frequently not the case, and thus the real specific gravity of a compound differs from that assigned by the process of Art. 435. For example, take the specific gravity of copper as accurately 8·78, and that of zinc as accurately 6·86; and let 14 pounds of copper be mixed with 7 pounds of zinc. Theoretically the specific gravity of the compound should be 8·14; but by experiment it is found to be about 8·6. The volume of the compound is thus a little less than the volumes of the copper and zinc.

440. Hitherto we have considered the specific gravity of any substance to be the proportion of the weight of that substance to the weight of an equal volume of the standard substance. But we may shew that this is the same thing as the proportion of the volume of the standard substance to the volume of an equal weight of the substance considered. For example, suppose that the weight of a certain

substance is equal to the weight of $\frac{2}{5}$ of the same bulk of water. Thus the weight of a cubic foot of the substance is $\frac{2}{5}$ of the weight of a cubic foot of water, and the specific gravity of the substance is $\frac{2}{5}$. And $\frac{2}{5}$ is also the proportion of a volume of water to the volume of an equal weight of the substance.

441. To find the specific gravity of a solid which is lighter than water we may proceed thus. Take a vessel with vertical sides, and having one side carefully marked with horizontal straight lines so that we know how much of the vessel is occupied by any liquid put into it by noting the line to which the level rises. Fill the vessel with water up to any of these lines, and note the line. Put the solid to float on the water; in consequence of this the level will rise: note the line at which the level stands. Again push the solid entirely under the surface, and note the line at which the level of the water stands. Thus we can determine the volume of the portion of the solid immersed when floating in equilibrium, and also the whole volume of the solid; the former divided by the latter gives the specific gravity of the solid. For example, suppose the side of the vessel to be marked with equidistant horizontal lines; and let the level of the water rise from its first position through $2\frac{1}{2}$ divisions when the solid floats; and let the level of the water rise from its first position through 4 divisions when the solid is entirely immersed. Thus the two volumes are in the proportion of $2\frac{1}{2}$ to 4; and the specific gravity of the solid by Art. 440 is $\frac{2\frac{1}{2}}{4}$, that is $\frac{5}{8}$.

442. Or we might determine the specific gravity of a solid which floats on water in the following way. First weigh the solid. Then attach one end of a string to the solid, immerse the solid completely in water, and let the string pass under a pully at the bottom of the water, then rise vertically and have its other end attached to the arm of a balance. By this means we ascertain what is the weight of water equal in bulk to the solid, diminished by

the weight of the solid itself; and by adding the known weight of the solid itself we obtain the weight of an equal bulk of water. Divide the weight of the body by the weight of an equal bulk of water, and the quotient is the specific gravity of the body. For example, suppose that a solid weighs 5 ounces; when the solid is kept completely immersed in water by a string which passes under a pully and then rises vertically, let the force which the string exerts be equal to 7 ounces: then the weight of water equal in bulk to the solid is 12 ounces, and therefore the specific gravity of the solid is $\frac{7}{12}$.

443. There is still another method for determining the specific gravity of a solid which floats on water. Attach the solid to a second so dense that both together sink in water; this body may be called the *sinker*. Weigh the two together both in the water and out of the water; the difference is the weight of water equal in bulk to the two solids. Also determine separately the weight of water equal in bulk to the sinker, and then by subtraction we know the weight of water equal in bulk to the first solid. Weigh this solid separately; then its specific gravity is the quotient of this weight by the weight of an equal bulk of water. For example, a piece of wood and iron together weigh 138 ounces, and in water 8 ounces; so that 130 ounces is the weight of water equal in bulk to the two. Again, the iron alone weighs 78 ounces, and in water 68 ounces, so that the weight of an equal bulk of water is 10 ounces; and the weight of water equal in bulk to the wood is therefore 120 ounces. Moreover as the wood and iron together weigh 138 ounces, and the iron alone weighs 78 ounces, the wood weighs 60 ounces. Hence finally the specific gravity of the wood is $\frac{60}{120}$, that is $\frac{1}{2}$.

444. We have made repeated use of the important principle that when a solid is immersed in a liquid the weight is diminished by the weight of an equal bulk of the liquid, but there is one curious application of the principle to which we have not yet drawn attention. The air is a

fluid and possesses the property of buoyancy which all liquids have. Hence any body in air loses weight equal to that of an equal bulk of air. Thus if we put any body into one scale of a balance and a counterpoise into the other, we must not in general take the counterpoise as representing the *exact* weight of the body. The fact is that the true weight of the counterpoise, diminished by the weight of an equal bulk of air, is equal to the true weight of the body diminished by the weight of an equal bulk of air. If the body and the counterpoise have the same volume the true weight of the counterpoise is exactly equal to the true weight of the body; but if not, the true weight of the counterpoise is less or greater than the true weight of the body according as the volume of the counterpoise is less or greater than that of the body. The correction thus required to the weight of a body when estimated in the usual way is too small to be of importance in ordinary matters, though it must be regarded in scientific investigations.

445. The specific gravities of some substances have been given in Art. 403; the following are selected from an elaborate Table in Dr Young's *Lectures* which extends to four places of decimals:

Diamond	3·52	Lignum Vitæ	1·33
Flint glass	3·33	Heart of Oak	1·17
Slate	2·67	Mahogany	1·06
Salt	2·13	Walnut	·67
Sulphur	2·03	White fir	·57
Newcastle coal	1·27	Poplar	·38
Ice	·93	Cork	·24

The specific gravities of the woods must be taken as *average* values, for the results will vary according to the character of particular specimens.

XXXVII. SPECIFIC GRAVITY OF LIQUIDS.

446. One of the most obvious methods of finding the specific gravity of a liquid is by actually determining the weight of an assigned volume of it. Let a flask be provided with a stopper which accurately fits it, and weigh the

flask and stopper. Also fill it with water and weigh it again. Then by subtraction we know the weight of water which would exactly fill the flask. We are now prepared to find the specific gravity of any liquid whatever. For fill the flask with the liquid and weigh it; subtract the weight of the flask, and the remainder is the weight of the liquid which would exactly fill the flask. Divide this by the weight of the water which would exactly fill the flask, and the quotient is the specific gravity of the liquid. For example, suppose that the water which would exactly fill the flask is found to weigh 20 ounces, and that the liquid which would exactly fill the flask is found to weigh 18 ounces; then the specific gravity of the liquid is $\frac{18}{20}$, that is $\frac{9}{10}$.

447. Or we might determine the specific gravity of a liquid by *immersing* the same solid successively in the liquid and in water. The weight lost in the first case is the weight of the liquid equal in bulk to the solid, and the weight lost in the second case is the weight of water equal in bulk to the solid: divide the former by the latter, and the quotient is the specific gravity of the liquid. For example, a piece of glass when immersed in sulphuric acid is observed to lose 185 grains of its weight, and when immersed in water is observed to lose 100 grains of its weight: hence the specific gravity of sulphuric acid is $\frac{185}{100}$, that is 1·85.

448. Or we might determine the specific gravity of a liquid by floating the same solid successively on the liquid and on water. The volume immersed in the first case is the volume of the liquid equal in weight to the solid, and the volume immersed in the second case is the volume of water equal in weight to the solid; divide the latter by the former and the quotient is the specific gravity of the liquid: see Art. 440. For example, a solid floats on oil and the volume immersed is found to be 25 cubic inches; and when it floats on water the volume immersed is found to be 23 cubic inches: hence the specific gravity of the oil is $\frac{23}{25}$.

449. Liquids are readily combined so as to form a new liquid, and when the specific gravities of the components are known we can determine the specific gravity of the mixture formed of assigned quantities of them, assuming that the volume of the mixture is equal to the sum of the volumes of the components. For example, suppose that a pint of water is mixed with a pint of alcohol of which the specific gravity is $\cdot 8$, and we want to know the specific gravity of the compound. We may if we please work with cubic feet instead of pints, and our language will then become more simple.

A cubic foot of water weighs 1000 ounces;
a cubic foot of alcohol weighs 800 ounces.

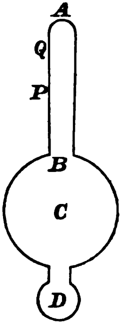
Hence the two cubic feet of the mixture weigh 1800 ounces, therefore one cubic foot of the mixture weighs 900 ounces, and therefore the specific gravity of the mixture is $\frac{900}{1000}$, that is $\cdot 9$.

In practice however it is often found that the volume of a mixture of fluids is not equal to the sum of the volumes of the components: see Art. 439.

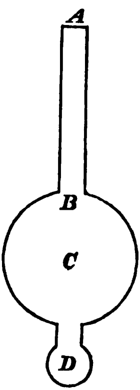
450. All the spirits which are used in the arts and in ordinary life consist of mixtures of alcohol and some other substances, of which water is the principal. It is often important to know what proportion the alcohol is of the whole in a certain mixture; or in ordinary language to know the *strength of the spirit*. The more water is mixed with the alcohol the greater the specific gravity of the mixture becomes. When the mixture has about the same specific gravity as oil it is called *proof spirit*, so that all spirit which will float on oil is said to be above proof. Thus the process of finding the specific gravity of a liquid becomes one of practical interest, and various instruments are used for the purpose called *Hydrometers*. They all depend on the principle that when a body floats on a liquid it displaces a quantity of the liquid equal in weight to its own.

451. *The common Hydrometer.* AB is a hollow cylindrical stem; C and D are two hollow spheres, which have their centres so situated that the axis of AB if pro-

duced would pass through them. D is loaded with lead, so that the centre of gravity of the whole instrument may be below the centre of gravity of the fluid displaced when the instrument floats with the cylindrical stem vertical and upwards; see Art. 428. When the hydrometer floats in water suppose that the surface of the water meets the stem AB at P; and when it floats in the liquid which we are examining suppose that the surface of the liquid meets the stem AB at Q. Then the specific gravity of the liquid is the proportion which the volume of the part of the instrument below P bears to the volume of the part of the instrument below Q; see Art. 440. The volume of the part below P is the volume of the whole instrument diminished by the volume of the stem from P upwards; and the volume of the part below Q is the volume of the whole instrument diminished by the volume of the stem from Q upwards: thus these volumes may be readily determined.

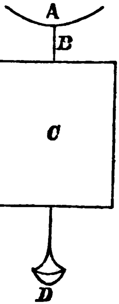


452. *Sikes's Hydrometer.* This instrument differs from the preceding in two respects; the stem AB is a very thin flat bar, and there is a series of weights capable of being attached to the part of the stem below the large sphere. These weights are in the form of round discs with notches cut in them by which they can ride on the stem. The weights are of such magnitude that if the instrument would float in a liquid with the whole of its stem above the surface the addition of one weight would sink it nearly to A. By the use of the weights the instrument is in fact capable of being converted into a *series* of hydrometers. So long as we keep the *same* number of weights attached below C, the mode of obtaining from the instrument the specific gravity of a liquid is the same as in the preceding Article. But



if we have to use more or fewer of the weights when the instrument floats on a liquid than when it floats on water the matter is not quite so simple. This hydrometer is employed by the excise officers under the authority of government to determine the specific gravity of spirits, with the view of fixing the amount of duty to be paid; it is accompanied with a Table properly calculated which gives the specific gravity of a liquid as soon as the number of weights attached to the stem is known and the depth to which the stem sinks has been observed.

453. *Nicholson's Hydrometer.* C is a hollow cylinder or ball; A is a dish supported by a slender wire B, the direction of which is the same as the axis of C. From the lower extremity of C a heavy dish D is suspended. The weights of the various parts of the instrument are so adjusted that when 1000 grains are placed in the dish A, the instrument will sink in water to a point marked on the stem B near the middle of it. Therefore the weight of so much water as would be equal in volume to the instrument below the marked point is equal to 1000 grains together with the weight of the instrument. Now put the hydrometer in the liquid which is to be examined, and by increasing or decreasing the weight in the dish A make the instrument sink again to the marked point. Thus we know the weight of so much of the liquid as is equal in volume to the instrument below the marked point. Divide this weight by the corresponding weight in the case of water, and the quotient is the specific gravity of the liquid.



454. Nicholson's Hydrometer may also be used for finding the specific gravities of solids. Place the solid, reduced to a convenient size, in the dish A, and let additional weights be placed in the dish until the instrument will sink in water to the marked point. Then the weight of the solid together with the additional weights which have been used must amount to 1000 grains, and so the weight of the solid is known. Next remove the solid from A,

place it in D, and as before add weights in A until the instrument will sink to the marked point. Then the weight of the solid in water, together with the weights in A, must amount to 1000 grains, and so the weight of the solid in water is known. Thus we know the weight of the solid, and also its weight in water; and therefore, by subtraction, we find the weight lost in water: divide the weight of the solid by this, and the quotient is the specific gravity of the solid. Of course any other weight might be adopted throughout instead of the 1000 grains which we have taken for simplicity.

455. "The wire which supports the dish A in this instrument is so thin, that an inch of it displaces only the tenth part of a grain of water. The accuracy of its results depending therefore on the coincidence of the mark on the wire with the surface, which can always be ascertained to a very small fraction of an inch, will come within the limit of a very minute fraction of a grain. Specific gravities may thus be obtained correctly to within a hundred thousandth part of their whole value, or to five places of decimals."

456. The hydrometer might be usefully employed to detect adulteration in various liquids which are used in ordinary life. For instance, the specific gravity of milk is greater than that of water, being about 1·03. By mixing water with milk the specific gravity is made less than that of milk, and the greater is the proportion of water used the more is the specific gravity diminished. Thus a very accurate hydrometer would enable us to find the proportion of water to milk in a mixture of the two.

457. A gallon is such a measure of volume as will just hold ten pounds Avoirdupois of pure water. Hence if we multiply the specific gravity of a liquid by 10 we obtain the weight in pounds of a gallon of it.

458. The following are the specific gravities of some liquids:

Sea water	1·027	Alcohol	·795
Linseed oil	·940	Naphtha	·753
Olive oil	·915	Æther	·724

XXXVIII. SPECIFIC GRAVITY OF GASES.

459. We have still to consider the specific gravity of gases, and we will give a Chapter to the subject here, although we shall have to allude to various matters which will be more fully explained in some subsequent Chapters, treating on Pneumatics.

460. Let us confine our attention first to one of the gaseous bodies, namely common air, which surrounds us altogether and which we continually breathe. Now although air may at first seem to have no weight, yet it really has; and we shall see hereafter that this gives rise to many important results. Here we need only say that if a flask be filled with air it will weigh more than when empty, shewing that the air has weight.

461. A very remarkable property of gaseous bodies is that they may be *compressed* to almost any extent. Thus air being put into a strong vessel we may compress it into half or a quarter of its original bulk. Moreover if we keep air in a vessel under a certain amount of pressure it will expand by the application of heat and contract by the withdrawal of heat. Again, the weight of an assigned volume of air or of any gas will depend to some extent on the quantity of watery vapour which is mixed with the air or gas. Instruments called *hygrometers* are constructed to shew the amount of this vapour. It follows from what has been said, that in speaking of the specific gravity of any gaseous body there are many circumstances which must be regarded in order to fix the exact condition of the body.

462. We may now state the facts with respect to *air* with sufficient accuracy for our purpose. Let the temperature be that of the freezing point of water; let the air be *dry*, that is free from watery vapour; let the air be in what we may call its natural state of pressure, namely, the state at which it is at the level of the sea on an ordinary day. Then it is found that a cubic foot of air will weigh nearly $1\frac{3}{10}$ ounces; thus taking water for the standard, the specific

gravity of air is $\frac{1\frac{3}{10}}{1000}$, that is ·0013. Or we may say that water is about 768 times as heavy, bulk for bulk, as air in the state just explained. A more accurate statement is the following: 100 cubic inches of air at the temperature of 60 degrees of Fahrenheit's thermometer, and under a pressure denoted by 30 inches in the height of the barometer, weigh 31·0117 grains.

463. The specific gravities of gases are usually referred to common air as the standard; they may be referred to water if necessary by means of the facts stated in the preceding Article. The subject however is not sufficiently elementary to be pursued here; indeed the various gases are not things with which we are so familiar as we are with solids and liquids: the gases require the aid of chemistry to make them known to us. The following Table gives the ratio of the specific gravities of some of these bodies to the specific gravity of air at the same temperature and under the same pressure.

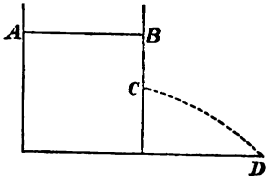
Hydrogen	·0688	Carbonic acid	1·5245
Vapour of water	·6201	Vapour of alcohol	1·6133
Nitrogen	·976	Chlorine	2·4403
Oxygen	1·1026	Sulphuric acid	2·7629.

XXXIX. EFFLUX OF LIQUIDS.

464. The subject of the motion of liquids is one of great difficulty, and though theory and experiment have been much employed on it the knowledge gained up to the present time is far from complete. We shall consider only some simple cases.

465. *Velocity of issuing liquid.* If a small hole be made in the side of a vessel which is full of liquid the liquid will escape with a certain velocity. The forces which produce the motion are the weight of the liquid itself and the pressure of the surrounding liquid; these

would be in equilibrium if there were no hole. Let AB be the surface of the liquid in the vessel, C the point at which the hole is made. Then it is found by theory that the velocity with which the liquid spouts out at C is the same as would be acquired by a body falling freely down the space BC. This supposes that the surface AB and the orifice at C are exposed to the *same* pressure, as for instance that of the atmosphere, which will be explained hereafter. If the pressure at the level AB is *greater* than at C, the effect is the same as if the height BC were increased to the extent which would correspond to this excess of pressure; and similarly if the pressure at the level AB is *less* than at C the height BC must be supposed diminished to a corresponding extent. Each particle of liquid on leaving the vessel will describe a *parabola* by virtue of the principles of Chapter XX.; and thus by the continuous stream of particles we obtain a visible representation of the parabolic course.



466. If we suppose the hole to be in the shape of a horizontal pipe the liquid will issue in a horizontal direction, so that the particles start from the highest point of their course and afterwards continually descend. But we may if we please insert at C a short pipe inclined to the horizon upwards, and then the fluid will ascend obliquely to some height before it begins to descend. Or the short pipe may be first horizontal for a brief space, and then turn vertically upwards: in this case the liquid spouts vertically upwards, and according to theory would rise to the level of AB.

467. Although the theory on which the preceding Article depends is beyond the range of the present work, yet there is one part of the result involved in it of which the reasonableness may be rendered tolerably evident; and this process well deserves attention. The liquid at C issues with a certain velocity, namely, with that which would be acquired in falling freely down BC. Hence if

we want the liquid to issue with twice this velocity we must make the hole, not at *twice* the depth of *C* below the surface, but at *four* times this depth: that is, we have as it were to provide *four* times the pressure in order to secure *twice* the velocity. But the apparent difficulty is soon removed. For since the velocity at the lower hole is to be double that at the higher hole, each particle issues from the lower hole with double the velocity with which it issues from the higher hole; and moreover supposing the holes to be of the same size, double the number of particles will issue in the same time from the lower hole as from the higher hole. Thus, in all, we have at the lower hole four times the effect produced which is produced at the higher hole, corresponding, as might be expected, to the circumstance that the pressure at the lower hole in equilibrium is four times that at the higher.

468. There are two cases of the motion considered in Art. 466, namely, that in which the liquid in the vessel is always maintained at the same level, and that in which it is not. In the latter case the value which theory gives for the velocity does not agree with observation when the level of the descending fluid comes near the hole. But in both cases, so long as the hole is not too near the surface of the liquid the actual velocity of the issuing liquid does not differ much from the value assigned by theory. But when the liquid is made to spout vertically upwards it does not reach the level of the liquid in the vessel; the velocity of the issuing fluid is diminished by the friction against the sides of the pipe or opening through which it escapes, and the resistance of the air also produces a sensible effect.

469. If we know the size of a hole and the velocity with which liquid is escaping through it, we can calculate the amount of liquid which will flow out in an assigned time. But in making such calculations and comparing the results with observation it is found that the theoretical estimate is too large. Some curious phenomena have been noticed in connexion with this subject. We will suppose that the hole is very small, that it is in the base of the vessel, and that the base is very thin; this special

case has been examined with much attention. At the hole the particles of liquid do not move *vertically downwards* so as to form a cylindrical column, but the lines of direction of the motion are inclined towards each other as if they were about to meet at a point. Thus the stream of issuing liquid is narrowest at a short distance from the hole, and this part of the stream is called the *vena contracta* or *contracted vein.* The area of a section of the *vena contracta* is equal to about five-eighths of the area of the hole. If in calculating the amount of liquid which would pass in a given time through a hole in the base of a vessel we take the area of the *vena contracta* instead of the area of the real hole, the result is found to agree reasonably well with observation.

XL RESISTANCE OF LIQUIDS.

470. The resistance which a solid body experiences in moving through a liquid is a matter of great importance in practice; but the subject is not one which admits of elementary exposition, and we shall confine ourselves to a few simple remarks.

471. Suppose that a flat board is urged through a liquid which is itself at rest; suppose the board to move with uniform velocity in a direction at right angles to its plane. Then it is found by theory that the resistance which the board experiences from the liquid is at right angles to the board, and is equal to the weight of a column of the liquid which has the board for base, and for height the space through which a body must fall freely in order to acquire the velocity. The height by Art. 127 is equal to the *square* of the velocity divided by 64. But this theoretical result is not very exactly confirmed by experiment.

472. If the preceding result be accepted as correct, we see that we must apply to the board a force equal to the weight of the column there mentioned in order to keep it moving uniformly. For then the force which we apply just balances the resistance, and the board continues to move with uniform velocity according to the First Law of Motion. One fact involved in this result deserves to be

explicitly noticed: suppose a force to be applied just sufficient to keep the board moving at a certain uniform rate, then if we wish to have the velocity *doubled* we must exert *four* times as much force, if we wish to have the velocity *tripled* we must exert *nine* times as much force, and so on. For according to the statement of Art. 471, if the velocity is doubled the resistance becomes four times as great, and so on. Moreover some reason may be given in explanation. If the velocity of the moving board is doubled then the board strikes against twice as many particles of liquid as before in a given time, and also strikes each particle with twice the velocity it did before. Thus the board may naturally produce four times the movement in the liquid which it did before, and so may itself experience four times the resistance which it did before.

473. Next suppose that the board is urged through the liquid in a direction which is not at right angles to its plane. Suppose for instance that the board faces North East, but that it is urged in the direction from South to North. In this case the resistance of the liquid is exerted as before at right angles to the board, and its amount is found by resolving the velocity of the board into two components, namely, one at right angles to the board, and the other along the board; the former component is alone regarded, and the resistance at right angles to the board is the same as would be experienced by the board if it were moving in this direction with this component velocity. When we have thus obtained the resistance in the direction at right angles to the board, we may often have to consider only that part of it which acts in the direction of the motion of the board. The whole process is somewhat beyond the range of this book; but the important principle still holds that if the velocity is *doubled* the resistance becomes *four* times as great, and so on.

474. We can thus understand the difficulty which occurs in attempting to give a very great velocity to bodies moving in the water, as ships or steam-boats. As long as we keep to the same steam-boat then in order to double the velocity, supposed uniform, we must apply four times the force, and so on. Much may be done by trial in devising the most favourable shape for the steam-boat in order to diminish

the resistance, but still if we attempt to obtain a very great velocity the resistance becomes too great to be overcome with due economy in the use of force.

XLI. GASEOUS BODIES.

475. We have hitherto been explaining the properties of *liquids*, that is of fluid bodies which, although not absolutely incompressible, yet retain their dimensions practically unchanged under all forces to which they are usually exposed. In liquids the two opposing principles, cohesion and repulsion, may be said to be nearly balanced. In air and other gaseous bodies the repulsive principle prevails, so that cohesion seems scarcely to exist. The constituent particles of the body fly asunder if left unconfined, and require to be constrained completely in some manner if we wish to keep them before us for examination. They can be compressed by suitable force to almost any extent, and when the force is withdrawn they return to their original dimensions. They are frequently called *elastic fluids*.

476. The distinction between *solid*, *liquid*, and *gaseous* is not so much a distinction between bodies as a distinction between the different states which the same substance may assume. We know for instance that the same substance may be solid in the state of ice, liquid in the state of water, and gaseous in the state of steam. Chemists have strong reasons for believing that all bodies can be made to pass into these three states, and that the state assumed depends principally on the quantity of heat which is present. Gaseous bodies are sometimes divided into two classes; to one of these the term *gases* is more peculiarly appropriated, and to the other the term *vapours*. A vapour is a gaseous body which passes easily by a reduction of temperature, or an increase of pressure, into the liquid state; thus steam is a vapour because by a slight cooling it is reduced to water. A gas, strictly so called, retains that form under all ordinary conditions of temperature and pressure; thus carbonic acid is a gas because it is only by special means that it can be reduced to a liquid: and common air is a still more eminent example.

477. The passage from the liquid to the gaseous state is usually accompanied by a large increase of volume. Thus a cubic inch of water is converted by boiling into about 1700 cubic inches of steam; so that the cubic inch of water becomes nearly a cubic foot of steam. If we suppose that the substance consists of a large number of particles, placed at nearly equal distances, then we may imagine that in passing from the state of water to that of steam the average distance between two adjacent particles becomes about twelve times as great as at first.

478. The changes of state take place at different temperatures for different bodies. Thus to cause water to take the solid state of ice the temperature must be reduced to 32 degrees of Fahrenheit's thermometer; while if the temperature is raised to 212 degrees the water becomes steam. Mercury freezes at about 40 degrees below the zero of Fahrenheit's thermometer, and boils at about 650 degrees. Thus we see that one substance, as mercury, may remain in the liquid state at a temperature so low that another substance, as water, becomes solid; and at a temperature so high that another becomes gaseous.

479. There is a curious fact connected with the passage of bodies from the solid to the liquid state, and from the liquid to the gaseous; it is expressed by the statement that in these changes heat becomes latent. Suppose that a pound of water at 32 degrees of heat as measured by Fahrenheit's thermometer is mixed with a pound of water at 174 degrees; it is found that the temperature of the mixture is 103 degrees, which is half the sum of 32 and 174 degrees: the hotter water has lost, and the colder has gained, 71 degrees of temperature. But now suppose that a pound of ice at 32 degrees is put with a pound of water at 174 degrees; after a time the ice will all be melted, and the temperature of the mixture will be only 32 degrees. The water has lost 142 degrees of temperature, and the ice has been melted without any apparent increase of temperature: the heat thus lost by the water is said to be *latent* in the melted ice. Thus we see that in the process of converting a solid into a liquid a large quantity of heat is required which is in some manner absorbed by the liquid and does not become apparent by a rise of temperature.

In like manner when water is converted into steam a large quantity of heat becomes latent. The amount is greater than in the case of liquefied ice, being now about 900 degrees instead of 142. These numerical values have been differently assigned by various experiments; but extreme accuracy is unnecessary for our purpose.

480. The term *latent heat* has been long in use and perhaps does not often lead to any confusion or error; but there is always a danger that such descriptive terms should be made to suggest more than they are actually intended to convey. It might be objected in the present case that heat cannot be properly said to be *hidden* because its influence is manifested in the remarkable change of state, namely, from the solid to the liquid state, or from the liquid to the gaseous.

481. Common air is the most obvious and the most important of the gaseous bodies, and we shall in the main confine ourselves to the properties of air, though the mechanical results obtained are applicable in general to gases and vapours. The science which relates to the mechanical properties of the air is called *Pneumatics*. It belongs to Chemistry to treat of the special properties of each gas.

XLII. AIR A SUBSTANCE.

482. The atmosphere is a thin fluid which surrounds the globe, and is necessary for the support both of animal and vegetable life. Although before attention has been drawn to its properties it might be imagined that air is scarcely a form of *matter*, yet on due consideration it will be found to be such, though in a very rarefied condition.

483. The air is generally supposed to be *transparent*, but when we look at a cloudless sky we recognise a blue colour which may be attributed to the air. The fact that this colour is not visible when we inspect a small quantity of air by itself is consistent with other facts of a similar kind. Thus sea-water in a large mass presents a greenish tint, but a small quantity of it seems without colour. So also wine in a very slender glass appears much paler in tint than in a wider glass.

484. One of the most obvious properties of matter is *weight*, and air may be shewn to possess this. It would seem a natural process to test this by first weighing an empty bladder, and then weighing this bladder full of air; Aristotle is said to have done so, and, finding the same result in the two cases, to have inferred that air has no weight. But here we have the operation of a cause of error to which we drew attention in Art. 444; the additional weight of air in the bladder is counterbalanced by the buoyancy of the atmosphere exerted on the inflated bladder. The experiment must then be made in a manner which avoids this cause of error. Take a flask of glass or metal, and exhaust it of air by the aid of a machine to be described hereafter, called the air-pump; then weigh the exhausted flask. Admit the air to the flask and weigh it again. Then the difference between the two results gives us the weight of the air which the flask will hold. As we have said in Art. 462 the weight of a cubic foot of air under ordinary circumstances is about $1\frac{3}{10}$ ounces. We spoke of exhausting the flask of air; but in practice we cannot draw out *all* the air, though we may contrive to leave only a quantity which is quite inappreciable. Again, the experiment may be carried a step further. For not only can we draw air out of a vessel, but we can force into it any quantity of air we please. Thus we can increase the amount of air in the vessel, and we shall find that as we do so we increase the weight of the air in the same proportion.

485. Again, the resistance which air opposes to motions through it is an evidence that it has the properties of matter; we are very sensible of this resistance when we run. The reaction of the air when they strike it with their wings enables birds to fly; in a space void of air they could not fly. *Wind* is air in motion, and the powerful effects of high winds are merely the consequences of matter in violent motion.

486. It is usual to remark that air possesses the property of matter which we call *impenetrability*. Invert a tumbler and press it below the surface of water; then it is easy to see that the water does not get to the highest part

of the tumbler. If a small cork be floating on that part of the water over which the tumbler was placed, the cork will not reach the highest part of the tumbler. The air in the tumbler is indeed compressed into less space than it originally occupied, and so the water occupies part of the tumbler; but the air remains in the upper part of the tumbler and excludes the water from it.

XLIII. PRESSURE OF THE ATMOSPHERE.

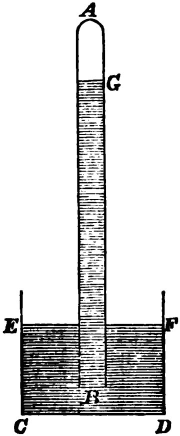
487. If we put air in a vessel furnished with a moveable piston we find that we can push in the piston and compress the air to any extent we please. If we wish to keep the air in this compressed state we must retain the piston in its place by a suitable force; if we diminish that force the air pushes the piston back through some space, and if we remove all the force the air resumes its original dimensions. There must be some relation then between the force which we apply to the piston, and the volume occupied by the compressed air; this relation we shall consider in the next Chapter after some necessary preliminaries in the present.

488. We know that air requires the exercise of some constraint to confine it within the space it occupies, and so we naturally suppose that there must be some pressure acting on the apparently unconstrained air around us, and we soon find that this pressure must be supplied by the atmosphere itself; any stratum of air has to support the pressure produced by the weight of all the strata above it. A very important experiment serves to demonstrate the existence of the pressure of the atmosphere, and to measure its amount.

489. *To measure the pressure of the atmosphere.*

Take a glass tube a yard long, open at one end and closed at the other; fill it with mercury and place a finger over the open end to prevent the escape of the mercury. Invert the tube, put the end closed by the finger below the surface of a vessel containing mercury, and withdraw the finger. Some of the mercury will fall out of the tube,

leaving a vacuum, that is an empty space, at the top of the tube. In the diagram let AB denote the tube, EF the surface of the mercury in the vessel, and G the surface of the mercury in the tube. It is found that the height of G above the level of EF is about 30 inches, so long as the place at which the experiment is made is not much above the level of the sea; but even at the same place the height is always fluctuating slightly according to the state of the temperature and the weather. The column of mercury above the level of EF is supported by the pressure of the atmosphere on the surface of the mercury in the vessel; this pressure is transmitted through the mercury in the vessel, and into the tube by means of the end B. The principle is the same as in Art. 420; we may imagine two tubes, one containing mercury of about 30 inches high, and the other extending upwards as far as the atmosphere extends, and the columns of mercury and of air would produce the same pressure at their lowest points: the column of mercury must be supposed to be in a tube closed at the top so as to relieve it from the pressure of the atmosphere above it.



490. As we ascend to a height above the level of the sea the pressure of the atmosphere diminishes, and so the height of the column of mercury diminishes. If the atmosphere were throughout of the same density there would be a diminution of about one inch in the mercury for every 900 feet of ascent; but the fact is that the higher we ascend the less is the density of the atmosphere, and so the diminution of the column of mercury is not in exact proportion to the ascent.

491. We see then that the pressure of the atmosphere under ordinary circumstances *on a square inch of surface* is equal to the pressure of a column of mercury of the height 30 inches standing on one square inch as base: thus

the pressure is equal to the weight of 30 cubic inches of mercury, that is about 15 pounds.

492. In the propositions which we have given with respect to liquids in equilibrium in open vessels we have supposed that no pressure was exerted on the upper surfaces. But we now see that the atmosphere will exert a pressure, which under ordinary circumstances is about 15 pounds on a square inch; and hence it is necessary to advert to the principal results formerly obtained, in order to ascertain whether they still hold.

(1) The upper surface of a liquid will still be a horizontal plane as in Art. 358; for the pressure of the atmosphere being of the same amount on every square inch of the surface will not disturb the horizontal surface.

(2) Suppose a small area taken inside a vessel containing liquid; then the pressure will be the weight of a certain column of liquid extending up to the surface, increased by the amount of pressure due to the atmosphere; see Arts. 362 and 363.

(3) In Arts. 376...378 we have found the pressure of liquid on a vertical side of a vessel, and the point at which the pressure may be supposed to act. Now if we consider the pressure of the atmosphere we must observe that it will act in the same manner on the *two faces* of the vertical side; on one face the atmosphere would be in contact with the vertical side, and on the other face the pressure will be transmitted through the liquid. Thus on the whole the pressure of the atmosphere merely supplies two equal opposing forces which balance each other and leave our former result unaffected.

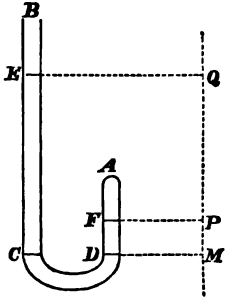
(4) The principle of Chapter XXX. that *liquids stand at a level* will still hold when we regard the pressure of the atmosphere.

(5) The result obtained at the end of Art. 420 will still hold when we regard the pressure of the atmosphere; for practically the pressure of the atmosphere at the levels AB and CD will be the same, supposing these levels only slightly different.

XLIV. RELATION BETWEEN PRESSURE AND VOLUME.

493. Two important principles were established with respect to liquids in Chapters XXVI. and XXVII.; namely, that pressure applied to the surface of a liquid in a vessel is transmitted unaltered in amount throughout the liquid, and that the pressure at any point is the same in all directions round the point. Now these two principles hold for air and gases as well as for liquids, as may be shewn by the same reasoning and experiments as have been already used; thus they hold for *all fluids*.

494. We are now about to explain the relation which holds between the volume and the pressure for the case of *compressible fluids*, that is for the case of air and the gases.

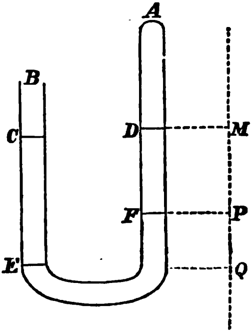


Take a glass tube and bend it so that the two branches shall be parallel. Let A, the end of the shorter branch, be closed, and B, the end of the longer branch, open. Pour into the tube a small quantity of mercury, and, by withdrawing air or adding mercury, make the mercury in the two branches stand at the same level C and D; let this level meet the vertical straight line MPQ at M. Thus in AD we have a quantity of mercury sustaining the pressure of the atmosphere; for the pressure at C is produced by the atmosphere, and this pressure is transmitted to D. We will suppose this pressure to be of its ordinary amount, so that it is measured by 30 inches of mercury. Pour more mercury into the tube at B, and suppose the mercury to rise to E in the longer branch and to F in the shorter branch. Let the level of E meet the vertical straight line MPQ at Q, and let the level of F meet this straight line at P. Then the air which formerly occupied the space represented by AD is now compressed into the space

represented by AF. The pressure at F is now the pressure of the atmosphere *increased* by the pressure measured by the height PQ of mercury. Suppose for instance that PQ is 18 inches; then the pressure at F is measured by $30+18$ inches, that is by 48 inches, of mercury. It is found by trial that the volume represented by AF bears the same proportion to the volume represented by AD as 30 bears to 48; and a similar result holds whatever may be the relative positions of P, Q, and M. Thus in the language of arithmetic the volume of the air *varies inversely* as the pressure exerted on it. For example, if the pressure were increased to 60 inches of mercury the volume would be reduced to half of the original volume; and if the pressure were increased to 90 inches of mercury the volume would be reduced to one third of the original volume; and so on.

495. The preceding experiments and results are of great importance in the subject, and a few remarks should be made on some incidental points. A condition must be carefully regarded to which we have not yet adverted; namely, the temperature must be the *same* throughout the experiment. For if the temperature of the air be changed the volume of the air will be changed on that account, while the pressure remains the same. The sudden compression of the air, when mercury is poured in, will raise the temperature of the air slightly, so that time must be allowed for the air in the shorter branch to cool down to its original temperature. We have used 18 inches for the sake of an example to measure the additional pressure, but the experiment may be varied by the use of more or less mercury, so as to introduce other numbers in the place of 18. Also we have taken 30 inches to measure the pressure of the atmosphere at the time and place of the experiment; but the real number may be somewhat greater or less than this, and would have to be accurately determined on the occasion. The volumes represented by AD and AF must be estimated with accuracy. If we are sure that the tube is of the same bore throughout we may take the volumes to be in the proportion of the lengths of the portions of the tube; but if this is not the case, we may determine the volumes accurately by weighing the quantities of mercury which they will hold.

496. In the experiment we supposed the air to be first under the ordinary pressure of the atmosphere, and afterwards under a *greater* pressure; in order to establish the law universally we ought to examine also the case in which the air is put under a pressure *less* than that of the atmosphere.



Take a glass tube and bend it so that the two branches shall be parallel. Let A, the end of the longer arm, be closed, and B, the end of the shorter arm, be open. Pour mercury into the tube, and by withdrawing air or adding mercury make the mercury in the two branches stand at the same level C and D; let this level meet the vertical straight line MPQ at M. Then in AD we have a quantity of air sustaining the pressure of the atmosphere; we will suppose this to be measured by 30 inches of mercury. Withdraw some of the mercury from the tube, and let the mercury sink to the level E in the shorter branch, and to the level F in the longer branch. Let the level of E meet the vertical straight line MPQ at Q, and let the level of F meet the same straight line at P. Thus the air which formerly occupied the space represented by AD is now expanded into the space represented by AF. The pressure at F is now the pressure of the atmosphere *diminished* by a pressure measured by the height PQ of mercury. Suppose, for instance, that PQ is 9 inches; then the pressure at F is measured by $30-9$ inches, that is by 21 inches, of mercury. It is found that the volume represented by AF bears the same proportion to the volume represented by AD as 30 bears to 21.

497. The result which is established in Arts. 494...496 may be put in other ways which are equivalent to the statement that the volume varies inversely as the pressure. Thus since the density of a given body varies inversely as

its volume we may state the law thus: *the density of air at a given temperature varies directly as the pressure exerted on it.* Or again, the pressure to which air is subjected is resisted by the air, and in the state of equilibrium the resistance is equal to the pressure; now this resistance is ascribed to the elasticity of the air, and thus the law is sometimes expressed thus: *the volume of the air is inversely proportional to its elasticity.* The law itself is sometimes called *Boyle's Law*, and sometimes *Mariotte's Law*, from the names of two philosophers by whom it was discovered. The law was long held to be absolutely true for all gaseous bodies, as experiments had been made in which the pressure was carried on to an amount equal to twenty-seven times that of the atmosphere, and the results seemed to agree with the law. In more recent times however, in consequence of closer scrutiny, it has been found that the law is not absolutely true; for the gaseous bodies which until very recently were considered as practically not liquefiable, such as air, hydrogen, and nitrogen, the deviations from the law are almost insensible; but in the case of liquefiable gases, as carbonic acid, the deviations may be considerable: in all cases gases are rather *more* compressible than Boyle's law would indicate, but hydrogen is a remarkable exception, being *less* compressible.

498. We have alluded in Art. 495 to the fact that when the *pressure remains unchanged* the volume of air or a gas changes when the temperature changes. The law on which this depends may be stated with sufficient accuracy for our purpose thus: add 450 to the number of degrees in the temperature as expressed by Fahrenheit's thermometer, the volume is proportional to the sum. Thus, for an example, suppose that the pressure is kept unchanged and that the temperature has been increased from 50 degrees to 100 degrees; $450+50=500$, and $450+100=550$. Then the volume at the higher temperature bears the same proportion to the volume at the lower temperature as 550 bears to 500, that is, as 11 bears to 10.

XLV. THE BAROMETER.

499. The Barometer is an instrument for measuring the pressure of the atmosphere; we have already explained the principle of the instrument in Art. 489, and we have now to add a few practical remarks. The principle of the Barometer is that a column of mercury has a vacuum above it, and is exposed to the pressure of the atmosphere at its base. In the construction of the instrument it is necessary to be careful in securing as far as possible this vacuum above. Now it is found that mercury in its ordinary state frequently contains air or other elastic fluid combined with it; and moreover particles of air and moisture are sometimes adhering to the glass tube when the mercury is poured into it. Then when the pressure of the atmosphere is removed from the top of the column of mercury the moisture becomes vapour, and that and the air rise to the top of the tube, and occupy the space which ought to be a vacuum. In consequence of this there is a pressure at the top of the mercury which tends to force it down, and so the height of the column is less than it ought to be. To guard against this defect it is found advantageous to heat the tube before the mercury is put in; thus the particles of air become expanded and their elastic force is increased and they escape: also the moisture is converted into vapour and escapes. The mercury is boiled, and this process expels from it any air or other elastic fluid which may have been combined with it. After all these precautions have been taken the portion of the glass tube above the mercury will be practically a vacuum; it is indeed highly probable that vapour may arise from the mercury itself and occupy this space, but it does not appear that this will exert any sensible pressure.

500. In the instrument, as we have described it in Art. 489, it is necessary to observe the level of the mercury at two points, namely, the place where it is exposed to the atmosphere, and the top of the column; and from the two observations we deduce the height of the column.

Various methods however have been devised in order to obviate the necessity of the *two* observations.

501. If the area of a section of the vessel in which the lower end of the tube is immersed is very large compared with the area of a section of the tube, it is obvious that the level of the mercury in the vessel will remain almost unchanged when the column in the tube rises or falls a little. Hence we may consider this level as fixed, and measure from it upwards the height of the column. A small brass scale divided into inches and tenths of an inch may be fixed close to the glass, so that by looking at the mark nearest to the top of the mercury we may ascertain the height of the column; the brass scale need not extend beyond the heights between 28 and 31 inches, which is practically inclusive of the range under ordinary circumstances.

502. Another method of avoiding the necessity for two observations is to have the numbers recorded on the brass scale really *exact*. That is, the maker of the instrument must ascertain for any position of the upper end of the column what is the true distance between the level of that end and the level in the vessel, and must record it on the scale. We may conceive that this is done by actual examination of every case at which a mark is to be recorded; but practically the maker can assist himself by an easy principle. If the sides of the vessel are vertical, and the bore of the tube uniform, there will be a precise relation of a simple character between the changes of level in the vessel and the tube. Suppose, for instance, that the area of a section of the vessel, excluding the part occupied by the tube, is 100 times the area of a section of the bore of the tube; then when the level in the tube rises 1 inch, the level in the vessel will sink $\frac{1}{100}$ of an inch, and therefore the height of the column will be increased by $1\frac{1}{100}$ inches. Therefore a length which is actually 1 inch on the brass scale must be marked as $1\frac{1}{100}$; and so on in the same proportion. Hence by reading off the mark opposite to the top of the mercury we learn the accurate height of the column which measures the pressure of the atmosphere.

503. There is still a third method of avoiding the necessity of two observations in order to know the height of the barometer. The bottom *CD* of the vessel denoted in Art. 489 is made so that it can move up and down without any leakage of mercury. Then when the instrument is to be consulted, the bottom is moved up or down so as to bring the surface of the mercury *EF* to one fixed level; this is effected by having a fine point projecting from the side of the vessel near its top, and making the surface of the mercury just touch this point.

504. Any other fluid might theoretically be used instead of mercury for the construction of a barometer, as for example water. The height of a column of water supported by the pressure of the atmosphere will be about $13\frac{1}{2}$ times the height of the column of mercury, because the weight of mercury, bulk for bulk, is about $13\frac{1}{2}$ times that of water. Hence the height of the water barometer would be rather less than 34 feet on an average; and this great height would obviously render the instrument somewhat unwieldy compared with the mercury barometer. Theoretically the water barometer would have the advantage of enabling us to discriminate more accurately the ever varying pressure of the atmosphere; for to a change of one-tenth of an inch in the height of the mercury barometer will correspond a change of more than an inch and a quarter in the water barometer. Thus in fact the scale of fluctuation is magnified as it were $13\frac{1}{2}$ times, so that minute changes become much more conspicuous. On the other hand, water is more susceptible of evaporation than mercury, so that the space which ought to be a vacuum is practically less so for the water barometer than for the mercury barometer. A water barometer was formerly in use at the apartments of the Royal Society of London, but is not now retained.

505. Pressures are often roughly estimated by taking the ordinary pressure of the atmosphere as the unit. Thus engineers may speak of a *pressure of three atmospheres*, by which they mean a pressure three times as great as the ordinary pressure of the atmosphere, that is a pressure of about 45 pounds on the square inch.

506. We see from Art. 504 that the pressure of the atmosphere will sustain a column of water of the height of about 34 feet. The ancients had observed that when air was withdrawn from a vessel exposed to water the water would rise in it; and they give what they considered an explanation of this and kindred facts by saying that *nature abhorred a vacuum.* But during the life of Galileo it was found that a column of water of more than 34 feet would not be sustained, so that a vacuum would be left above; this shewed the absurdity of the old doctrine. Torricelli a pupil of Galileo appears first to have suggested that the height sustained would be inversely as the density of the liquid employed, and thus he virtually constructed what we now call a *barometer.* Pascal first proposed that experiment which at once serves as a test of the truth of the theory of the instrument and furnishes one of its most valuable applications. He predicted that the column of mercury would be shorter on the top of a mountain than at its base; and he requested a friend to verify the prediction by trial on the mountain Puy de Dôme in Auvergne. The success of the experiment was complete, and Pascal afterwards repeated it on a high tower in Paris.

507. It is obvious that at the top of a mountain the pressure of the atmosphere will be less than at the bottom, for the superincumbent mass of air is much diminished. If we observe the height of the barometer at the bottom of the mountain and also at the top, we can by the aid of theory find the height of the mountain. The exact rule for the purpose is rather complicated, and requires us to know the temperature at the bottom and the top, and also the latitude of the place. But a rough rule may be given which will hold reasonably well so long as one station is not more than 3000 feet above the other. *Observe the heights of the barometer at the bottom and at the top of the mountain; divide the difference of the heights by the sum, and multiply the result by* 52428; *this will be the height of the mountain in feet.* For example, the height of a barometer on Carnarvon Quay was 30·2 inches, and on the top of Snowdon 26·5 inches. Here the sum is 56·7, and the difference is 3·7; multiply together 52428 and 3·7, and divide the product by 56·7; thus we obtain 3421. The real height of the

mountain found by careful measurement was 3555 feet. In the rough rule which we have given no attention is paid to the temperature of the air; the rule really assumes that the *sum* of the two readings of Fahrenheit's thermometer at the top and bottom is 64 degrees, or, as we may say, that the *average* temperature is 32 degrees. The result found by the rule is improved if we add a *thousandth part* for every degree in the sum of the temperature above 64. In the preceding example the temperature at the lower station was 60 degrees, and at the upper station 49 degrees; the sum is 109 which exceeds 64 by 45. Thus the result should be increased by the $\frac{45}{1000}$ part. Now $\frac{45}{1000}$ of 3421 is about 150, so that we obtain 3571, which differs very little from the true value. The process may be easily applied to a very lofty mountain by dividing the whole height into convenient separate portions; for example, if the mountain is supposed to be about 6000 feet high, we may by the rule determine with sufficient accuracy the height of a nearly midway station above the bottom, and the height of the top above this station.

508. From the level of the Thames to the top of St Paul's Cathedral in London is about 500 feet; and the difference in the height of the barometer at the two stations is about half an inch. Mont Blanc is about 15000 feet high; the barometer at its summit stands at about 15 inches, indicating that half the mass of the atmosphere is below the level of the top of Mont Blanc.

509. As we ascend in the atmosphere the density becomes less because the pressure of the superincumbent air is less; it is probable that the atmosphere extends to more than 50 miles from the surface of the earth, becoming at last excessively rare. It is a point of some interest to determine what would be the *height of the atmosphere supposed homogeneous.* This means that we want to know what would be the height of a column of fluid supported by the atmosphere, supposing the fluid incompressible and of the same density as the atmosphere is. We know that the height of the water barometer would be

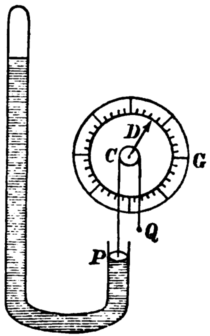
about 34 feet; now water is about 768 times as dense as air under the ordinary circumstances: see Art. 462. Hence the height of the homogeneous atmosphere would be 34×768 feet, that is about 26000 feet, that is about 5 miles. It should be noticed that this result holds for any place whatever. Thus at the top of Mont Blanc the pressure and the density of the air are just half what they are respectively at the surface of the earth: thus the pressure *there* would be able to support a column of incompressible fluid five miles high of the same density as the air has *there*.

510. If at a given place the force of gravity were susceptible of change, the height of the homogeneous atmosphere would vary in the inverse proportion. Suppose for example the force of gravity became increased. The height of the water barometer and the height of the mercury barometer would remain unchanged; the 34 feet of the one would still balance the 30 inches of the other, but the pressure measured by them would be increased in the same proportion as the force of gravity. Hence the density of the air at the surface of the earth would be increased in this proportion, and therefore the height of air of that density corresponding to the height of the water barometer would be less than the height of the original homogeneous atmosphere.

511. The following statement given by Dr Young supplies a striking notion of the wide limits between which the density of air may theoretically range; "at the distance of the earth's semidiameter, or nearly 4000 miles, above its surface, the air, if it existed, would become so rare, that a cubic inch would occupy a space equal to the sphere of Saturn's orbit: and on the other hand, if there were a mine about 42 miles deep, the air would become as dense as quicksilver at the bottom of it."

XLVI. BAROMETER FOR COMMON USE.

512. In speaking of the barometer hitherto we have had in view its construction for scientific purposes; we ought however to take some notice of a popular form in which the instrument frequently appears, which is called the *Wheel Barometer*.



A tube of uniform bore is taken closed at one end and open at the other. It is bent so as to form two parallel branches, the shorter branch being that with the open end. After being filled with mercury it is placed so that the branches may be vertical; the distance between the levels in the two branches of the tube shews the pressure of the atmosphere as represented by the height of a column of mercury. On the surface which is exposed to the atmosphere a small iron ball floats; this is denoted by *P*. To this ball a string is attached which passes round a groove in the circumference of the wheel *C*, and has at the other end a weight *Q* rather lighter than *P*. Suppose the mercury in the shorter arm to rise a little; then *P* ascends and *Q* descends; the friction of the string round the wheel is sufficient to turn the wheel, and the amount through which it has been turned is shewn by a pointer *D* attached to it, the end of which moves over a graduated circle *G*. Similarly if *P* descends then *Q* ascends, and the pointer moves in the contrary direction. Suppose, for example, that *P* ascends through one inch; then the mercury in the longer branch of the tube descends through one inch, and the height of the barometric column is diminished by two inches. Let the circumference of the wheel *C* be 2 inches, and let the circumference of the graduated circle *G* be 40 inches. Then when the barometric column is diminished by two inches one inch of string passes over the wheel *C*, so that the wheel turns half round, and the pointer goes half round, that is, over 20 inches. Thus for a change of 2 inches in the height of the barometric column there is a change of 20 inches in the position of the end of the pointer; so that the scale of fluctuation is magnified as it were *ten* times.

513. The wheel barometer serves for common use though it is not accurate enough for scientific purposes. The iron ball *P* is somewhat heavier than the counterpoise

Q, and thus there is really a slight force in addition to the pressure of the atmosphere exerted to sustain the column of mercury, namely, the excess of the weight of P over the weight of Q. Moreover when the pressure of the atmosphere is diminished, so that the mercury in the shorter branch has a tendency to ascend, the ball must be raised; and thus a change of pressure so minute as not to be sufficient to raise P would not be exhibited by the instrument.

514. It is said that during the great earthquake at Lisbon in 1775 the mercury in the barometer in England fell so low as to disappear from that portion of the tube which is usually left uncovered for observation. The rapid fall of the barometer at sea has sometimes given warning of a coming storm when the most experienced sailors would not otherwise have had any suspicion of danger. The barometer is often popularly supposed to serve as a weather glass, and the makers of wheel barometers are in the habit of putting such words as *Rain*, *Fair*, *Changeable*, against parts of the graduated circle over which the end of the pointer ranges. But experience shews that there is no very close correspondence between the weather actually occurring and the contemporaneous *position* of the pointer. By Arts. 508 and 512 it is easy to see that there may be 5 inches difference on the circumference of a wheel barometer between the position of the end of the pointer at the same epoch at the level of the Thames and at the top of St Paul's Cathedral, while the weather is probably of the same character at the two stations. The *changes* of the atmospheric pressure may however be made to give some suggestions as to the weather likely to follow such changes, especially with regard to a particular place, if observations are carefully made and studied there. The following rules are given in Dr Lardner's *Treatise on Hydrostatics and Pneumatics* as in general fairly trustworthy.

(1) *Generally* the rising of the mercury indicates the approach of fair weather; the falling of it shews the approach of foul weather.

(2) In sultry weather the fall of the mercury indicates coming thunder. In winter, the rise of the mercury indi-

cates frost. In frost, its fall indicates thaw, and its rise indicates snow.

(3) Whatever change of weather suddenly follows a change in the barometer may be expected to last but a short time. Thus, if fair weather follow immediately the rise of the mercury there will be very little of it; and, in the same way, if foul weather follow the fall of the mercury it will last but a short time.

(4) If fair weather continue for several days, during which the mercury continually falls, a long succession of foul weather will probably ensue; and again, if foul weather continue for several days, while the mercury continually rises, a long succession of fair weather will probably succeed.

(5) A fluctuating and unsettled state in the mercurial column indicates changeable weather.

515. One of the most general laws which has been observed in meteorology is expressed thus: the barometer usually falls when the thermometer rises, and the barometer usually rises when the thermometer falls. The reason is simple. When the thermometer rises the air expands and consequently overflows into the neighbouring regions; and so the pressure of the atmosphere is diminished, and the barometer falls. On the other hand, when the thermometer falls the air contracts, and this produces an influx from the neighbouring regions, and consequently an increase of pressure, and so the barometer rises.

516. An instrument called the *Aneroid Barometer* has been introduced in recent years for measuring the pressure of the atmosphere. It consists of an elastic metallic chamber either in the form of a flat box or of a short tube; the air is exhausted from the chamber which is then closed so as to be air-tight. When the pressure of the atmosphere increases the chamber slightly contracts, and when the pressure of the atmosphere decreases the chamber slightly expands. These changes are transmitted by a system of wheels and levers to a pointer which moves over an index; this index is graduated by trial, by the maker of the instrument, so that the reading against the pointer in any position records the corresponding pressure of the

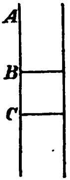
atmosphere. The instrument is recommended by its portability, for it can be made to take up little more room than a watch; but it is liable to gradual changes, so that it ought to be compared from time to time with a good mercury barometer. An *Aneroid* is a serviceable companion for an Alpine tourist, as it enables him to form a good estimate of the height he has reached. Suppose the tourist to make several observations with it in the course of a day; then even if the instrument is a little wrong the error will probably be the *same* at every station, so that the *difference* of the heights of any two stations will be correctly given: and the amount of error of the instrument at the time will be known if one of the observations is made at a station of which the actual height has been well settled by other means.

517. It appears as we have stated in Art. 491, that the pressure of the atmosphere is about 15 pounds on every square inch; and thus it may at first sight seem very strange that we are not conscious of this great pressure. But the fact is that the air is all around us; and also the internal parts of the body are filled with fluids, in the liquid or gaseous state, which exert a pressure from within equal to that of the atmosphere from without.

518. The applications of the pressure of the atmosphere are numerous and important. As we shall see hereafter, pumps for raising water act by means of this pressure. In a common pair of bellows when the upper board is raised the pressure of the atmosphere forces air through a hole in the lower board; a small valve prevents the air from escaping through this hole when the upper board is pressed down, and thus the air is driven through the nozzle of the bellows. Most persons must have seen the large constructions in which the gas used for lighting the shops and houses of a town is stored for consumption. They consist of pits lined with iron and containing water; the gas is confined between the water and a large covering vessel, in the same manner as the air in the experiment of Art. 486 is retained in the upper part of the inverted tumbler. When beer is drawn from a cask it flows at first because a little air is left at the top of the cask, and this, though expanding as beer is drawn out, still exerts for a time sufficient pressure, with

the assistance of the weight of the liquid, to overcome the pressure of the air which opposes the issue of the beer from the tap. After a time the air at the top of the cask is so much expanded that it no longer exerts sufficient pressure, and so the beer will not flow. Then a little more air is let in through a hole at the top of the cask provided for the purpose, called the *vent-peg*, which is kept closed until it is thus necessary to open it for a short time. The gurgling sound which is heard when we pour water from a decanter that is nearly full, arises from the air forced in by the pressure of the atmosphere to supply the place of the water withdrawn; the sound continues as long as the neck of the decanter is choked by the water escaping. But as the water is gradually withdrawn room is obtained in the neck of the decanter for water to pass out through part of the neck and for air to enter through the rest of the neck.

519. An interesting process for estimating the magnitude of the *pores* of bodies as compared with that of the *solid* parts depends on the use of the pressure of the atmosphere. Some substances, as charcoal and pumice stone, contain an immense number of small cavities, and to these the process may be applied. Take a long glass tube open at both ends, and fill a portion AB with charcoal, supporting the charcoal at B by a perforated partition which will allow air to pass through. Plunge the tube in a vessel of mercury to the level B, then cover the end A, and withdraw the upper part of the tube from the mercury. If there had been no air in the cavities of the charcoal the mercury would remain in the tube at the usual height of 30 inches above the level of the mercury in the vessel. But when the pressure of the atmosphere is diminished the air in the cavities of the charcoal issues from the cavities and expands, and by its elastic force compels the mercury to stand at a lower height than it would otherwise reach. Suppose the mercury stands at C at the height of 15 inches above the level in the vessel. Then the air in the pores and in BC, being under half the atmospheric pressure, occupies just double the space it did formerly; and thus the

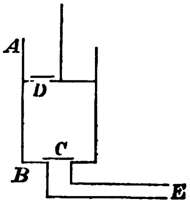


space denoted by BC is just equal to the volume of all the pores. It is found in this way that the solid part of charcoal is really about four times as heavy as water, bulk for bulk, although charcoal is usually taken to be about half as heavy as water, bulk for bulk. The solid matter of pumice stone is found to be as heavy as marble, bulk for bulk.

XLVII. AIR PUMPS.

520. It is important to be able to examine the consequences which result when bodies are withdrawn from the influence of the pressure of the atmosphere. Accordingly machines are constructed by the aid of which we can withdraw the air almost entirely from certain closed vessels, and perform various interesting experiments in the empty space. These machines are known as *Air Pumps*.

521. The construction of Air Pumps may vary a little as to details, but the principles are the same in every case. A plate of brass or other metal, made exactly plane, is provided, and on that is placed a strong glass bell with its mouth downwards; this vessel is called the *Receiver*. The glass at the mouth is ground very smooth, so that it may fit exactly on the metal plate. To ensure that the contact between the two shall be air-tight, it is usual to smear the mouth of the glass with lard or some other unctuous substance. The air is then withdrawn from the glass vessel by a pipe which passes through the metal plate; and we shall now describe the way in which this is effected. AB is a cylindrical vessel in which a piston can move up and down. At the bottom of the cylinder there is a valve C which opens upwards. There is also a valve D in the piston which opens upwards. A pipe E passes from the bottom of the cylinder and communicates with the receiver. Suppose the piston to be at the bottom of the cylinder, and that the receiver and the pipe contain air of



the density of the atmosphere. Raise the piston to the top of the cylinder; the valve D is kept closed by the pressure of the atmosphere above it, while the pressure of the air in the receiver and the pipe opens the valve C, and the air diffuses itself throughout the receiver, the pipe, and the cylinder. Push the piston down to the bottom of the cylinder; then the valve C closes and the air in the cylinder is expelled through the valve D. Then the operation may be repeated. When the piston ascends the air in the receiver and the pipe is diffused through the receiver, the pipe, and the cylinder; and when the piston descends so much of the air as was in the cylinder is expelled. Thus the air in the receiver is gradually diminished.

522. A valve is a contrivance which allows a current of fluid to pass through a tube or aperture in *one* direction but not in the other. The valves in the air pump are commonly formed of a triangular piece of oiled silk, stretched over a grated orifice in a piece of metal, to which the corners of the triangle are fastened. When air presses on the upper surface of the silk it is brought into contact with the edge of the orifice, and the passage of air prevented; when air presses on the lower surface of the silk it is raised from the edge of the orifice, and air is allowed to pass.

523. It must be observed that we cannot remove *all* the air from the receiver. Let us suppose for example that the volume of the receiver and the pipe together is nine times that of the cylinder. Then when the air is diffused through the receiver, the pipe, and the cylinder, that in the cylinder is $\frac{1}{10}$ of the whole. Thus by the up and down stroke of the piston we can remove $\frac{1}{10}$ of the air originally in the receiver and the pipe; and therefore we leave $\frac{9}{10}$. Thus we can never remove all the air; for at the end of an up and down stroke of the piston we leave in $\frac{9}{10}$ of what there was at the beginning of the stroke. It is easy to find by arithmetic what fraction of the original quantity will

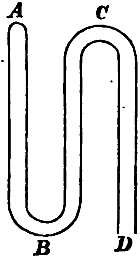
remain after any number of what we will call *operations*. At the end of the first we have left $\frac{9}{10}$ of the original quantity; at the end of the second $\frac{9}{10}$ of what there was at the end of the first, that is $\frac{9}{10} \times \frac{9}{10}$ of the original, that is $\frac{81}{100}$ of the original; at the end of the third operation we have left $\frac{9}{10}$ of the quantity at the end of the second, that is $\frac{9}{10} \times \frac{81}{100}$ of the original quantity, that is $\frac{729}{1000}$ of the original quantity; and so on.

524. Thus we see that even if there were no practical difficulties in the machine itself we could never draw out all the air from the receiver; but there are various practical difficulties which also limit the degree of exhaustion attainable. Thus however light the valve may be made it has some weight, and when the air in the receiver and pipe becomes so attenuated that it has no longer sufficient force to raise the valve *C* the exhaustion of the receiver cannot be carried further. Again, the valve *D* has the pressure of the atmosphere above it; if the piston could be pushed down to the bottom of the cylinder the air between *D* and *C*, however attenuated it might be at the beginning of the downward stroke, would become sufficiently condensed to overcome the pressure of the atmosphere. But practically the piston cannot be pushed *close* to the bottom of the cylinder, and hence it might happen that the valve *D* would finally remain closed, and so prevent the exhaustion of the receiver from being carried further. There are two ways in practice by which this difficulty is met. One way consists in closing the top of the cylinder, leaving only a valve opening upwards and a hole through which the piston rod works in an air-tight collar. In consequence of this the valve *D*, when it descends, is relieved from the pressure of the atmosphere, and so can be opened by a very small force from below. Another advantage gained is that the removal of the pressure of the atmosphere from

the upper surface of the piston diminishes the labour of the upward stroke. The air pump with this modification is called *Smeaton's Air Pump*. There is another way of securing the same advantage as by Smeaton's Air Pump. Instead of the cylinder being open to the atmosphere at the top it communicates with the receiver of an auxiliary air pump; and then by occasionally giving a few strokes to this we can always keep the pressure above D considerably less than that of the atmosphere.

525. We have thus sufficiently explained the principle of the air pump; in practice various details are regarded for the sake of convenience, at least when the machine is on a large scale. Thus we have spoken of *one* cylinder, but there are usually *two*, side by side; by means of a toothed wheel and rack-work the two pistons are moved simultaneously, one going up while the other goes down, so that the exhaustion proceeds twice as rapidly as with a single cylinder. Moreover the labour of working the pump is diminished; for while one piston is being drawn up the pressure of the atmosphere above it produces a great resistance to be overcome; but when two pistons are used this resistance is balanced by an equal pressure on the surface of the descending piston, which assists the motion. Thus the pump may be worked by a force which is sufficient to overcome the friction together with the difference of the pressures on the *lower* surfaces of the ascending and descending pistons. Instead of the valve C some instrument makers substitute a stopper, which is raised when necessary by a rod passing through the piston and working tightly in it, so as to be carried up and down by the motion of the piston-rod. In spite of all the care with which the instrument is made it is found that there is always some leakage at various parts, and although the quantity of air which thus enters is small compared with that drawn out by the early operations, yet it may be as much as is drawn out by the later operations; so that finally the exhaustion reaches a point beyond which it could not be carried were it for this reason alone. It is found that a diminution of the density of the air to one thousandth of its original value is practically almost as much as can be obtained.

526. The air pump is usually furnished with an appendage by which the degree of exhaustion can be ascertained. One such appendage is called the *barometer gauge.* The upper end of a barometer tube, instead of being closed, is allowed to be open and to communicate with the receiver. If all the air could be withdrawn from the receiver the mercury in the barometer gauge would then stand at the ordinary height; but some air will always remain, and thus the mercury in the barometer gauge will not reach to the ordinary height. Suppose, for example, that it stands at the height of 28 inches instead of 30 inches; this shews that the pressure of the air in the receiver is measured by the height of 2 inches of mercury, so that the density of the air in the receiver is $\frac{2}{30}$ of the density of the ordinary air. The *siphon gauge* is another contrivance for ascertaining the degree of exhaustion. This is a bent tube $ABCD$ closed at A, and communicating with the receiver at D. The whole of AB and part of BC is filled with mercury at first; as the exhaustion proceeds the mercury sinks in AB and rises in BC. If the air could be entirely removed the mercury would stand at the same level in AB and BC. If the mercury in AB stands at a level three inches higher than in BC then the density of the air in the receiver is $\frac{3}{30}$ of the density of the ordinary air.



XLVIII. AIR-PUMP EXPERIMENTS.

527. Numerous interesting experiments are performed by the aid of the air pump; they enable us to understand the important functions of the atmosphere by shewing us how very different the phenomena would be if that atmo-

sphere were removed. The experiments have the great merit of being very successful; the spectator can easily watch them and will admit that their testimony is decisive.

528. The experiment called the *guinea and feather experiment* is intended to shew that if the resistance of the air is removed all bodies will fall to the ground from the same starting point in the same time. A tall receiver is provided, furnished with a small platform at the top on which a coin and a feather are placed. After the receiver has been sufficiently exhausted of air the platform is removed by turning a screw provided for the purpose; the coin and the feather fall, and reach at the same instant the plate which supports the receiver.

529. The pressure of the atmosphere is illustrated in the following way. A jar open at both ends is converted into a receiver by fastening a piece of bladder over one end, and the other end is placed on the plate. After one or two strokes of the air pump the bladder becomes much stretched and bent inwards, so as to take a cup-like shape; the pressure of the atmosphere above is not fully balanced by the pressure of the attenuated air below the bladder, and so the bladder is forced inwards. By continuing the exhaustion the bladder is urged still further, until at last it bursts.

530. Let a little air be put into a bladder, and let the bladder be closed in an air tight manner and placed under the receiver of an air pump. As the receiver becomes gradually exhausted the air inside the bladder, having little pressure to constrain it, expands, and the bladder swells and appears to be fully inflated. In like manner some fruits when dried and shrivelled retain within them a little air which expands when the pressure is removed from their surfaces; thus when a shrivelled apple is placed under a receiver and the air withdrawn it is restored apparently to a plump and fresh condition; raisins in like manner expand to the size of the grape from which they were originally derived. On the re-admission of air to the receiver the fruits become again shrivelled as at first.

531. When we say that water or any other liquid *boils* we mean that it passes from the liquid to the gaseous

state; it is found that the temperature at which this change of state takes place diminishes when the pressure of the atmosphere is decreased. Under the ordinary pressure of the atmosphere, water boils at 212 degrees of Fahrenheit's thermometer; if the pressure is reduced so as to be measured by 23½ inches of mercury, water boils at 200 degrees. Thus if water which is hot, though much below the ordinary boiling temperature, be placed under a receiver and the air exhausted the water soon begins to boil furiously. By observing the temperature at which water will boil on the top of a mountain we may form a good idea of the height of the mountain, supposing that we have a Table in which are recorded the results of trials already made at various elevated places. Thus at the summit of Mont Blanc water boils at 187 degrees; so that if we observed water to boil on any mountain at that temperature we might assume the height to be equal to the height of Mont Blanc. It is found that a diminution of about one tenth of an inch in the height of the barometer corresponds to a diminution of about one sixth of a degree in the temperature at which water boils. It is obvious that when water boils at a low temperature inconvenience may arise from the fact that we cannot easily obtain water at so high a temperature as we require; it is said that the monks at the monastery of St Bernard cannot make good soup or good tea, because on account of their high situation water boils at too low a temperature. But by boiling water in *closed* vessels it is possible to produce so great a pressure as to carry the boiling point far above the ordinary temperature of 212 degrees. Other processes besides that of boiling are promoted by diminishing the pressure of the atmosphere; if a bottle of champagne is opened on the top of a high mountain the wine may burst forth and be almost entirely lost.

532. The experiment of making water boil at a low temperature, by diminishing the pressure on its surface, can be performed in a striking manner without the aid of the air pump. Water is put into a glass flask, so as to occupy about half of it; then the water is boiled by placing a lamp beneath the flask, so that the upper part of the flask becomes full of steam, the air being expelled.

The flask is now stopped with a cork, removed from the lamp, and allowed to cool down to a temperature below 212 degrees. By pouring cold water on the upper part of the flask the steam is cooled and some of it is condensed, so that the pressure on the surface of the water is much diminished; and in consequence the water begins to boil again. The experiment requires great care to prevent accidents.

533. A celebrated illustration of the pressure of the atmosphere is called the *experiment of the Magdeburg hemispheres.* Two hollow hemispheres are constructed of brass; they fit accurately together so as to be air tight and to form a hollow sphere: the two parts can however be separated with ease. A small pipe furnished with a stop-cock is fixed to one of the hemispheres; this pipe can be connected with an exhausting cylinder, such as we have described in Art. 521, and so the air can be withdrawn from the interior of the hollow sphere: the stopcock is then closed to prevent the entrance of fresh air, and the sphere may be removed from the cylinder. Now if we attempt to pull the hemispheres apart we find that there is a great resistance to prevent the separation; this is due to the pressure of the atmosphere on the external surface, and its amount may be readily assigned. We must find the number of square inches in the area of a section of the sphere through its centre, and multiply it by 15 to obtain the pressure in pounds. The experiment was devised by Otto Guericke of Magdeburg, the inventor of the air pump. He constructed such a pair of hemispheres one foot in diameter; the area of the section in this case is about 113 square inches, and multiplying this by 15 we obtain about 1700 for the number of pounds. Thus if we hang up the sphere when the air is exhausted, and attach a weight of about 1700 pounds to it, the two hemispheres will not be separated. This supposes the air to be *completely* exhausted, but even with only partial exhaustion a very great force is necessary in order to separate the two hemispheres. The rule we have given for estimating the amount of the pressure which urges one hemisphere against the other may be easily justified. Imagine one hemisphere placed mouth downwards on a

smooth horizontal table, and the air exhausted from the space between it and the table; then the resultant pressure on the external surface of the hemisphere is in fact the weight of the column of the atmosphere which stands on the portion of the table covered by the hemisphere, and reaches up to the limit of the atmosphere. The amount of this we know to be 15 pounds on each square inch of the circular area of the table which is covered by the hemisphere.

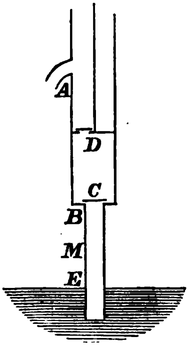
534. The *air pump* is a machine for *withdrawing* air from an enclosed space; there is also a machine called the *condenser* by which air may be *forced into* an enclosed space to any amount we please. But this instrument does not furnish us with any very important experiments, and so a brief notice of it will suffice. Take the diagram of Art. 521, and suppose the valves to open *downwards* instead of *upwards*. Let the piston be in its highest position; then when it is forced down, the pressure of the air between D and C opens the valve C, and being greater than that of the atmosphere keeps the valve D closed. Thus when the piston has reached the bottom, the air which was originally in the cylinder has been forced through C into the pipe E and the receiver with which the pipe is connected. While the piston is being drawn up, the valve C is closed by the pressure of the air below it, while the valve D is opened by the pressure of the atmosphere. Thus when the piston is at the highest point the cylinder is again full of air, and the whole process may be repeated. Every complete operation forces into the receiver and pipe as much air as would fill the cylinder under the ordinary pressure. Suppose, for example, that the volume of the pipe and the receiver together is nine times the volume of the cylinder; then at each descent of the piston, air equal in quantity to $\frac{1}{9}$ of that originally in the pipe and the receiver is forced through C. Thus at the end of five operations the air in the pipe and the receiver consists of the original air together with $\frac{5}{9}$ more; and at the end of nine operations there is just twice the original quantity of air in the pipe and the receiver.

535. The *air gun* is an instrument of no practical importance, but which may be noticed as its action depends on the condensation of air. A strong chamber is constructed into which air is condensed until the elastic force of the whole is very great. The chamber is connected with a tube in which a bullet is placed; by opening a valve the condensed air rushes out and sweeps the bullet along the tube, from which it issues with great velocity: the force which drives the bullet along the tube is the excess of the pressure of the condensed air behind, above the ordinary pressure of the atmosphere in front.

XLIX. PUMPS.

536. There are various machines for raising water from one level to another which is higher; and we will now describe some of them.

537. The *common pump* sometimes called the *suction pump*. AB is a cylinder having at the bottom a valve C opening upwards. A piston works up and down in the cylinder, having a valve D opening upwards. A pipe BE passes from the bottom of the cylinder, and the end of it is below the surface of the water in a well; let E denote the level of the water in the well. Suppose the piston to be at C, and the pipe to be full of air. Let the piston be raised to A; then the pressure of the atmosphere keeps the valve D closed, and the pressure on the valve C being lessened the air in the pipe opens this valve and fills the cylinder below the piston. The pressure of the air in the pipe is now less than that of the atmosphere, and accordingly the pressure of the atmosphere on the surface of the water in the well forces water up the pipe EB to such a height as to make the pressure at E equal to that of the atmo-

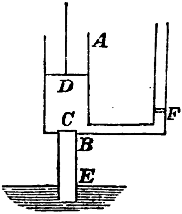


sphere. When the piston descends the valve C closes, and the air between C and the piston escapes through D. The water will rise in EB each time this operation is repeated until at last it passes through C; and now when the piston descends to C the water passes through D and is then carried up by the piston as it ascends and discharged through the spout at A.

538. It will be observed that the ascent of the water consists in general of two distinct processes. The water is raised from E to B by the pressure of the atmosphere, and in consequence of this EB must not be higher than the column of water which this pressure would support, that is about 34 feet. But the length AB may be as great as we please, provided that we have a cylinder and a piston rod of sufficient strength, and force enough to do the requisite work. For when the water reaches to a point in the cylinder the height of which above E is greater than the standard 34 feet, the pressure of the atmosphere will take it no further, and it must be lifted by the ascending piston from this point up to the spout. If the height of the spout above E is not greater than the standard 34 feet, then we have not the two processes but only the first of those just noticed.

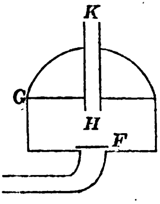
539. An error is frequently made with respect to the amount of force which must be used to work the piston; it seems to be imagined that the pressure of the atmosphere renders, or ought to render, any application of force to the piston unnecessary. Let us suppose that the piston is at some point between A and B, and that the water in the pipe has risen to the level M; so that between M and the piston there is air. Then above the piston we have the pressure of the atmosphere; and below the piston we have this pressure diminished by so much as corresponds to the height of the column EM. Thus on the whole the piston is urged down by a pressure which is measured by the height of the column EM of water; and so force must be applied sufficient to overcome this. But if the height of the piston above E is greater than the standard 34 feet, then below the piston there is a vacuum, and the pressure above it is the pressure of the atmosphere increased by the weight of the water which is to be lifted.

540. The *forcing pump.* AB is a cylinder having at the bottom a valve C opening upwards. A piston D works up and down in the cylinder. A pipe BE passes from the bottom of the cylinder, and the end of it is below the surface of the water in a well; let E denote the level of the water in the well. Just above C a tube BF passes from the cylinder and has a valve at F opening upwards. Suppose the piston to be at C, and the pipe to be full of air. Let the piston be raised; then the pressure on the valve C being lessened, the air in the pipe opens this valve and fills the cylinder below the piston. The pressure of the air in the pipe is now less than that of the atmosphere, and accordingly the pressure of the atmosphere on the surface of the water in the well forces water up the pipe EB to such a height as to make the pressure at E equal to that of the atmosphere. When the piston descends the valve C closes, and part of the air between the piston and C is forced through the valve F. The water will rise in EB each time this operation is repeated until at last it passes through C; and now when the piston descends some of the water is forced through the valve F. As in the common pump EB must not be greater than the standard 34 feet; but the ascending tube may be as long as we please, and if the pump be of sufficient strength and the force enough for the work, we may raise water to any height we please.



Sometimes instead of a piston D there is a solid cylinder working through a water-tight collar at A.

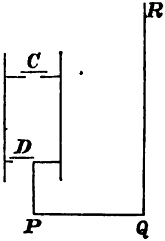
541. The stream of issuing water may be made continuous by connecting the tube BF with a large vessel having a pipe HK which reaches nearly to the bottom. Suppose the water to be forced into this vessel and to reach the level G. Then above G there is condensed air which formerly occupied all the vessel above the level H, and the pressure of this condensed air on the



water in the lower part of the vessel forces out the water through HK in a continuous stream.

542. The fire engine is a forcing pump with the appendage just described; there are usually two cylinders worked simultaneously, so that one ascends while the other descends in the manner mentioned in Art. 525 with respect to the air pump.

543. There are various other contrivances for raising water, but we need not delay long upon them, as they do not involve any new principle. We may just notice a pump called the *lifting pump*, which differs from the suction pump in having the fixed valve *above* the piston instead of *below* it. The piston is moved up and down by a frame-work of which PQR represents part. When the piston descends the valve D in it opens, and water rises above it; and when the piston ascends it lifts this water through the fixed valve C to any height that may be desired. This contrivance is said to avoid the inconvenience arising "from the length of the barrel through which the piston rod of a sucking pump would have to descend in order that the piston might remain within the limits of atmospheric pressure."

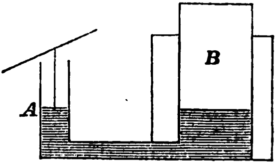


544. It will be observed that in all the pumps which we have described the pressure of the atmosphere discharges a very important function. There are however processes for raising water in which this pressure is not concerned. A simple example is that of drawing water from a well by the aid of a bucket. The *chain pump* is of the same kind; through a vertical cylinder moveable bottoms or pistons are drawn one after another lifting the water above them. In the *plunging pump* a long hollow cylinder having at its lowest part a valve opening upwards is inserted in water; the cylinder is so long as to reach considerably above the surface of the water. The water enters through the valve, and rises to the same level inside

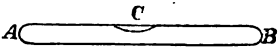
the cylinder as outside. A solid cylinder somewhat less in diameter than the hollow cylinder is plunged into the hollow cylinder; the water having no other escape is driven up in the hollow cylinder, and may be conducted through a spout provided at the highest part. Such a pump has been constructed with two cylinders, and two plungers working simultaneously, one ascending as the other descends; a plank moveable about its middle point has its ends connected with the plungers, and a man walking backwards and forwards on the plank continually by his weight supplies the force necessary to raise one plunger and depress the other.

545. *The Screw of Archimedes* is a machine for raising water, which is said to have been invented by that ancient philosopher for the purpose of enabling the inhabitants of the low grounds of Egypt to clear away the stagnant water left by the Nile after its inundations. This machine may be presented in slightly different forms, and we will confine ourselves to the simplest. A hollow tube is bent into the form of a corkscrew and placed inclined to the horizon. The screw can be turned round in the manner of a corkscrew and is so fixed that its lower end alternately dips below the surface of the water and rises above it as the screw is turned round. Then during each turn of the screw water enters at the lower end; and in successive turns of the screw the water thus entering passes on up the screw until at last it issues from the top. The fact that the water will thus pass along the screw is not very easy to establish by the aid of diagrams so as to be intelligible to the early student; but it becomes clear on examining a model of the machine. The screw must obviously not be inclined at too great an angle to the horizon: for instance, if it is placed vertically no water will be raised. Let the screw be at rest, and suppose that as we pass along it we find points such that the screw *rises on each side of them*, then the proper inclination is not exceeded. Moreover, of the two directions in which the screw can be turned round only one is suitable, namely, that in which a corkscrew would be turned round in order to penetrate a cork occupying the place of the water.

L. VARIOUS INSTRUMENTS.



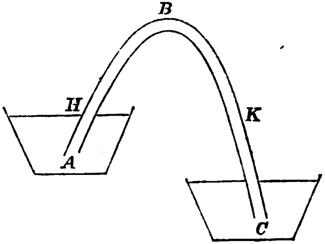
646. *Bramah's Press* or the *Hydrostatic Press.* A piston A can be moved up and down a small cylinder by the aid of a lever; a piston B can be moved up and down a large cylinder. The two cylinders communicate by a channel filled with water, which also occupies the cylinders up to the pistons. If any pressure is exerted on the smaller piston A an equal pressure is transmitted to every portion of the piston B of equal area. Thus if the area of a section of the piston B is a hundred times the area of a section of the piston A, then when a pressure of 10 pounds is exerted on the piston A the piston B is urged up by a pressure of 1000 pounds. The pressure on A may be applied by means of a lever. These machines are used very extensively in practice where great force is required, as in testing the strength of iron chains, or in raising enormous weights; they may be constructed so as to exert a pressure of three hundred tons. They have been employed in some of the greatest works of modern engineering, as in launching the *Great Eastern* steamship, and in raising the *Britannia Bridge.* It has been said that if Archimedes had been acquainted with the Hydrostatic Press he would have preferred it to the Lever for his proposed feat of moving the world. The principle of the machine has been known since the time of Pascal, but it long remained undeveloped on account of the difficulty of making the piston B work in a water-tight manner in its cylinder. Bramah invented a peculiar leather collar which fits more tightly as the pressure on the piston increases.



547. The *Spirit Level.* It is sometimes necessary to determine if a certain plane surface is accurately horizontal. A small tube is almost filled with a liquid which is as nearly as possible a perfect fluid, for instance spirit; as the tube is not quite

full of the spirit a small bubble of air remains. The tube is apparently straight, but is really an arc of a very large circle. It is mounted on brass work, and is so adjusted that when placed on a strictly horizontal plane the bubble occupies the middle of the tube. If the plane on which the instrument is placed is not horizontal this is shewn by the circumstance that the bubble, which always tends towards the highest point of the tube, moves away from the middle point.

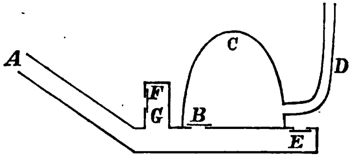
548. The *Siphon* is a contrivance for transferring liquid from a vessel through a tube which rises to a higher level than the liquid in the vessel. The siphon is a bent tube ABC open at both ends. Let it be filled with liquid and the ends closed; and let it be put with A, the end of the shorter arm, beneath the surface of the liquid in a vessel; let H and K denote points



in AB and BC which are in the same level as this surface. Open the end A of the tube; then the pressure at H and at K will be equal to the pressure of the atmosphere. Thus the liquid contained between K and C is urged down by its own weight, and by a pressure at K equal to the pressure of the atmosphere. Open the end C; then as there is only the pressure of the atmosphere there, the liquid in KC will not be supported and will flow out. More liquid is then forced by the pressure of the atmosphere up HB to supply the vacuum which would otherwise be formed, and thus a continuous flow is maintained, until the level of the liquid in the vessel sinks below the end A. It is necessary that the height of B above the level HK should be less than the height of a column of the liquid which the pressure of the atmosphere would sustain; thus if the liquid is water this height must be less than 34 feet. For if the height were greater than this the pressure of the atmosphere could not force liquid up to B so as to maintain a flow through BC.

549. We have supposed the siphon to be filled with the liquid at first, but other means may be used to set it in operation. For instance, the end A may be put under the liquid, and then the air may be withdrawn from the tube by suction at C; the pressure of the atmosphere will force liquid up AB to take the place of the air withdrawn.

550. *Montgolfier's Ram*, or the *Hydraulic Ram.*



A is a pipe by which water descends obliquely from a reservoir; the water by means of a valve B, which opens upwards, can enter into an air vessel C, and it can leave this through an ascending tube D. At E there is a valve opening downwards by which water may escape without entering the air vessel. Suppose the pipe, and its continuation below the air vessel, to be full of water; also suppose the valve B closed by its own weight, and the valve E supported in its place. Let the valve E be set free; then its own weight draws it down; the water flows out at this point, and by its motion carries up the valve E until the orifice becomes stopped. At this instant the water in the channel from A to E, being suddenly checked in its motion, exerts a very great pressure on the surface which constrains it, forces open the valve B, enters the air vessel and the ascending tube, and at the same time compresses the air in the upper part of the air vessel. As soon as the water comes to rest B closes, then E sinks by its own weight, and the action is renewed. In this manner more water passes through B at each stroke, and by the reaction of the compressed air in the upper part of C the water is forced up the ascending tube to any required height, where it is discharged.

551. A small auxiliary chamber is sometimes added. The upper part F of this contains air, the lower part contains some of the water of the pipe. When for an instant the water comes to rest the compressed air in F recoils

and occupies a larger space than it did originally; thus the pressure below *E* is lessened so that the valve descends more readily. A portion of the air in *C* and *D* is taken up by the water, which absorbs a considerable quantity of air under high pressure: to supply the waste thus caused a valve is put at *G*, which opens and allows air to enter during the recoil of the water; and some of it finds its way through *B* to the vessel *C*.

552. The hydraulic ram is easily constructed, and it is durable as the valves are of a very simple character, so that it may be advantageously applied whenever there is a stream or a reservoir of water which would not otherwise be used. But much of the water is wasted by flowing through *E*, and it must depend upon local circumstances whether this is an economical method for using an available supply of water power.

553. The form and use of a *balloon* are well known from familiar observation. A large bag is constructed of silk, and filled with light gas so that it takes a globular form. It weighs much less than an equal volume of common air, even when a car occupied by two or three persons is attached to it; and so it rises in the atmosphere on being released from its fastenings. Balloons were invented by two brothers named Montgolfier; they filled their balloon with air which was kept heated by a small fire, and owing to the heat was lighter, bulk for bulk, than the atmosphere. Afterwards hydrogen gas was used which is only about one fourteenth as heavy as the atmosphere, bulk for bulk. At the present time the gas which serves for lighting streets and houses is used: this is much heavier than hydrogen, being about half as heavy as the atmosphere, bulk for bulk; but it has the great advantage of being very easily obtainable. It is important that the balloon should not be quite filled at first, because the pressure of the atmosphere diminishes as the balloon rises, so that the gas within expands and tends to burst the silk. The balloon is urged upwards by a force equal to the excess of the weight of the air displaced above the weight of the balloon and its adjuncts. The weight of the air displaced by the balloon alone, without its adjuncts, remains constant during the ascent until the balloon has swollen out

to its extreme size. For suppose, for example, the balloon to have ascended so high that the density of the air is half its value at the surface, then the pressure is half what it originally was, and so the balloon swells out to twice its original size, and therefore the weight of the air which it displaces remains unchanged. Bags of sand are usually taken in the car which may be emptied when an increase of the upward velocity is desired. For the upward force is the weight of the displaced air diminished by the weight of the balloon and its adjuncts, and so the upward force is increased by throwing out the sand. Moreover the mass to be moved is diminished by the same operation. Thus on both accounts the upward velocity will be increased: see Art. 132. By means of a valve the persons in the car can allow some gas to escape, and so diminish the upward force on the balloon when they wish to descend.

554. Ascents in balloons have been made for the purpose of determining the temperature and the pressure of the atmosphere at different heights above the surface of the earth. In some cases balloons have risen to a height of more than five miles. But they have not hitherto been of use in passing from one assigned place to another, on account of the want of means for constraining their course. They are carried along in the direction of the wind; and even this is not a definite course which can be known beforehand, because it is found that in different strata of the atmosphere there are currents which tend in different ways. Scientific men seem now to be turning their attention to the construction of flying machines, yet up to the present time balloons are the only contrivances for practically moving through the air. It is said that M. Giffard, an eminent French engineer, has recently obtained some success in controlling the course of a balloon. He constructed an oblong pointed balloon, to the stern of which he attached a rudder, and in the car he carried a small steam engine of three horse power, which worked a screw formed of sails like a windmill. M. Giffard was able to make way through the air at about six miles an hour against the wind, and to give a circular motion to his balloon. *Quarterly Review*, July, 1875.

555. The *Diving Bell.* This is a large vessel closed at the top and sides, but open at the bottom. It was originally shaped something like a bell, and took its name from this circumstance; but it is now usually made square at the top and bottom, the bottom being a little larger than the top. The bell is lowered into the water with its mouth downwards, precisely as the tumbler is put into water in the experiment of Art. 486. The pressure to which the air in the bell is thus exposed forces it into a smaller volume, so that the water rises some way in the bell; yet there remains air enough in the upper part of the bell to enable persons to breathe at considerable depths below the surface. Moreover the water may be almost wholly expelled from the bell by forcing in air from above through a tube which enters the bell at its mouth; also the air may in this way be changed as often as it becomes unfit to be breathed.

556. We may easily find the space which the air in the bell will occupy when we know the depth below the surface of the water, assuming that the original stock of air has not been increased. Suppose for instance that the level of the water inside the bell is 20 feet below the surface of the water. The pressure of the atmosphere is measured by a column of water 34 feet high; and thus at the level of the water in the bell the whole pressure is measured by a column in height $34+20$ feet, that is 54 feet. Thus the air in the bell was originally exposed to a pressure measured by 34 feet, and afterwards to a pressure measured by 54 feet; therefore, by Art. 494, the volume of the air in the bell is $\frac{34}{54}$ of its original volume, that is $\frac{34}{54}$ of the volume of the bell.

557. The diving bell, as is well known, is employed in recovering objects from the sea, especially the stores or the treasures lost in a sunken ship. The bell is usually furnished with seats for the workmen, and shelves for their tools; there are also means of communicating signals between persons in the bell and others at the surface. The increased pressure of the air causes some inconvenience to the workmen in the bell, producing especially a painful

sensation in the ears; but no danger is incurred, provided there are trustworthy persons at the surface who will pay immediate attention to the signals made to them.

558. It is usual to give an account, in connection with Hydrostatical Instruments of two which are more closely related to *Heat* than to our present subject; and accordingly we shall briefly notice the *Thermometer* and the *Steam Engine.*

559. *The Thermometer.* Almost all bodies expand by heat and contract by cold. This circumstance is used to furnish the means of recording and comparing temperatures; the expansion or contraction of mercury combined with that of a glass vessel in which it is contained is usually employed for this purpose. The thermometer consists of a slender glass tube closed at one end and expanding at the other into a hollow globe called the *bulb.* The bulb and part of the stem contain mercury; the rest of the stem is a vacuum. The thermometer is made in the following manner. At first the end of the slender tube is open, and is placed below the surface of mercury in a vessel; the bulb is heated which partially expels the air from it. As the bulb cools mercury is forced by the pressure of the atmosphere through the tube into the bulb. The mercury in the bulb is then heated until it boils, the other end of the tube being still surrounded by mercury: thus the remaining air is expelled, and its place supplied by mercurial vapour: this condenses in cooling and more mercury enters the tube and fills it completely. When the temperature is lowered to the highest point which the instrument is intended to mark, the end of the tube hitherto open is closed. As the mercury continues to cool it contracts and leaves a vacuum at the upper part of the tube.

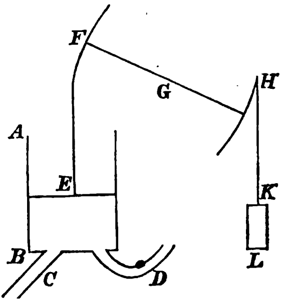
560. *To graduate a thermometer.* The instrument is put into melting snow, and a mark is made opposite to the end of the column of mercury in the tube; this is called the *freezing point.* Next the instrument is surrounded with the vapour of water boiling under the standard pressure of the atmosphere, and a mark is made opposite to the end of the column of mercury in the tube; this is called the *boiling point.* The space on the tube

between the marks which denote the freezing point and the boiling point may be divided into any number of equal parts which is convenient; these parts are called *degrees*. In the Centigrade thermometer the space is divided into 100 degrees, and 0 is put at the freezing point and 100 at the boiling point; this is the instrument commonly used for scientific purposes. In Fahrenheit's thermometer the space is divided into 180 degrees, and 32 is put at the freezing point and 212 at the boiling point: this is the instrument in popular use in England, and it is often called the *common thermometer*. In Reaumur's thermometer the space is divided into 80 equal parts, and 0 is put at the freezing point and 80 at the boiling point.

561. It is easy to pass from a reading on one thermometer to the corresponding reading on another. For instance, suppose that a Centigrade thermometer indicates 30 degrees, and that we require the corresponding reading on Fahrenheit's thermometer. The number 30 on the Centigrade thermometer indicates $\frac{30}{100}$, that is $\frac{3}{10}$, of the whole space between the freezing point and the boiling point; now in Fahrenheit's thermometer this space is divided into 180 degrees, and $\frac{3}{10}$ of 180 is 54: thus the reading in Fahrenheit's thermometer must be 54 above the freezing point, and as the reading for the freezing point is 32 the required reading is $32+54$, that is 86. Again, suppose that Fahrenheit's thermometer indicates 104 degrees, and that we require the corresponding reading on the Centigrade thermometer. Since $104-32=72$, Fahrenheit's thermometer indicates 72 degrees above the freezing point. Now 72 degrees above the freezing point means $\frac{72}{180}$, that is $\frac{2}{5}$, of the whole space between the freezing point and the boiling point. This space in the Centigrade thermometer is divided into 100 degrees; and $\frac{2}{5}$ of 100 is 40: thus the required reading is 40.

562. The temperature of melting snow is always the same. The temperature of boiling water is different for different states of the pressure of the atmosphere: see Art. 531. The exact definition of the boiling point of the Centigrade thermometer is the boiling point when the height of the barometer is $\frac{19}{25}$ of a *metre* at a place on the level of the sea in latitude 45 degrees North. The $\frac{19}{25}$ of a metre is about $29\frac{23}{25}$ inches. A variation of 1·045 of an inch in the barometer from the standard height causes a change of about one degree centrigrade in the temperature of steam.

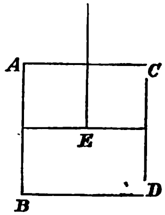
563. The *Atmospheric Steam Engine.* *AB* is a hollow cylinder into which a pipe *C* passes from a boiler. *D* is a pipe which communicates with a vessel of cold water. *E* is a piston which works up and down in the cylinder. The piston is connected with one end of a lever *FGH* which can turn round a fixed point *G*. From the other end of the lever is suspended a rod *HK*, by which the machinery connected with the steam engine is set in motion; this rod carries a weight *L* which is equal to half the atmospheric pressure on the upper surface of the piston *E*. An apparatus connected with the lever opens a cock in *C* when the piston is in its lowest position, and closes it when the piston is in its highest position. A cock in *D* is opened when the piston comes to its highest position, and is closed soon after the piston begins to descend. Suppose that the piston is in its lowest position; and let the pressure of steam in the boiler be a little greater than that of the atmosphere. When the cock in *C* is opened steam rushes into the cylinder; thus the pressure on the two surfaces of the piston is about equal, and the piston is made to rise by means of the weight *L* attached



to the end H of the lever. When the piston is at its highest point the cock in D opens, and a jet of cold water enters the cylinder; this condenses the steam and forms a vacuum below the piston. The piston is then forced down by the pressure of the atmosphere which is twice as great as the opposing weight L. The water introduced into the cylinder, together with that arising from the condensed steam, escapes through a valve provided for that purpose at the bottom of the cylinder; this valve opens when the piston is nearly at its lowest point.

564. The great defect of the atmospheric steam engine is that by the admission of the cold water the cylinder is cooled at every stroke, so that when steam again enters the cylinder part of it is condensed; this leads to a waste of fuel. Watt improved the engine by having the condensation carried on in a separate chamber. Thus instead of water entering through D to condense the steam in the cylinder, the steam escaped through D into a vessel of cold water and was there condensed. But further improvements were made, and thus the engine assumed the form now to be described.

565. *Watt's Steam Engine.* AB is a hollow cylinder closed at both ends; C and D are openings at the ends. A piston E works up and down in the cylinder by means of a rod which passes through a steam-tight collar in the upper end of the cylinder. A vessel of cold water, called the condenser, is placed near the cylinder. The openings at C and D are connected with appropriate pipes furnished with cocks, so that steam may be alternately admitted and expelled. When the piston is in its lowest position steam from the boiler enters through D, and at the same time a communication is made between C and the condenser, so that the steam above the piston passes away and is condensed while the piston is forced up by the pressure beneath it. When the piston is in its highest position steam from the boiler enters through C, and the steam below the piston passes



away through *D* to the condenser, so that the piston is forced down by the pressure of the steam above it. This engine is sometimes called the *double-acting steam engine* from the circumstance that the force of steam drives the piston alternately up and down.

566. *The High-pressure Steam Engine.* The construction is much the same as in Watt's steam engine, but there is no condenser. The steam has a pressure many times greater than that of the atmosphere, and instead of being condensed after each stroke it is permitted to escape into the open air. This is the form of steam engine used on railways.

567. We have given only a brief sketch of the steam engine; there are many important details connected with the subject, for an account of which the student must consult special treatises. One of the most remarkable contrivances due to Watt is called the *Parallel Motion.* In the atmospheric steam engine the ends of the lever are arched, and *chains* passing round them are connected with the ends of the rods which move up and down; thus the piston *E* can pull the end *F* down, but cannot push it up. Watt devised a system of jointed bars which allowed the piston rod to move vertically and *F* to describe an arc of a circle, while the piston rod could push as well as pull the end of the lever. The object is very important not only in the steam engine but in various cases where motion in a right line is to be transformed, as it were, into motion in a circular arc, and the contrary; attention has recently been drawn to this transformation by some fine researches of Professor Sylvester in relation to a method invented by M. Peaucellier.

LI. FAMILIAR APPLICATIONS.

568. In this Chapter the principles which have been already explained will be applied to some familiar examples, in some cases taken from well-known toys of children.

569. The *Kite* is memorable as having been a favourite toy with Newton; and the younger Euler, a well-known

mathematician, has devoted to it a memoir in the Transactions of the Berlin Academy for 1756. It is unnecessary to describe an object so well known as the kite; we will suppose it floating in the air and at rest. There are three forces which act and maintain equilibrium; the weight of the kite, including the tail; the force of the wind; and the tension of the string. The weight acts vertically downwards. The wind may be taken to blow horizontally, but its force must be supposed to be resolved into two components, one *along* the surface of the kite, and the other at *right angles* to the surface; it is only the latter which produces any effect on the kite, for the former would be like a wind gliding over the surface of the kite and not pressing it: see Art. 473. The tension of the string acts in the direction of the string at the point where it leaves the kite; but usually the string near the kite is, as it were, divided into two, one going to a point near the upper end of the kite, and the other to a point near the lower end: in this case the tensions of the two strings are equivalent to the tension of a single string the direction of which is that of the kite-string at the point where it is divided into two. The three forces which thus act on the kite must fulfil the proper conditions in order to produce equilibrium; this will require that their directions should meet at a point, and that their magnitudes should be in the proper proportion.

570. The kite then adjusts itself to a suitable inclination, and the tail adjusts itself to a suitable position, so as to bring about the precise circumstances necessary for equilibrium; but it would not be easy to state in words exactly what these must be. If we consider the kite alone we can find the situation of its centre of gravity by the experimental method of Art. 170; but when the tail is attached the situation of the centre of gravity of the whole will depend on the position taken by the tail. The weight of the kite alone, or of the kite and the tail, can easily be ascertained. If we consider the kite alone, the points at which the force of the wind on it may be supposed to act can be found. For we may conceive the force of the wind to consist of parallel pressures on all the portions of the face of the kite, the pressures being equal on equal

portions of the face. The point where their resultant acts will be the centre of gravity of the *face* of the kite; this is not necessarily the centre of gravity of the *whole kite*, for there is usually a straight piece of wood running down the middle of the kite, and a curved piece of wood at the top; but it would be the centre of gravity of the kite if these pieces of wood were removed. We might cut out a figure in pasteboard, or thick paper, of the shape of the kite, and then its centre of gravity would be practically coincident with the centre of gravity of the *face* which we require. But when the tail is taken into consideration also, since there must be some action of the wind on that, it is impossible to say at what point the resultant force of the wind may be supposed to be exerted. The *string* itself will be in equilibrium when all the system is at rest, and this gives rise to an interesting problem though too difficult for an elementary book. The forces acting on the string are its own weight, the pressure of the wind, and the tensions at the two ends, where the string may be considered to be held fast. It is obviously seen by trial that the string does not take the form of a straight line.

571. *The See-saw.* A plank is put across a log of wood, and one boy sits on one end of the plank and another boy on the other end. The plank turns round the part in contact with the log as a fulcrum, and so the boys move alternately up and down. If the boys are of unequal weight their positions must be adjusted according to the principle of the lever; the distance of the heavier boy from the fulcrum must bear the same proportion to the distance of the lighter boy from the fulcrum as the weight of the lighter boy bears to the weight of the heavier boy. The motion is kept up by each boy in his descent touching the ground with his feet, which diminishes his pressure on the plank, and gives to the weight of the other boy a momentary superiority. Or the descending boy may push firmly against the ground, which tends still more to send him up and bring the other boy down.

572. *The Swing.* This well-known contrivance bears a resemblance to a pendulum. The person in the swing

may have his motion kept up by receiving an occasional push to counteract the effects of the resistance of the air and the friction at the points of support. He may also keep up, and even increase, his own motion by crouching in the swing when at the highest point and rising in the swing when at the lowest point. It is not possible to establish this result strictly in an elementary manner, but we may give some explanatory remarks. When the man crouches in the swing he puts his centre of gravity further from the fixed point than it was before; thus the result is like that of augmenting the length of a simple pendulum while starting it at the same inclination to the vertical: therefore the centre of gravity descends through more vertical space and so has a *greater velocity* at the bottom than before: see Art. 318. Again, when the man rises he brings his centre of gravity nearer to the points of support; thus the result is like that of diminishing the length of a simple pendulum while starting it with the same velocity from the lowest point; therefore the pendulum must move through a *larger angle* than before, in order that by passing through the same vertical space as before it may lose the velocity with which it started. Thus by either crouching at the highest point or rising at the lowest, the motion is increased; and of course if *both* changes are made the effect produced is all the greater.

573. *The Top.* A few words must be devoted to this striking toy; the tops introduced of late years, which continue spinning for several minutes, are especially interesting. The reader will perhaps be disappointed by the remark that it is impossible to give any satisfactory account of the subject in an elementary book; but such is the fact; for the discussion of the motion is really a most difficult problem, requiring the highest mathematical resources. One peculiarity of the motion is the steady character which sometimes belongs to it when the top in popular language is said to be *sleeping*. We know that it is almost impossible to balance the top on its point when the top does not rotate, and the question then is, how can the top be kept from falling when it rotates rapidly? Suppose that at any instant the centre of gra-

vity of the top, instead of being vertically over the small base, is a little to the right-hand side; then before the top has time to fall over towards this side the rotation carries the centre of gravity round to the opposite side, and thus prevents the fall towards the right-hand side. Another peculiarity of the motion is the fact that under certain circumstances the top raises itself from an inclined position to a vertical position; this is done by the aid of the friction. It may suffice to convince the reader of the difficulty of the subject just to state that some reasoning given by Euler in explanation of this fact, and adopted by Dr Whewell, is pronounced by a well-qualified judge to be vague, inconclusive, and directly the reverse of the truth. A popular explanation has been given which assumes the existence of a *friction* acting vertically *upwards* at the point of contact with the ground; but in the first place friction acts horizontally and not vertically, and in the second place the effect of a vertical force would be to increase the inclination of the top to the vertical instead of diminishing it: see Art. 345.

574. *The Popgun.* This is a well-known toy. A pellet is put at each end of a hollow cylinder so as to keep the cylinder air-tight. One of the pellets is pushed forwards by a stick, and thus the air between the two pellets is compressed, and its elastic force increased. The other pellet then is pushed out as soon as the pressure of the compressed air behind is greater than that of the atmosphere in front, together with the friction between the pellet and the hollow cylinder.

575. *The Squirt.* A hollow cylinder is tapered off to a point where there is a hole; this end is put under the surface of water in a vessel, and a piston drawn nearly through the cylinder just as in the common pump: water is forced in by the pressure of the atmosphere. The water will not flow out of itself if the squirt is removed from the vessel, but may be expelled to some distance by driving the piston back rapidly.

576. *The Sucker.* A string is fastened to the middle of a circular piece of leather, and the leather is moistened

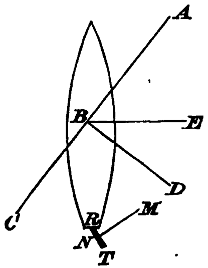
and pressed closely against the flat surface of a stone or other heavy body, so as to exclude the air. Then the heavy body may be raised by means of the string; the leather seems as if it were glued to the body, being really held in contact with it by the pressure of the atmosphere on the upper surface of the leather.

577. A *wheel-carriage* illustrates various mechanical principles. Compared with a sledge it has the great advantage of a diminished friction. In the sledge we have *sliding* friction between the sledge and the ground; in the carriage we have *rolling* friction between the wheels and the ground; rolling friction is less than sliding friction: see Art. 328. In the carriage there is sliding friction between the axle and the nave of the wheel in which it turns; but the rubbing surface is small and so can be easily made smooth and kept greased. It is found on the whole that when a road is in good condition, the resistance on a wheel-carriage from friction does not exceed $\frac{1}{30}$ or $\frac{1}{45}$ of the load. The shock against obstacles in the road is much less in the case of a wheel-carriage than of a sledge; the wheel is pulled over the obstacle, turning round it as a fulcrum. Theory shews that other circumstances being the same a large wheel is pulled over an obstacle more easily than a small wheel. The forewheels of a carriage are made small, because as they can go under the body of the carriage *turning* is made easy. When the carriage is going *up* hill the pressure is greater on the hind wheels, which are large; and when going down hill the pressure is greater on the front wheels, which are small: thus in the former case we gain ease, and in the latter security. The *springs* of a carriage much diminish the shock of obstacles; they serve the same purpose as the powerful springs which are placed at the ends of railway carriages, and called *buffers:* the force of the shock at starting or stopping, when one railway carriage strikes up against another, is spent as it were in compressing these powerful springs instead of acting directly on the carriages and the passengers.

578. *The Rocket.* This consists of a long cylindrical tube which is filled with an inflammable mixture consisting chiefly of gunpowder. This mixture is ignited, and as it burns away the rocket is driven upwards vertically or obliquely, as it may have been pointed. The moment the powder is in flames a large quantity of gas is produced; this is to some extent confined by the atmosphere, which resists its escape, and so it presses against the end of the hollow part of the rocket and forces the rocket upwards. Thus the pressure of the atmosphere is necessary for the motion of the rocket; for if there were no atmosphere the gas would find no resistance to develope its elastic force, and would exert scarcely any pressure on the rocket. The same principle has been suggested as suitable for producing motion in a vessel; a steam engine was to force out water from the vessel into the surrounding sea or river, and the resistance of this sea or river against the issuing water was to force the vessel on. It is asserted that the animal world presents illustrations of the same principle; for the larva of a dragon-fly appears to swim forward by ejecting water from its tail, and the nautilus likewise.

579. The motion of a sailing vessel produced by the action of the wind on the sails, and guided by the action of the water on the rudder, will afford good illustrations of mechanical principles: we begin with the action of the wind. Suppose that a ship has to sail from the South towards the North, and that the wind is blowing exactly in the same direction; then if a sail is stretched right across the vessel, that is at right angles to the length of the vessel, it catches the full pressure of the wind, and the vessel is urged on. In this case it would be of little use to put a second sail parallel to the first just behind or before it; for the sail nearest the bow would be sheltered by the other from the wind, and so would experience only a slight pressure. Next let us suppose that the ship is to sail from the South to the North as before, but that the wind blows from East to West. Let the sail now be stretched, not at right angles to the length of the ship, but in some direction intermediate between the proposed course of the vessel and that of the wind; say towards

the North-East and South-West. Let ABC denote the position of the sail, and EB the direction of the wind. Suppose the velocity of the wind resolved into two components, one along AB and the other along DB at right angles to AB; the latter only is effective in producing a pressure on the sail: see Art. 473. Thus we get a force acting on the sail in the direction DB; this force is not in the direction in which the ship is to move, but we may suppose it resolved into two components, one acting from the stern to the bow and the other at right angles to this. The former component urges the ship in the required direction; the latter would tend to urge the ship sideways, but it produces little effect because of the resistance opposed by the water to the large surface which the ship presents sideways; what effect it does produce must be counteracted from time to time by the action of the rudder.



580. It will be seen that two or three sails may be used parallel to each other; thus in the diagram a second sail, as large as the first, might be placed near the stern, so that neither of them should intercept the wind from the other. We have supposed that the angle between the direction of the wind and that of the course is a right angle; but this is not essential: the precise position in which the sails must be put in order to secure the greatest velocity will depend on the angle between the directions of the wind and the course, and can be determined in every case by trial. A ship cannot sail *directly against the wind*, but it may approach very nearly to such a course. For example the wind might be from the North East, and yet the ship sail from South to North. If the wind is directly, or nearly directly, opposed to the desired course the ship must adopt a zigzag course. Thus, for instance, if the ship cannot sail directly from South to North it may sail towards the North East for some dis-

tance, and then change its direction and sail for some distance towards the North West; thus by one or more such *tacks* the ship may reach its proposed port.

581. We proceed to explain the use of the *Rudder*. Suppose the ship sailing from South to North, and let the rudder be in its middle position, that is let the rudder and the keel be in one plane. Then the resistance of the water acts on one side of the ship precisely in the same manner as on the other, and so does not tend to change the direction of the motion. Suppose then that in order to avoid an obstacle, as for instance another ship, it is desirable to change the direction of the ship by turning the bow towards the right hand side: to effect this the rudder is turned towards the right-hand side. Let RT denote the position of the rudder thus turned to the right-hand side; by reason of the passage of the rudder through the water, with the ship, a resistance is exerted on the rudder in the direction MN which is at right angles to RT. One effect of this would be to push the whole ship in the direction MN; but a more important effect is to give the ship a rotation in the horizontal plane round its centre of gravity; see Arts. 344 and 345. Thus in virtue of this rotation the bow of the ship turns towards the right hand, and the stern towards the left hand. When the proper change of direction has been produced the rudder is put back again into its middle position.

582. *Rowing Boat.* In Art. 198 we just alluded to the oar of a boat as affording an example of a Lever of the second class; but it will be instructive to consider the forces which act on the boat. The man who rows exerts by his hands a certain force on the oar or the pair of oars which he grasps: we will suppose that he holds a single oar, and we will denote the force he exerts on it by P. In consequence of this effort of the man a certain force is exerted by the oar on the row-lock and by the row-lock on the oar; this we will denote by Q. If the man holds two oars P is the sum of the forces he exerts on the oars, and Q the sum of the forces at the row-locks. Now Q is greater than P by the nature of the lever, for the oar may be considered to form a lever of the second class with the fulcrum at the blade in the water. It might

at first sight seem that Q is the force which the man applies to the boat to maintain it in motion. But we must remember that he cannot bring his strength to bear on the oar unless he pushes with his feet against the boat in the opposite direction. The force thus exerted is practically equivalent to the force which he exerts by his hands, that is to P. Hence he really exerts on the boat the force $Q - P$. It is only through having the external water as a fulcrum that he is able to bring the force Q *which is greater* than P to act in the contrary direction, and thus to leave $Q-P$ to be effective on the boat. Children in a railway carriage may sometimes be seen pushing at the front of the carriage in order to start it; they do not know that, since action and reaction are equal, the force of their hands is balanced by that of their feet in the contrary direction.

583. The preceding Article suggests the remark that in a Mechanical Problem there may be more than one distinct body involved, and that in order to discuss the equilibrium or the motion we may have to consider each body separately. Thus if a man row a boat with a pair of oars there will be four bodies, namely the boat, the man, and the two oars, each acted on by its special system of forces. We will illustrate the matter by considering the case of a wheelbarrow which contains a load, as a stone, and is held by a man in the position just previous to motion so that the whole is in equilibrium. The stone is acted on by its own weight downwards, and by the resistance of the wheelbarrow at the point or points of contact. The wheelbarrow consists of two distinct parts; namely, the trough-part including the legs and arms, and the wheel-part consisting of the wheel with its axle. The trough-part is acted on by its own weight, by the pressure of the stone, by the action of the wheel-part at the ends of the axle, and by the force exerted by the man's hands. The wheel-part is acted on by its own weight, by the action of the trough-part at the ends of the axle, by the resistance of the ground in a vertical direction at the points where the wheel touches the ground, and by the friction of the ground at the same point in a horizontal direction. The man is acted on by his own weight, by the pressure of the

wheelbarrow on his hands, by the resistance of the ground in a vertical direction, and by the friction of the ground in a horizontal direction. By the law of the equality of action and reaction the pressure of the stone on the wheelbarrow is equal and opposite to that of the wheelbarrow on the stone; the action of the trough-part on the wheel-part is equal and opposite to that of the wheel-part on the trough-part, and the action of the man on the wheelbarrow is equal and opposite to that of the wheelbarrow on the man. We suppose the ground to exert a friction, which it will do in practice; but it is theoretically conceivable that there is no friction. In this case as the man's weight and the action of the ground are both vertical, so must the action of the wheelbarrow on him also be, in order that he may be in equilibrium; that is, he must pull the wheelbarrow in a vertical direction upwards and be pulled by it in a vertical direction downwards. This would require him to lean backwards so as to throw his centre of gravity behind the points at which he presses the ground; for the upward force on him must fall between the two downward forces that he may be in equilibrium. In this case all the other forces which act on the trough-part being vertical the action from the wheel-part must also be so; then the other forces on the wheel-part being vertical there can be no friction on it.

LII. WORK.

584. In modern treatises on Practical Mechanics the term *Work* is employed in a peculiar sense; and various useful facts and rules are conveniently stated by the aid of the term in this sense. We propose accordingly to give some explanations and illustrations which will enable the reader to understand and apply such facts and rules.

585. The labour of men and animals and the power furnished by nature in wind, water, and steam, are employed in performing operations of various kinds, such as drawing loads, raising weights, pumping water, sawing wood, and driving nails. In these, and similar operations, we may perceive one common quality which is adopted

as characteristic of Work, and suggests the following definition; *Work is the production of motion against resistance.*

586. This definition will not be fully appreciated at once; the beginner may be inclined to think that it will scarcely include every thing to which the term *work* is popularly applied: he will find however as he proceeds that the definition is wide enough for practical purposes. According to this definition a man who merely supports a load does not work; for here there is resistance without motion. Also while a free body moves uniformly no work is performed; for here there is motion without resistance.

587. Work, like every other measurable thing, is measured by a unit of its own kind which we may choose at pleasure. The unit of work adopted in England is the work which is sufficient to overcome the resistance of a force of one pound through the space of one foot: or we may say practically that the *unit of work is the work done in raising a pound weight vertically through one foot.*

588. The term *foot-pound* is often used instead of the term *unit-of-work:* so that *foot-pound* may be considered as an abbreviation for *one pound weight raised vertically through one foot.*

589. Some English writers prefer to use the French system of weights and measures instead of our own; this system is explained in the *Mensuration for Beginners.* In this system the standard weight is the *kilogramme* which corresponds to about 15432 English grains, that is rather more than two pounds Avoirdupois; the standard length is the *metre* which is about 39·371 English inches. The unit of work is that done in raising one kilogramme through a vertical height of one metre; it is called a *kilogrammetre.*

590. The term *horse-power* is used in measuring the performance of steam engines. Boulton and Watt estimated that a horse could raise 33000 pounds vertically through one foot in one minute; this estimate is probably too high, on the average, but it is still retained, so that a horse power means a power which can perform 33000 units of work in a minute.

591. The term *duty* is also used with respect to steam engines; it means the quantity of work which can be obtained by burning a given quantity of fuel. In good ordinary engines the *duty* of one pound weight of coal varies between 200000 and 500000. A horse power will yield about 2000000 units of work in an hour, so that the consumption of coal per horse power per hour varies between 10 pounds and 4 pounds. Sanguine persons look forward to the possibility of so improving the steam engine as to obtain a horse power per hour with the consumption of one pound of coal. And already the results obtained by some engines much exceed what we have given as the duty of good ordinary engines; so that 70000000 is taken as the duty of one bushel of coals, the bushel containing about 87 pounds. Another estimate is 100000000 as the greatest duty at present obtained from one bushel of coals. The average daily labour of a man working under the most favourable circumstances may be put at 2000000, and of a horse at 10000000, so that a bushel of coals consumed daily can perform, on the highest estimate of duty, the work of 50 men or of 10 horses.

592. Some writers have found pleasure in constructing examples of the virtue contained in a bushel of coals. The following, taking 70000000 as the duty of a bushel of coals, are from Herschel's *Discourse on the Study of Natural Philosophy.*

"The ascent of Mont Blanc from the valley of Chamouni is considered, and with justice, as the most toilsome feat that a strong man can execute in two days. The combustion of two pounds of coal would place him on the summit.

"The Menai Bridge, one of the most stupendous works of art that has ever been raised by man in modern ages, consists of a mass of iron, not less than four millions of pounds in weight, suspended at a medium height of about 120 feet above the sea. The consumption of seven bushels of coals would suffice to raise it to the place where it hangs.

"The great pyramid of Egypt is composed of granite. It is 700 feet in the side of its base, and 500 in perpendicular

height, and stands on eleven acres of ground. Its weight is therefore 12760 millions of pounds, at a medium height of 125 feet; consequently it would be raised by the effort of about 630 chaldrons of coal, a quantity consumed in some foundries in a week."

But the first example is stated in a note to be not quite a fair estimate. Such an example as the second also is open to the charge of exaggeration; for although the seven bushels of coals might be able to raise a weight *equivalent* to that of the Menai bridge, provided this weight were in a convenient situation, this is altogether different from raising the actual bridge.

593. By our definition of Work its amount is measured by the *product* of the number of pounds in the force into the number of feet in the space through which it is exerted. Thus the Work is the same so long as this product remains the same. For example, the Work is the same to raise 5 pounds vertically through 4 feet, or 10 pounds vertically through 2 feet, or 20 pounds vertically through 1 foot.

594. Observations have been made of the amount of work which can be performed by men and by animals labouring in various ways; and the results are given in treatises on Practical Mechanics. The following Table is an example: the first column states the kind of labour, the second column the number of hours in a day's labour, the third column the number of units of work performed in a minute.

Man raising his own weight on a ladder	8	4230
Man raising a weight with a cord and pulley	6	1560
Man turning a windlass	8	2600
Man lifting earth with spade to the height of five feet	10	470

Besides these general statements particular facts have been given as the result of special experiments: thus a man ascended a mountain 9000 feet high in 9 hours, so that, his weight being 14 stone, his work was at the rate of 3270 units in a minute; in a boat race it was calculated that the work done by a rower was at the rate of 7500 units per minute.

595. There is one mode in which the labour of men and animals is employed which is not directly comparable with the application of a force to raise a weight, namely, that of carrying burdens along a horizontal road. It seems that a portion of the labour is spent in merely supporting the burden, and this portion does no *work* in the sense in which the term is used here: the remainder of the labour does the work of carrying the burden. By observation we may find the amount of useful effect which can be produced by this mode of labour; thus for example, it is said that a porter walking with a burden on his back through a day of seven hours long can carry a weight of 90 pounds through 145 feet in a minute. But this is not the *work* in the sense in which we have used the word. Nor must it be taken to measure the whole labour of the porter; for, as we have said, some labour is spent in merely supporting the burden: moreover some labour is also spent by the porter in carrying the weight of his own body. It will be seen in the Table of Art. 594, that there is a great difference apparently between the amounts of work which are performed in various ways. This is doubtless partly owing to the weight of the man's body or limbs. In the first case mentioned in the Table the whole of this weight is taken into account; in the last case it is not, and yet labour is spent by the man in the perpetual elevation of a portion of his body.

596. The main part of the labour of walking arises from the fact that at each step the centre of gravity is *raised* and made to describe a small portion of a curve. It has been calculated that at every step the centre of gravity is raised through a perpendicular height equal to about one *eleventh* of the length of the step; thus a person in walking for eleven miles would raise his body through a succession of lifts together equal to a mile: therefore if his weight were 160 pounds he would in this way alone perform 160×5280 units of work. Other estimates however put a much smaller fraction in place of the one *eleventh*.

597. A large number of examples may be proposed which consist merely of the application of Arithmetic to the measurement of work; but the importance of the subject is more obvious when we combine it with some principles

already explained, and in the first place with one which occurs so often in Mechanics, namely, that of the centre of gravity.

598. The following is a very important proposition: *When weights are raised vertically through various heights the whole work is the same as that of raising a weight equal to the sum of the weights from the first position of the centre of gravity to the last.* Some part of this proposition is obvious. For if we have a solid body and move it vertically upwards so that each point describes a vertical straight line of the same length for every point, then the centre of gravity also describes a vertical straight line of the same length; and the proposition thus merely makes the whole work just equal to the sum of its obvious parts. But the proposition includes something more. Suppose for instance that a heavy chain is lying on the ground; let one end be taken and lifted up until the whole chain hangs vertically with its other end just touching the ground. Then the different parts of the chain have been raised through different spaces; the top of the chain through a space equal to the length of the chain, and the bottom of the chain through no space at all. The proposition then says that the whole work is measured by the product of the weight of the chain into the height of its centre of gravity in the last position above the ground. If the chain be of the same kind throughout the centre of gravity will be at the middle point when the chain hangs vertically. In this case the proposition appears to be very natural, and almost self-evident. It can be *demonstrated* by the aid of a little mathematics; but the reader can take it as a fact which may be verified experimentally. As another illustration we may mention the case of raising a Venetian blind.

599. *The Work done in raising a heavy body along a smooth Inclined Plane is equal to the Work done in raising the same body through the corresponding vertical space.* The weight of the body may be resolved into two components, one down the Plane, and the other at right angles to the Plane; the former is the part which resists motion up the Plane. Now by Art. 246 we know that the former component is to the whole weight as the height

of the Plane is to its length. And thus it follows that a resistance equal to this component exerted throughout the length of the Plane gives just the same amount of Work as the weight of the body exerted through the height of the Plane.

600. *If a body be dragged along a rough horizontal plane the Work done is the product of the weight into the coefficient of friction and into the space.* This follows at once from principles already explained. For when a heavy body is dragged along a horizontal plane the resistance is not the Weight of the body, but the product of the weight into the coefficient of friction; see Art. 325. Then the *Work* is the product of this resistance into the space.

601. Next suppose a body is drawn *up a rough Inclined Plane;* the Work done consists of the sum of two parts. First there is that part which would be done if the Plane were smooth, and this is just the same as that done in raising the whole weight through the corresponding vertical space. Next there is the part which depends on friction; and it is found that this is equal to the product of the weight into the coefficient of friction into the *horizontal length* described. For the resistance is found by multiplying the coefficient of friction into the pressure at right angles to the Plane; and, by Art. 247, this pressure bears the same proportion to the whole weight as the base of the Plane does to its length. Thus the product of this pressure at right angles to the Plane into the length of the plane is equal to the product of the whole weight into the base of the Plane. Hence we obtain the required result.

602. Next consider the case of a body drawn *down a rough Inclined Plane*, the Plane being too rough for the body to slide down by itself. Here the *Work* owing to friction is as before the product of the weight into the coefficient of friction into the horizontal length. And to obtain the resultant Work we must *diminish* this by the product of the weight into the vertical space described; for tendency down the Plane has not now to be overcome by force applied, but is really a force furnished by nature which assists in urging the body down the Plane.

603. In cases which occur in practice the *length* of an Inclined Plane is scarcely perceptibly different from the *base* in size. Thus suppose we have on a railway a rise of 1 foot in 100; then it will be found that the difference between the length and the base of such an incline will be at the rate of about 1 foot in 4 miles.

604. The preceding four Articles may be applied to the case of carriages drawn along a common road or a railroad; in this case there is indeed a rotatory motion of the wheels which is not contemplated in our propositions: but the weight of the wheels will in general be small compared with the weight of the whole mass which is moved, and we will assume that no important error can arise from neglecting the rotatory motion. The numerical value of the *coefficient of friction* will depend on various circumstances. Take the case of a cart on a common road; then observation indicates that the value of the coefficient depends on the size of the wheels, and on the velocity of motion, as well as on the nature of the road. For a cart having wheels four feet in diameter, drawn with a velocity of six miles an hour along a good road, the value of the coefficient may lie between $\frac{1}{30}$ and $\frac{1}{40}$. Again, take the case of a train drawn along a railroad; then observation indicates that the value of the coefficient depends on the velocity. For a velocity of 30 miles an hour the value will be about $\frac{16}{2240}$; that is the friction is about 16 pounds per ton, estimated on the whole weight of the engine and the load. There is however besides this the resistance of the air, which depends on the square of the velocity and the area of the frontage of the train.

605. In an account of the Mechanical Powers we have frequently drawn attention to a general principle that in the state of motion *what is gained in power is lost in speed.* Now it is really the same principle which in discussions on *Work* reappears under the following form: *the Work applied to a machine is equal to the Work done by the machine.* In this form it is called the *Principle of Work.* In fact when the parts of a machine move *uniformly* it is found that

the proportion between the Weight and the Power is the same as for equilibrium; and from this the proposition is derived: the reader may take it as a result established by theory.

606. But the statement of the Principle of Work requires to be modified in practice. The whole work done by a machine may be divided into two parts, the *useful* part and the *lost* part. The useful part is that which the machine is designed to perform. The lost part is that which is not wanted but which is unavoidably performed; such for example as moving the parts of the machine itself which are necessarily heavy, or overcoming friction or the want of flexibility of springs. It is still true that the whole work applied to the machine is equal to the whole work, useful and lost, done by the machine; and consequently the *useful* work alone is always less than the work applied to the machine. A good example of the distinction between useful and lost work is furnished by a steam vessel; the useful work is the moving the vessel itself, and the lost work is the setting in motion a large mass of water by the strokes of the paddle-wheels.

607. The proportion of the useful work done by a machine to the work applied to the machine is called the *efficiency* of the machine, and sometimes the *modulus* of the machine. The efficiency or modulus is thus a fraction; and it is of course the object of inventors and improvers to bring this fraction as near to unity as possible.

608. Take for example a high pressure steam engine, working without expansion or condensation. Up to 10 horse-power the *modulus* for an ordinary engine is ·4, and for the best engines ·5. The modulus increases as the horse-power increases, and when the engine is of above 40 horse-power the modulus is ·56 for an ordinary engine and ·7 for the best engines. For an undershot water-wheel the modulus lies between ·25 and ·3; and for an overshot water-wheel under favourable circumstances the modulus lies between ·7 and ·75. For a common pump the modulus lies between ·25 and ·3. In the common pump work is wasted in the friction of the solid parts, in generating the velocity with which the water is discharged, and in producing eddies in the water; and we must also add the leakage of the water.

609. *Accumulated Work.* If a body is moving it is said to have work *accumulated* in it; in fact if a body possesses any velocity it can be made to do work by parting with that velocity. For example, a cannon-ball in motion can penetrate a resisting substance, and water flowing against a water-wheel can turn the wheel. Suppose that a body is moving with any velocity; that velocity might have been gained by falling through a suitable vertical height, and if the body be started up with such velocity it will just reach that height. This suggests the following as a measure of accumulated work: the work accumulated in a moving body is measured by the product of the weight of the body into the space through which it must fall to acquire the velocity. Instead of the space through which the body must fall we may take the quotient of the square of the velocity by 64: see Art. 127.

610. An example will illustrate this. The force necessary to drive in an ordinary nail may be taken as 200 pounds; suppose that the nail is to be driven to the depth of $\frac{1}{10}$ of a foot: then the Work to be done is $200 \times \frac{1}{10}$, that is 20. Let us take a hammer weighing 2 pounds and give to it the velocity which would be gained in falling through 1 foot; then the work accumulated in the hammer is 2×1, that is 2. Hence ten blows of the hammer on the nail would theoretically do the work of driving the nail in. Practically however a large part of the work accumulated in the hammer is *lost* at each blow, as will be understood from the next Chapter.

611. The term *labouring force* was used by Dr Whewell, in his *Mechanics of Engineering*, nearly in the same sense as we have here used *Work;* the former term suggests rather the *cause* and the latter the *effect:* but as the cause is measured by the effect the terms are practically equivalent. "In our towns in which large manufactories exist, such establishments often generate by their machinery more labouring force than they need; and the surplus (transferred by an axis, or in some other way, to a distance) is hired by other persons, and employed for the purposes of the most

varied kind of work. In such towns, we often read advertisements of 'power to be disposed of to a large amount.' The *power* here spoken of is labouring force. The cost is proportional to the quantity of labouring force so bought and sold."

612. There are various sources of *labouring force.* They may be enumerated as the powers of water, wind, steam, man, and animals. To these may be added magnetism, electricity, and chemical agencies, which, however, up to the present time, have contributed very little to the general stock of force.

613. In England, in recent times, wind and water have been comparatively much less used than in earlier times; and at present steam is the main source of our labouring force. Or, to speak more correctly, we may say that the stores of *coal* really contain our available labouring force. It is of course possible that in future means may be invented for availing ourselves of other natural forces. For instance, by the *tides* an enormous mass of water is raised twice a day through a considerable height; so that if this water could be stored up in any way its fall would supply a vast amount of labouring force: but no practical method of turning the tides to this account seems to have as yet been proposed.

LIII. ENERGY.

614. We have spoken in the preceding Chapter of Work and of *Accumulated Work;* now the term *Energy* has been recently introduced and employed in the same sense as we have used Accumulated Work. Thus by Energy we mean the capacity which a body has, when in a given condition, for performing a certain measurable quantity of Work. The Energy of a moving body is equal to the product of its weight into the height through which it must fall to acquire the velocity which it has. Moreover we shall always express the weight in pounds and the height in feet. Instead of the height through which the body must fall we may take the quotient of the square of the velocity by 64.

615. A beginner is not in a position to judge of the advantage which will follow from the use of certain definitions, or of the propriety of these definitions; but he may anticipate, after reading the preceding Chapter, that interesting and important facts can be stated and applied by the use of the word *Energy* as now explained. We may add that these facts are not confined to the Mechanical sciences with which we are occupied, but extend to other subjects of great interest as Heat, Electricity, and Chemical Action. We shall be able to give but a brief outline here.

616. A simple example will shew at once that *Energy* presents itself naturally to our attention. Suppose that a ball of certain weight fired with a certain velocity will just go through a plank *one* inch thick. Let a similar ball be fired with twice the former velocity; then it is found by trial that this will go through a plank of about *four* inches thick; so that the penetrating power of the ball changes, not in the same proportion as the velocity, but as the *square* of the velocity, that is as the *Energy*. Again, suppose a soldier to discharge a ball from his rifle; it is well known that the rifle recoils, and would give a severe blow if it were not held firmly against the shoulder. Now it follows from the Law of the equality of Action and Reaction that in *one sense* the backward stroke of the rifle on the shoulder is equal to the stroke which the ball would inflict on an obstacle just as it left the rifle. The two are equal in this respect that the *momentum* of the one is equal to the momentum of the other. Suppose that the rifle weighs 10 pounds, and the ball one ounce, that is $\frac{1}{16}$ of a pound; and suppose that the ball starts from the rifle with a velocity of 800 feet per second: then the rifle recoils with a velocity of $\frac{800}{10 \times 16}$ feet per second, that is with a velocity of 5 feet per second. But the *Energy* of the ball is measured by $\frac{1}{16} \times \frac{800 \times 800}{64}$; and that of the rifle by $10 \times \frac{5 \times 5}{64}$; so that the Energy of the ball is 160 times that

of the rifle. Thus we need not be surprised that in spite of the inconvenience of the recoil the rifle is a powerful weapon for the soldier's purpose; for although there is the backward stroke yet the Energy of this is inconsiderable compared with that of the ball.

617. By the fall of a heavy body we gain Energy, and hence it follows that if a heavy body be in a position from which it can fall we may regard it as a store of Energy. In other words, we may apply the term *Energy of position* to a body in such a situation. Thus if a mass of water is so placed that we can if we please allow it to fall and turn the wheel of a water mill, we may say that the water is a *store of Energy* or has an *Energy of position*. When the spring of a watch is wound up there is a store of Energy which suffices to keep the watch in motion for several hours. When the string of a bow or a cross-bow is pulled back there is a store of Energy which suffices to propel the arrow.

618. In Chapter XVIII. we have discussed various cases of the Collision of bodies. It will be found on examining the results there obtained that there is always a *loss of Energy* by the collision of two bodies unless the bodies are *perfectly elastic.* For example, suppose two equal inelastic balls to move with equal velocities in opposite directions and come in contact. The energy of each ball is the same before impact, and therefore the Energy of the two is double that of one of them. By the impact the balls are reduced to rest, and so the Energy is destroyed. Again, suppose that one inelastic ball impinges on an equal inelastic ball at rest. After impact the two balls move with *half* the velocity of the impinging ball before impact. Thus the Energy of each ball after impact is one fourth of the Energy of the impinging ball before impact; and therefore the energy of the system after impact is half of the energy before impact. Other examples may easily be constructed.

619. Again, we have noticed in Art. 606 that in using any machine there is always a large proportion of the Work *lost*, owing to friction and other causes. Some of the Work lost appears in the form of motion given to bodies which it was not the object of the machine to move; but this does

not apply to all that is lost, especially to that which is lost by the friction of solid parts.

620. The question then arises what becomes of the Energy lost in such cases as those of Art. 618, and of the Work lost in the use of machines. Modern science shews that it is in some way turned into heat; that it is possible to measure the amount of heat which corresponds to a given amount of Energy; and that if we make a strict calculation of the amount of Work done by a machine, and of the amount of heat developed, we shall find that the two together balance the Work applied, so that there is no destruction of Energy. The fact is called the principle of the *Conservation of Energy*.

621. That there is some connection between motion and heat must have been long known. Savages are said to kindle a fire by rapidly rubbing one piece of wood against another. A workman after sawing a log or filing a nail, could not fail to observe that his tool became warm. Towards the end of the last century the celebrated chemist Sir Humphry Davy shewed that two pieces of ice might be nearly melted by rubbing them together, when by reason of the arrangements he made the heat could not have been obtained from the surrounding bodies. Shortly before Count Rumford had observed that in the process of boring cannons a large amount of heat was developed. What was now necessary was an exact determination of the relation between the quantity of mechanical work performed and the equivalent quantity of heat generated; this in recent times has been ascertained by careful experiment, principally by Dr Joule of Manchester. The final result may be thus stated: if water be allowed to fall through 1391 feet, and its motion suddenly stopped, sufficient heat will be produced to raise the temperature of the water one degree of the Centigrade thermometer.

622. If we take as our unit of heat the heat necessary to raise the temperature of one pound of water one degree of the Centigrade thermometer we see that 1 unit of heat is equivalent to 1391 units of work; where the unit of work is as usual the *foot pound*. If we take as our unit of heat the heat necessary to raise the temperature of one pound of

water one degree of Fahrenheit's thermometer, one unit of heat is equivalent to $\frac{5}{9}$ of 1391 units of work, that is to 772 units of work. Moreover the heat required to raise the temperature of one pound of water by a given amount is not *quite* the same for all original temperatures, though the difference is very slight. To be precise then we may say that the unit of heat is the quantity of heat required to raise the temperature of water by one degree, starting from the temperature of 60 degrees of Fahrenheit's thermometer.

623. It should be noticed that water requires more heat than most substances in order to raise its temperature by a given amount: the same quantity of heat which would raise *one* pound of water one degree of temperature would raise about *nine* pounds of iron one degree.

624. We may add to the examples which we have already given of the conversion of motion into heat some cases of sudden blows: thus a blacksmith can make a piece of lead hot by repeated blows, and a cannon-ball striking against an iron target may make it red hot. Other cases less immediately obvious may be noticed. When a bell is put into vibration, by a stroke of the clapper, part of the energy of the vibration is communicated to the air, and by the aid of this the sound of the bell is heard. The state of motion communicated to the air passes on with the known velocity of sound, but it no doubt becomes at last converted into heat. Also a portion of the energy of the vibration remains in the bell, and this is ultimately converted into heat.

625. Suppose for an example that an iron ball weighing 9 pounds, and moving with a velocity of 1000 feet per second, enters a mass of water and is brought to rest; the Energy of the ball is equal to $\frac{9 \times 1000 \times 1000}{64}$, that is to 140625. Divide this by 772; the quotient is 182, so that 182 units of heat will be produced by taking the velocity from the ball. This heat will raise the temperature of the water and the iron ball. Suppose for instance there are 90 pounds of water; the 9 pounds of iron count as 1 pound of water in the demand for heat, by Art. 623; so that the 182 units of

heat may be supposed given to 91 pounds of water, and will therefore just suffice to raise the temperature of the ball and the water 2 degrees of Fahrenheit's thermometer.

626. The principle of the Conservation of Energy is not confined to the two subjects which we have considered and here brought into connexion, namely mechanical work and heat; it is extended by philosophers so as to include chemical action and electrical action: the principle asserts that in all transformations of Energy from one kind of action to another the amount of Energy remains unchanged.

627. The earlier researches into the subject of Energy related chiefly to the conversion of Work into Heat; more recently attention has been given to the conversion of Heat into Work. Sir W. Thomson has been led to a principle which is called the *Dissipation of Energy*, meaning however something different from what the words would at first naturally suggest. It is found that although we can easily convert Work into Heat, we cannot get all the Heat back again into the form of Work. In consequence of this it is held that the mechanical Energy of the universe is becoming every day more and more changed into Heat; and so science looks forward "to an end when the whole universe will be an equally heated inert mass, and from which every thing like life or motion or beauty will have utterly gone away." Two treatises have been published on the subject of Energy to which the student may refer for more information; *The Conservation of Energy*... by Professor Balfour Stewart, and *An Elementary Exposition of the Doctrine of Energy*, by D. D. Heath.

LIV. ELASTICITY.

628. In the theory of Mechanics we suppose for simplicity that we are concerned with *rigid* bodies, that is with bodies which retain always the same shape and size. But a body is never really rigid; it always changes more or less its shape and size under the action of force, and when the force is withdrawn the body resumes, at least to some extent, the original shape and size: the property by virtue of which this resumption takes place is called *Elasticity*.

629. Gaseous bodies and liquids may be said to be elastic inasmuch as they regain their original *size* when any pressure to which they have been exposed is withdrawn; but we now propose to confine ourselves to the case of solid bodies, in which the *shape* as well as the *size* have to be considered.

630. A solid is said to be *perfectly* elastic which returns *exactly* to its original size and shape, when any constraint to which it has been subjected is removed; and it is said to be *imperfectly* elastic when this is not the case. Strictly speaking no solid is perfectly elastic, though some solids possess the property of elasticity in a very high degree, as for example, Indian rubber, ivory, glass, and marble; other solids, as lead and clay, have very little elasticity. If a ball of ivory be allowed to fall on a slab of polished marble it will rebound to nearly the original height. It is believed that during the brief time of collision the ball was at first slightly flattened, and then resumed its original form; and that the rebound is occasioned by the effort to resume its original form.

631. Practically speaking almost every solid body may be considered perfectly elastic *up to a certain point.* That is, there is generally a limit of constraint for every body to which it may be exposed and from which it will recover when the constraint is removed, the recovery being complete so far as our means of observation extend. But if the constraint is carried beyond this limit the body undergoes some appreciable lasting change of shape or of size, or of both; in technical language the body receives a *permanent set.* For degrees of constraint beyond the limit the body is imperfectly elastic. It is obvious that in practice it will be necessary to pay great attention to the limit of elasticity, so as to ensure safety and durability in constructions. The perfect elasticity of some bodies within certain limits is shewn by obvious facts; thus a steel watch-spring, or the spring by which a pen-knife is closed, will continue to work for years without any appreciable change. We proceed to consider three different modes of constraint to which bodies may be exposed, and to state the laws which determine the behaviour of bodies

under the influence of such constraint and their own elasticity.

632. *Extension.* If forces are exerted on rods and wires tending to lengthen them the elasticity of the substances will be called into action. Experiments are conducted by fixing one end of a wire to a firm support; then the constraint may be exerted at the other end along the direction of the wire by means of a lever: or the wire may be put in a vertical position and weights attached to the free end. The amount of lengthening thus produced is carefully observed; and the following laws are found to hold so long as the limit of elasticity is not exceeded.

(1) Rods and wires are perfectly elastic, that is they resume their original lengths as soon as the stretching force is removed.

(2) For the same substance and the same diameter the lengthening is proportional to the original length and also to the stretching force.

(3) For rods and wires of the same substance under the same stretching force the lengthening is inversely proportional to the square of the diameter of the rod or wire.

633. The second of the preceding three laws is sometimes called *Hooke's Law,* from the name of the person who first obtained it; the law does not hold quite strictly however, as we shall see by some numerical results given in the next Chapter.

634. Both calculation and experiment shew that when bodies are lengthened by a stretching force their volumes increase. Thus if a wire is pulled out, and so lengthened, the area of a section of the wire will at the same time diminish, but not so much as to leave the volume just what it was before. It appears in general that all causes which increase the density of a body increase the elasticity, and those which diminish the density diminish the elasticity. Thus the elasticity of metals diminishes continuously as the temperature rises from 59 degrees to 392 degrees of Fahrenheit's thermometer; but iron and steel form exceptions, for their elasticity increases as the temperature rises to 212 degrees, and then diminishes.

635. *Compression.* In like manner experiments are made on bars or rods by subjecting them to the action of forces in the direction of their length which tend to *shorten* them. Laws similar to those of Art. 632 now hold with respect to the shortening and the compressing forces.

636. *Torsion.* Experiments on the elasticity called into action when wires are *twisted* are conducted by means of what is called the *Torsion Balance.* One end of a wire is fixed; the wire hangs vertically, and to the other end a needle is attached at right angles to the wire. Immediately below the needle there is a graduated horizontal circle having its centre in the same vertical line as the wire. By turning the needle round in the horizontal plane, through any angle, the wire is twisted; the angle through which the needle is turned is called the *angle of torsion*, and the force necessary to retain the needle in the position to which it has been turned is called the *force of torsion.* When the needle is left to itself after having been turned through any angle it oscillates for some time, to and fro, like a pendulum, until at last it comes to rest in its original position. The elasticity of torsion for *stout rods* has also been investigated, but by a method different from that used for wires. Both for wires and rods the following laws are found to hold so long as the limit of elasticity is not exceeded.

(1) The oscillations for the same rod or wire are, like those of a pendulum, performed in nearly the same time, whether the angle of torsion be greater or smaller.

(2) For the same rod or wire the angle of torsion is proportional to the force of torsion.

(3) With the same force of torsion, and with rods or wires of the same diameter and of the same substance, the angle of torsion is proportional to the length of the rod or wire.

(4) If the same force of torsion is applied to wires of the same length and the same substance the angle of torsion is inversely proportional to the *fourth power* of the diameter, that is to the *square of the square* of the diameter.

637. A solid when cut into a rod or a thin plate, and fixed at one end, after having been more or less bent

strives to return to its original position. This kind of elasticity is of frequent application in the arts, as for instance in carriage-springs, watch-springs, and springs for measuring weights. The elasticity of hair, wool, and feathers is of service in pillows and cushions.

638. The importance of the elasticity of bodies, especially of the metals, for the ordinary concerns of life is forcibly stated in the *Illustrations of Mechanics* by the late Professor Moseley. "With the elasticity of metallic bodies every one is conversant. It is a property which, as it belongs to steel, iron, and brass, contributes eminently to the resources of art, and ministers largely to the uses of society. Were it, indeed, not for this property, it would be *in vain* that the metals should be dug out of the earth and elaborated into various utensils. Infinitely more brittle than glass, they would immediately be dashed to pieces by the slight shocks to which every thing is more or less subject; a shower of hail, or even of rain, would be sufficient to *indent* their surfaces, and the impact of the minute particles of dust blown against them by the wind would be sufficient permanently to destroy their polish."

LV. STRENGTH OF MATERIALS.

639. In all questions of practical engineering it is of the utmost importance to ascertain how far we can rely on the materials we employ to support the strains or pressures which may be brought to bear on them. To a great extent the necessary information consists in numerical results connected with the principles of the preceding Chapter on Elasticity.

640. *Modulus of Elasticity.* Suppose a given rod or bar held fast at one end and stretched by a force at the other; then by *Hooke's Law* the amount of lengthening is proportional to the stretching force: see Art. 633. This Law indeed holds only so long as the amount of lengthening is slight; but let us assume for the moment that the Law holds for any amount of lengthening. Then by the

application of a proper force, the rod or bar could just be *doubled in length;* this force expressed in pounds Avoirdupois per square inch is called the *Modulus of Elasticity*. The term was introduced by Dr Thomas Young.

641. The values of the *Modulus of Elasticity* have been determined by experiment for almost every solid substance of importance, and will be found in works on Practical Mechanics, such as Rankine's *Applied Mechanics*. We give here a selection from these values.

Material.	Modulus.
Brass, cast	9170000
Brass, wire	14230000
Copper wire	17000000
Iron, cast	17000000
Iron, wire	25300000
Steel	29000000 to 42000000
Elm	700000 to 1340000
Fir	900000 to 1900000
Oak, European	1200000 to 1750000
Teak, Indian	2400000

642. The meaning of the foregoing Table will be seen from an example. Suppose a rod or bar of cast brass, one square inch in section; then if a weight of one pound were hung at the end the bar would be lengthened $\frac{1}{9170000}$ part of its original length; by two pounds it would be lengthened double this amount; by three pounds triple this amount; and so on. This holds so long as the lengthening is not very great; if it held for any amount of lengthening the length would be just doubled by a weight of 9170000 pounds. If the area of the section of the brass rod is more or less than a square inch, more or less weight will be required in proportion, to produce the same amount of lengthening; thus if it be *half* a square inch in section *half* the weight will be required; if it be three square inches in section three times the weight will be required; and so on.

643. *Tenacity*. Suppose we take a rod or bar of any material; and stretch it by a weight. As the weight is increased so also the amount of lengthening increases, but at last, when the weight becomes sufficiently great, the rod or bar breaks. The breaking weight is taken as a measure of the *tenacity* of the bar or rod; it is determined by experiment and expressed in pounds Avoirdupois per square inch.

644. The following Table gives the *Tenacity* of various materials.

Material.	Tenacity.
Brass, cast	18000
Brass, wire	49000
Copper wire	60000
Iron, cast	16500
Iron, wire	70000 to 100000
Steel	100000 to 130000
Elm	14000
Fir	9000 to 14000
Oak, European	10000 to 19800
Teak, Indian	15000

645. The following Table also gives in a convenient shape information respecting the tenacity of various metals in the form of a wire. Suppose wires one-sixteenth of an inch in diameter formed of different metals; then the numbers of pounds which they would support are determined by experiment to be approximately the following:

Iron	512	Gold	140
Copper	282	Zinc	102
Platina	256	Tin	32
Silver	175	Lead	25

646. When iron is stretched beyond the elastic limit the character of the phenomena will depend altogether on the nature of the iron. If the iron is soft and ductile it will be reduced to a much smaller size in the neighbourhood of the point where the fracture ultimately takes place; the area of a section may thus be reduced to three-fourths of the original area. This peculiarity is sometimes called *toughness;* it is in many respects of great value,

because the iron thus affords warning that it is about to break. But cast-iron on the whole is comparatively an uncertain metal, and frequently breaks with little or no warning.

647. *Endurance.* The resistance of a bar to a force which *stretches* it is called tenacity; writers on Practical Mechanics have not fixed on a term which shall denote the resistance of a bar to a force which *compresses* it: we shall here use *Endurance.* A bar is taken and compressed in the direction of its length by a weight which is gradually increased until the rod is crushed. This crushing weight is taken as a measure of the *endurance;* it is determined by experience and expressed in pounds Avoirdupois per square inch.

648. The following Table gives the *Endurance* of various materials.

Material.	Endurance.
Brass, cast	10300
Iron, cast	112000
Iron, wrought	36000 to 40000
Elm	10300
Fir	5375 to 6200
Oak, British	10000
Teak, Indian	12000
Brick	550 to 1100
Granite	5500 to 11000
Limestone	4000 to 5500
Sandstone	2200 to 5500

649. Crushing, that is breaking by compression, is not so simple an operation as tearing asunder; according to Professor Rankine there are five different forms which crushing assumes in different substances.

(1) Crushing by *splitting.* This consists of a breaking up into fragments, with the surfaces of separation nearly parallel to the direction of the pressure; it occurs with hard substances of a uniform glassy texture, such as vitrified bricks.

(2) Crushing by *shearing or sliding.* This consists of a breaking where the surfaces of separation are inclined to

the direction of the pressure; it occurs in substances of a granular texture, such as cast-iron and most kinds of stone and brick.

(3) Crushing by *bulging*. This consists of a lateral swelling and spreading out of the materials; it occurs in substances which are ductile and tough, such as wrought iron.

(4) Crushing by *buckling or rippling*. This is characteristic of fibrous surfaces, and consists in a lateral bending and wrinkling of the fibres, sometimes accompanied by a splitting of them asunder; it occurs in timber, in plates of wrought iron, and in bars longer than those which give way by bulging.

(5) Crushing by *cross-breaking*. This is the mode of breaking of columns in which the length greatly exceeds the breadth; the columns yield sideways and are broken across like beams under a transverse force.

650. When substances are crushed by splitting or shearing the *endurance* considerably exceeds the *tenacity:* thus for cast-iron the endurance is rather more than six times the tenacity. In the case of crushing by bulging the endurance is in general less than the tenacity, sometimes much less; for wrought iron the endurance is from two-thirds to four-fifths of the tenacity. The endurance of most kinds of dry timber is from one-half to two-thirds of the tenacity; the endurance of moist timber is only about half that of dry timber.

651. In modern engineering iron and steel are the most important materials; they have to a great extent superseded the wood which was formerly employed. Hence in stating some facts with respect to the strength of materials we shall confine ourselves to iron and steel. Experimental results with respect to the behaviour of bodies, as for example rods, under the influence of tension or pressure, apply strictly speaking to the precise bodies on which the operations are performed; and caution will always be necessary in extending them to other bodies apparently of precisely the same kind: owing to some internal defect, or other cause, a bar may be considerably weaker than we should have been led to expect from observation on ano-

ther bar of apparently the same kind. In some specimens of iron cut from a large mass the elastic limit was found to be under 4 tons of strain per square inch, while in general the limit was from 8 to 12 tons. The average tenacity of cast-iron may be taken at 7 tons; but in some specimens it is only 5 tons, while in others it reaches 14 or even 15 tons. The *length of time* during which the constraint is applied has a considerable influence on the result produced; in one experiment a weight at first stretched a bar less than ·2 of an inch, but in the course of 17 hours stretched it nearly ·24 of an inch.

652. In consequence of the great variety in the merit of cast-iron engineers are compelled to adopt means for testing the quality of the material furnished to them by iron founders. One method is to cast in a mould a bar of one square inch in section and four feet long, and to test it by supporting it on its ends in a horizontal position and observing how much deflection is produced by a weight hung in the middle. A good bar ought to sustain a load of more than 600 pounds with a deflection of about half an inch. The engineer may also test the tenacity of the bar under extension, and its endurance under compression. Sometimes instead of examining specimens of the same iron the engineer will order more beams or bars than he requires, and will take at random a certain number of these and test them up to the highest strain to which his structure will be exposed: then if any of these fail he will reject the whole supply.

653. We will now give some numerical details. A convenient fact to remember is that a bar of wrought iron, one square inch in area, if stretched by a weight of one ton will be lengthened nearly $\frac{1}{10000}$ part. The change of temperature from winter to summer will produce a lengthening of about $\frac{1}{2000}$ part. Hence it follows that if the ends of such a bar are attached to fixed obstacles so that the bar exerts no pressure on them in summer, it will in winter exert a force of 5 tons to draw them together.

654. In one series of experiments cast-iron bars were taken one square inch in section and 10 feet long. They were *stretched* by various weights ranging from a little over 1000 pounds to 17000 pounds; the latter weight on an average broke the bars. A *permanent set,* indicating the limit of the elasticity, was obtained by about one-tenth of the breaking weight. When the stretching weight was a little over 1000 pounds the rod was lengthened ·009 of an inch; when it was about 9500 pounds the rod was lengthened ·1 of an inch; and when it was nearly 15000 pounds the rod was lengthened nearly ·2 of an inch.

655. In another series of experiments similar bars were submitted to *compression* by various weights ranging from a little over 2000 pounds to a little over 37000 pounds; the latter weight on an average greatly injured the bars. A *permanent set,* indicating the limit of the elasticity, was obtained by about one-seventeenth of the injuring weight. When the compressing weight was a little over 2000 pounds the rod was shortened about ·02 of an inch; when it was nearly 21000 pounds the rod was shortened rather more than ·2 of an inch; and when it was about 37000 pounds the rod was shortened about ·4 of an inch.

656. It will be seen that *Hooke's Law* does not hold very exactly in the case of either of the series of experiments given in Articles 654 and 655. But we must observe that when the constraining weight is not extremely large the lengthening which it produces by stretching is numerically very nearly equal to the shortening which it produces by compression. Thus for example the constraining weight being about 4200 pounds both the lengthening and the shortening were about ·039 of an inch. As the constraining weight increases the shortening becomes sensibly less than the lengthening; and this is in accordance with the statement of Art. 650 that the endurance of cast-iron is much greater than the tenacity.

657. The material which is most extensively employed in the arts is wrought iron; it is obtained directly from cast-iron by a process which removes the greater part of the impurities. The tenacity may be taken to be on an

average 23 tons per square inch; and the *limit of elasticity* as approximately half the tenacity.

658. Experiments give the following results with respect to the stretching of wrought iron bars one square inch in section and ten feet long. The stretching weight was at first 1262 pounds, and was successively increased by this amount until at last it was 30 times the original weight. Under the first weight the lengthening was rather more than ·005 of an inch; under a weight 20 times as great it was about ·11 of an inch: throughout this range the lengthenings followed *Hooke's Law* pretty closely. As the weight was increased beyond this point the deviations from *Hooke's Law* became very large: until under a weight 30 times as great as the first the lengthening was about 2·9 inches, that is about eighteen times as great as it would have been according to *Hooke's Law.*

659. The strength of wrought iron is not much affected by the increase of the temperature up to 350 degrees of the common thermometer. There has been a difference of opinion as to the influence of extreme frost; direct experiment does not seem to make out that the strength is less in cold weather; but there exists a popular notion that iron and steel are rendered more brittle by frost, and this receives some confirmation from the fact that the accidents on railways arising from the breaking of the rails and of the axles of the carriages are most frequent in winter.

660. An opinion prevails, and apparently on good grounds, that some change takes place in the constitution of wrought iron when it has been subjected to incessant jars for a long time; and that in consequence of this the strength is much diminished. The axles of railway carriages, and the chains of cranes, are cited as examples of this great deterioration. With respect to chains it is well established that in the course of time they change for the worse, and it is a rule in the War Department that all chains are to be passed periodically through the fire, and thereby annealed: thus the quality is restored, and the duration of the chain prolonged.

661. Steel is a combination of pure iron and carbon; its tenacity is far greater than that of wrought iron, ranging

from 30 tons to 50 tons per square inch. Moreover, when steel is tempered in oil the limit of elasticity is fully three times as great as that of wrought iron; and the steel will stretch much before rupture takes place. A good serviceable quality of steel is now manufactured, by what is called the *Bessemer process*, in an economical manner, and this is applied to many of the purposes for which iron was formerly used, especially where strength is to be combined with lightness.

662. The engineer being furnished with information as to the strength of the materials which he has to use, must be guided by experience as to the greatest demand which he will make on that strength in his constructions. Of course for safety he will keep far below the extreme limit which is theoretically allowable. Thus the average tenacity of wrought iron may be taken as 23 tons to the square inch; but in practice it is not considered prudent to calculate on more than 5 tons: and in the case of chains which are liable to a sudden impulse, as the chains of cranes, it would be unwise to rely on as much as 5 tons. In practice there are three terms used for different degrees of strength of materials; namely *ultimate* strength, *proof* strength, and *working* strength. The *ultimate* strength may be taken to be measured by the constraint which will destroy or damage the material in a specified way; the *proof* strength as measured by the greatest constraint which is consistent with safety; and the *working* strength as measured by the greatest constraint allowed by prudence and experience. The ultimate strength may be 2 or 3 times as great as the proof strength, and 10 times as great as the working strength. Constraint equal to the proof strength might not produce any harm in a single short trial, but it might by long continuance or by frequent operation.

663. Numerical results slightly different from those adopted in the present Chapter may be found in works of good authority; and from the nature of the subject it cannot be expected that all experiments will be in exact agreement. The older writers of course could not attain the same accuracy as their successors; but it is difficult to account for the wide discrepancy sometimes to be found

between their statements and those which are at present received. For instance Dr Young says "The weight of the modulus of the elasticity of a square inch of steel...is about 3 million pounds..." The modern value is at the lowest nearly *ten* times this: see Art. 641. Again, he says "Oak will suspend much more than fir; but fir will support twice as much as oak..." According to modern authority oak will support nearly twice as much as fir: see Art. 648.

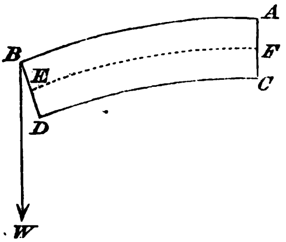
664. A few interesting remarks may be quoted from Dr Young. "The strongest wood of each tree is neither at the centre nor at the circumference, but in the middle between both; and in Europe it is generally thicker and firmer on the south east side of the tree. Although iron is much stronger than wood, yet it is more liable to accidental imperfections; and when it fails, it gives no warning of its approaching fracture...... Wood, when it is crippled, complains, or emits a sound, and after this, although it is much weakened, it may still retain strength enough to be of service. Stone sometimes throws off small splinters when it is beginning to give way: it is said to be capable of supporting by much the greatest weight when it is placed in that position, with respect to the horizon, in which it has been found in the quarry."

LVI. STRENGTH OF BEAMS.

665. We have spoken of the strength of *materials* in the preceding Chapter; in the next place it would be proper to enquire into the strength of *structures* formed of these materials: we shall however confine ourselves almost entirely to one simple case, that of beams placed in a horizontal position, either fixed at one end or supported at both ends. The engineers now use the term *girder* for a beam supported at both ends, and *cantilever* for a beam fixed at one end; the beam in both cases is supposed to be subjected to transverse strain, as for instance, to that produced by a weight.

666. Suppose a horizontal rectangular beam to have one end firmly fixed, and at the other end let a weight W be hung; we neglect the weight of the beam itself. By

the action of the weight the beam will be drawn out of the horizontal position; the diagram is intended to shew the new form of the beam: it is much exaggerated for the sake of distinctness. The beam is supposed to be built into a wall, on the right hand side of the vertical line AC, or to be otherwise fixed, and we are concerned with only the portion of the left-hand side of AC. The boundaries AB and CD both become curved, AB being longer, and CD shorter, than when they were both horizontal. We must understand distinctly what is meant by the *length*, the *depth*, and the *breadth* of the beam. The *length* is the distance from end to end, namely the distance from A to B in the original position. The *depth* is the distance from the upper surface to the lower; it is the straight line AC in the diagram. The *breadth* is the distance from the front to the back; it is not shewn in the diagram, being perpendicular to the plane of the paper.



667. Somewhere between AB and CD a line exists which was originally horizontal, and of the same length as it is in the bent form; we will denote it by EF. This line is called the *neutral line* in the surface $ABDC$. The assemblage of the neutral lines in all the sections parallel to $ABDC$ is called the *neutral surface*.

668. Now take the portion of the beam which stands out from the wall, denoted by $ABDC$, and consider the forces which keep it in equilibrium: these are the weight W, and the actions along the imaginary section represented by AC. The actions will be partly in the vertical plane, denoted by AC, and partly at right angles to it. The former must on the whole be equal to the weight W, and we shall not require to take any more notice of them; the latter are very important for our purpose. The part of the beam *above* the neutral surface is lengthened; its

elasticity is brought into play, and thus we have forces acting on the beam at points between A and F, all towards the *right-hand* side. In like manner the part of the beam *below* the neutral surface is shortened; its elasticity is brought into play, and thus we have forces acting on the beam at points between C and F, all towards the *left-hand* side. The two sets of forces must balance each other, because there are no other forces acting on the beam in the horizontal direction. If the lengthening and shortening are small the elasticity brought into play by the former is equal to that brought into play by the latter; and the neutral surface will then be *midway* between the upper and lower surfaces. The forces acting on the beam along FC tend to turn the beam round A in one direction; while the forces acting on the beam along FA, and also the weight W, tend to turn the beam in the opposite direction; but on account of their greater distance from A the moments round A of the forces along FC will be greater than the moments round A of the forces along FA. For equilibrium it is necessary that the excess should just be equal to the moment round A of the weight W.

669. The further we proceed from the neutral line, upwards or downwards, the greater is the extension or compression respectively; thus along AC the former is greatest at A, and the latter is greatest at C. The force between F and C on the beam is a compression; and therefore if we were to cut through FC with a fine saw, so as to remove extremely little of the material, the compression would be still exerted as before, and the equilibrium little if at all endangered. But, on the other hand, a very slight incision downwards at A would weaken the strength of the beam, and might be followed by a total fracture.

670. If the weight W is gradually increased until the beam breaks at A we obtain in the final value of the weight a measure of the strength of the beam. Now mathematical investigation, confirmed by experiment, shews that the strength of the beam is proportional to the product of the breadth into the square of the depth, divided by the length. This result may be easily justified by simple reasoning. First as to the *breadth*. If the breadth

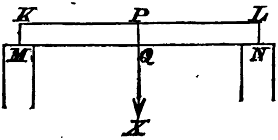
be doubled the effect is the same as if two beams of equal length and depth were placed side by side, and, as each singly would just sustain the same weight before breaking, the two together would sustain twice as much as one alone. Similarly we see that if the breadth is *tripled* the strength is *tripled;* and so on. Thus the strength increases in the same proportion as the breadth. Next as to the *depth.* If the depth be doubled then since AC is doubled we have *double* the force from elasticity which we had before; and the distance of any point from A is *double* the former distance of the corresponding point: and thus the moment round A is *four* times as great as it was before, and therefore the weight sustained before breaking will be *four* times as great as it was before. Similarly we see that if the depth is *tripled* the strength is *nine* times as great as it was at first; and so on. Thus the strength increases in the same proportion as the square of the depth. Finally as to the *length.* If the length be doubled the moment of W round A is doubled; and therefore if W be halved the moment remains the same as before: thus the strength is half what it was before. Similarly we see that if the length is *tripled* the strength is a *third* of what it was at first; and so on. Thus the strength diminishes in the same proportion as the length increases.

671. Along the neutral line we have neither extension nor compression; hence the beam may be pierced or reduced in size near the neutral surface without danger. For instance, if holes are to be bored parallel to the breadth, for the insertion of pins or rods, they should be made so as to pass close to the neutral surface. In like manner we may save material without great sacrifice of strength by giving up the solid rectangular form. This is almost always done in the case of iron girders; they often consist of two horizontal flat parts called *flanges*, connected by a vertical flat part called the *web.* The form in this case is something like what we should obtain by running two broad and deep grooves in the solid rectangular beam, one along the neutral line in the front, denoted by FE in the diagram, and one along the neutral line on the back. There are sometimes two webs, and then the girder becomes tubular with a rectangular section. In the tubular

bridges on some railway lines we have girders of such large dimensions as to allow the weight to be placed inside the girder instead of being borne on the outside in the usual manner: the weight in this case being the engine and train.

672. If we want to take account of the weight of the beam itself in Art. 666 we must suppose this to be collected at the centre of gravity. We shall no longer have the simple result of Art. 670.

673. Let us now consider the case of a beam which is supported at both ends, though not built into a wall; and let it bear a weight. Let *KLMN* denote the beam, placed on supports at *M* and *N*; and let a weight *X* be hung from the beam at *PQ*. The beam will bend under the load; the upper layers of the beam will be shortened, and the lower layers will be lengthened. This case may be referred to that of Art. 666, as we will now shew. The two supports will exert upward forces on the beam, say *R* at *M*, and *S* at *N*. These forces will be such as just to balance *X*, and their values might be found by Art. 165: for instance, if *PQ* is the middle of the beam, then *R* and *S* are each equal to half of *X*. Then consider the part *QMKP* of the beam; this may be supposed to take the place of *ABDC* of Art. 666, the letters corresponding so that *QM* corresponds to *AB* and *KP* to *DC*: also *R* takes the place of *W*. The strength of the beam is determined by increasing the weight *X* until the beam breaks at *Q*. The strength is found to follow the same rule as in Art. 670. If the beam is precisely the same as that of Art. 666, and the suspended weight the same, being now hung at the middle, the strength under the present circumstances is *four* times as great as in Art. 666: for the length *QM* is half of *AB*, and the force *R* is half of *X*. Thus if a beam *fixed* at one end can just sustain a certain weight at the other end without breaking, the same beam *supported* at both ends will just sustain four times that weight at its middle point.



674. The rule which assigns the strength of the beam is consistent with the well-known fact that a long beam which is to bear a weight should rest on an edge and not on a face. Suppose that a plank is one inch thick and six inches across; when it rests on an edge the breadth is one inch, and the depth is six inches, so that the product of the breadth into the square of the depth is 36; when it rests on a face the breadth is 6 inches, and the depth is 1 inch, so that the product of the breadth into the square of the depth is 6: thus the strength in the former case is six times as great as in the latter.

675. We will now shew by an example how we may determine the strength of a solid rectangular beam when the strength of another beam of the same material but of different dimensions is known. It has been found by experiment that the strength of a beam of teak, 7 feet long, 2 inches broad, and 2 inches deep, used in the manner of Art. 673, is 938 pounds; required the strength of a beam of teak 10 feet long, 1 inch broad, and 3 inches deep. It is $938 \times \frac{7}{10} \times \frac{1}{2} \times \frac{9}{4}$, that is 739 pounds nearly. Here the factor $\frac{7}{10}$ comes from the lengths, the factor $\frac{1}{2}$ from the breadths, and the factor $\frac{9}{4}$ from the squares of the depths.

676. The following Table gives the relative strengths of beams of various materials, used in the manner indicated in Art. 673. The beams are supposed to be one foot long, and the section to be a square inch; the beams are supported at their ends and loaded at the middle: the strength is expressed in pounds:

Material.	Strength.
Cast Iron	1830 to 2410
Blue Gum	880 to 1110
Fir	270 to 680
Oak	480 to 750
Teak	820 to 830
Sandstone	60 to 130

677. If the rectangular beam is square the breadth is equal to the depth; and the strength of the beam is proportional to the cube of the breadth, divided by the length. The strength of solid *cylindrical* beams in like manner is found to be proportional to the cube of the diameter, divided by the length.

678. When a beam is supported at the two ends, and bears any weight, it is bent out of its horizontal position as we have seen. We will suppose that the weight is borne at the middle point; then the distance through which the middle point is forced below its position before the weight was put on is called the *deflection*. It is found that the deflection varies as the product of the weight into the cube of the length, divided by the product of the breadth into the cube of the depth. By this we may determine the deflection of any beam under a given weight, when the deflection of another beam of the same material but of different dimensions is known. For example, suppose that for a rod of English oak one foot long, one inch broad, and one inch deep the deflection is one inch under a weight of 3360 pounds; required the deflection when the length is 20 feet, the breadth 5 inches, and the depth 8 inches, under a weight of 6000 pounds. The required deflection in inches is $\frac{6000}{3360} \times 8000 \times \frac{1}{5} \times \frac{1}{512}$; that is, about six inches. Here the factor $\frac{6000}{3360}$ occurs because the deflections are in the same proportion as the weights. The number 8000 is the cube of 20; this factor occurs because the deflections are in the same proportion as the cubes of the lengths. The factor $\frac{1}{5}$ occurs because the deflection diminishes in the same proportion as the breadth increases. The number 512 is the cube of 8; and the factor $\frac{1}{512}$ occurs because the deflection diminishes in the same proportion as the cube of the depth increases.

679. Experiments have been made as to the strength of columns, both solid and hollow, when employed to resist pressure in the direction of the axis. When the column is

solid the strength is found to depend very much on the secure fixing of the ends. Thus when both ends of the column are flat, the strength is three times as great as when both ends are rounded; and when one end is flat and the other end rounded, the strength is twice as great as when both ends are rounded. It has been long known that when an assigned quantity of material is to be formed into a column of assigned height, more strength is obtained by making the column hollow than by making it solid; and some authorities have stated that the column is strongest when the internal diameter is to the external diameter in the same proportion as 5 is to 11. Examples of the use of hollow rods and columns are frequent in nature; as in the bones of animals, the stiff part of feathers, and the stalks of corn and other plants.

680. Dr Young remarks: "It is obvious that when the bulk of the substance employed becomes very considerable, its weight may bear so great a proportion to its strength as to add materially to the load to be supported. In most cases the weight increases more rapidly than the strength, and causes a practical limitation of the magnitude of our machines and edifices. We see also a similar limit in nature: a tree never grows to the height of 100 yards; an animal is never strong enough to overset a mountain. It has been observed that whales are often larger than any land animals, because their weight is more supported by the pressure of the medium in which they swim." But it is easy to lay too much stress on such remarks, and we may therefore just draw attention to some matters of a contrary tendency. The great tubular bridges across rivers and straits, which the present generation has constructed, would have been considered almost impossible a few years since. A tree has been discovered in California which surpasses Dr Young's limit of 100 yards. The ostrich might have been deemed the largest bird that has existed on the earth if we had not received from New Zealand the bones of an extinct creature of far greater size.

On the subject of this and the preceding Chapter the reader may consult a treatise on *The Strength of Materials and Structures* by J. Anderson.

LVII. CAPILLARY PHENOMENA.

681. We have stated that the surface of a liquid in equilibrium is a *horizontal plane*, and that liquids *seek their level:* see Arts. 358 and 383: we have now to notice some phenomena which are exceptions to these general laws. They occur when solid bodies are placed in contact with liquids, and are called *capillary phenomena* because they are best seen in tubes the bore of which is not greater than the diameter of a hair.

682. Let a very fine glass tube open at both ends be plunged vertically in a vessel of water. The water is seen to stand at a higher level in the tube than in the rest of the vessel; and moreover the surface of the water in the tube is not plane but curved, with the concavity upwards. Again, if water be put into a vessel of any size the surface of the water is not horizontal, close to the vessel, but concave upwards, rising above the general level; and the same holds with respect to the water close to a solid which floats on the water. These phenomena are also observed in the case of other liquids which *wet* the surfaces of vessels or tubes in contact with them. But in the case of liquids which *do not wet* the surfaces in contact with them, the facts are different. Thus when a fine glass tube, open at both ends, is plunged vertically in a vessel of mercury, the mercury is seen to stand at a lower level in the tube than in the rest of the vessel; and moreover the surface of the mercury in the tube is not plane but curved, with the convexity upwards. Again, the surface of mercury close to any vessel which contains it, or any solid which floats on it, is not horizontal but convex upwards, sinking below the general level. In extremely fine glass tubes the surface of water and of other liquids which *wet* the glass is a concave hemisphere; and the surface of mercury is a convex hemisphere.

683. Water will in general *wet* a surface with which it is brought into contact; but it will not do so if the sur-

face is oiled or waxed. Thus if the inner surface of a fine glass tube be oiled, the phenomena, when it is plunged vertically in water, are like those which are seen when a tube is plunged in mercury. And the water round a ball of wax floating on it is depressed below the ordinary level and curved with its convexity upwards.

684. Thus the statement made in Art. 383 will require to be a little modified if one of the vertical tubes is very fine. Suppose the left-hand tube very fine, and the right-hand tube large; then if the liquid be water it will stand at a *higher* level in the left-hand tube than in the other, and if the liquid be mercury at a *lower* level.

685. Capillary *elevation* depends on the nature of the liquid; the nature of the tube is of scarcely any consequence, provided the precaution is taken of first *wetting* the tube with the liquid which is to be used in the experiment. In a tube of about ·04 of an inch in diameter water will be elevated about 1·2 inches, nitric acid about ·9 of an inch, alcohol about ·5 of an inch. Capillary *depression* depends on the nature of the tube as well as on that of the liquid. In a glass tube of ·08 of an inch in diameter mercury will be depressed about ·15 of an inch; in a glass tube of ·2 of an inch in diameter mercury will be depressed about ·06 of an inch; in a glass tube of ·4 of an inch in diameter it will be depressed about ·015 of an inch. The mercury in a barometer has its upper surface convex, and it is therefore necessary in reading the barometer always to regard the highest point of the surface. If however the mercury with which the barometer is filled has been boiled for a long time in contact with the atmosphere, it is found that the surface has undergone some chemical change, and is then a plane at right angles to the axis of the tube.

686. Capillary phenomena depend on the attractions which are exerted between the particles of the liquid itself, and also between the particles of the liquid and those of any solid close to the liquid. So long as the mutual distance of the particles is not extremely small the attraction follows Newton's law; see Arts. 71, 77, 79: but when the distance is extremely small this does not seem to be the case. The theory of capillary phenomena has been studied

by some great mathematicians, and it is one of the most abstruse in natural philosophy. It follows from these investigations that in the case of tubes not exceeding ·1 of an inch in diameter, the amount of elevation or depression is greater in the same proportion as the diameter of the tube is smaller; and this law has been verified by observation.

687. Capillary phenomena may be observed not only in tubes but in various cases in which solids and liquids are in contact. Thus, as we have already stated, liquid in a vessel experiences capillary elevation or depression close to the sides of the vessel. Let a flat plate of glass be placed vertically in water; then it will be found that close to the plate the water will rise to the height of about one-seventh of an inch above the general level. Again, let a flat plate of glass be placed vertically in mercury; then it will be found that close to the plate the mercury will sink to the depth of about one seventeenth of an inch below the general level. Take two plates and put them vertically in water; if the plates are parallel, and near together, the water rises between them; and so likewise it does when the two plates instead of being parallel are joined along a common vertical edge, in such a manner as to form a very small angle. It is on the principles of capillary attraction that water ascends in wood, sponge, sugar, blotting-paper, and other porous bodies generally. The same forces which produce these capillary phenomena also determine the form which a drop of water assumes when hanging or falling.

688. The wick of a candle or lamp feeds the flame by capillary attraction. The wick is a bundle of threads the surfaces of which are very nearly in contact; and thus the melted tallow, or the oil, rises between them in the same way as the water between the plates in the experiments of the preceding Article. If a short fine iron tube be inserted in a vessel of oil the oil will rise to the top of the tube by capillary attraction, and may there be lighted. Suppose a bundle of threads, such as form the wick of a lamp, to have one end dipped in a vessel of liquid, and to be passed over the edge of the vessel and allowed to hang down on the outside below the level of the liquid. The liquid rises

from the vessel between the threads by capillary attraction, and then issues from the other end in drops after the manner of liquid issuing from a siphon. Any impurity in the liquid remains in the vessel, and is not transmitted through the bundle of threads: the contrivance is appropriately called the *Siphon Filter*.

689. Let a fine tube open at both ends be plunged vertically in water, and then carefully withdrawn; a drop of water will hang at the bottom of the tube, and a small column of water will remain in the lower part of the tube: the length of this column will be greater than the height of the column in the tube above the general level of the surrounding water before the tube was withdrawn. Thus it is possible to construct a vessel which shall hold some water though the bottom is full of holes. The bottom may be made of wire gauze, of iron or brass; then the meshes of the wire, being very fine, serve as capillary tubes, so that below each mesh a drop of water may hang, and a little column of water be supported above the drop.

690. There are also other phenomena which seem allied to capillary phenomena and are usually connected with them. Small needles, if slightly greased and placed very carefully on the surface of water, will remain without sinking; and some insects can walk on the surface of water. The needles and the feet of the insects are not wetted by the water; a small depression is formed round them, and they are supported in the same way as bodies would be if they displaced just as much water as would fill these depressions.

691. Let two small balls of wood or pith be placed on the surface of water; each floats with the water close round it raised a little above the general level. Let the balls be so near each other that no portion of water at the general level occurs between the two raised portions: then the two balls attract each other and run together. If instead of the wood or pith balls we put two wax balls, the water close round is depressed a little below the general level; and, as before, the balls attract each other when they are brought very near to each other. But if we put a pith ball on water close to a wax ball they repel each other.

Two needles carefully placed on water near each other will attract each other; and in like manner two iron balls placed on mercury attract each other.

692. There are numerous processes in nature and art where the influence of the forces may be traced which are concerned in the production of capillary phenomena. Thus water is supposed to rise from wells and reservoirs below the surface of the earth to the roots of plants which are nearer the surface in the same way as it rises in fine tubes. Moisture deposited on the surface of fibrous bodies is transmitted through the interior by capillary attraction; and in consequence an increase of volume occurs which may lead to striking results. Thus let one end of a rope be fixed at a point, and the other end fixed to a weight vertically below the point; the rope being just stretched tight. Let the rope be wetted; then it swells in bulk, and in the act of swelling it shortens its length and raises the weight. Considerable weights may be raised in this manner. Another illustration is furnished by a process used in France for splitting off mill stones from a block; it is thus described in Sir J. Herschel's *Discourse on Natural Philosophy*. "When a mass of stone sufficiently large is found, it is cut into a cylinder several feet high, and the question then arises how to subdivide this into horizontal pieces so as to make as many mill-stones. For this purpose horizontal indentations or grooves are chiselled out quite round the cylinder, at distances corresponding to the thickness intended to be given to the mill-stones, into which wedges of dried wood are driven. These are then wetted, or exposed to the night dew, and next morning the different pieces are found separated from each other by the expansion of the wood, consequent on its absorption of moisture..."

693. *Endosmose.* There are phenomena somewhat resembling capillary elevation and depression, but which at present have not been well connected with them. The general fact involved is this: when two different liquids are separated by a thin porous partition, either of organic or inorganic substance, currents arise between them in opposite directions. M. Dutrochet having introduced into the swimming bladder of a carp a thin syrup, carefully closed up the aperture by which he introduced it, and placed the

bladder in a vessel of water. After a time he found that the weight of the bladder had increased considerably; the water had passed through the pores of the bladder, and become mixed with the syrup. In another experiment he filled the bladder with water, and then put it in a vessel of syrup; in this case the weight was diminished after a time: the water passed out of the bladder and became mixed with the syrup. He gave the name *endosmose* to the first process, and *exosmose* to the second. At present the former word alone is found sufficient to enable us to describe all the phenomena; and it is applied to the current which *increases* the volume: so that in both experiments there is endosmose from the water to the syrup.

694. The experiment is usually performed in the following way. Take a long tube open at both ends; to one end fasten a membranous bag containing a strong syrup: then immerse the bag in a vessel of water, supporting the tube in a vertical position. It is found that some of the syrup passes out into the vessel, but at the same time more of the water passes into the bag, so that the liquid will rise in the tube to the height of several inches. The experiment may be changed by putting water into the bag, and syrup into the vessel. Then again more water passes through the membrane than syrup: so that the level of the liquid in the vessel rises. In both experiments endosmose takes place from the water to the syrup. Instead of syrup other liquids may be used, as milk or albumen; and, in general, endosmose takes place towards the denser liquid.

695. For the production of endosmose the following conditions are necessary: (1) The liquids must be different but yet capable of mixing, as spirit and water; there is no endosmose between oil and water. (2) The liquids must be of different densities. (3) The membrane must be such that at least one of the liquids can pass through it. The ascent of the sap in plants seems to be a case of endosmose.

696. The phenomena of endosmose are seen in the case of gases. If two different gases are separated by a porous partition currents are produced both ways; and finally the composition of the mixture on both sides of the partition is the same.

LVIII. ANIMAL MECHANICS.

697. In the structure and in the movements of living creatures numerous interesting illustrations of mechanical principles have been pointed out by philosophers. In order to appreciate these fully some knowledge of anatomy and physiology would be required; but a few remarks may be made which will be easily intelligible.

698. The long bones of men are *hollow*, in agreement with the principle that they are stronger than solid bones of the same weight and length would be: see Art. 679. At a joint of two bones a tough elastic substance called *cartilage* is always interposed to break the force of shocks, like the *buffers* which are attached to railway carriages: see Art. 577. And moreover a joint is always provided with an apparatus by which a certain viscid liquid can be spread over the surfaces in contact. This somewhat resembles the white of an egg, and is hence called *synovia;* it is perpetually renewed as required, and acts like the oil and unguents which are used to prevent friction in machinery: see Art. 329.

699. Numerous examples of Levers of the third kind occur in the animal frame. One is found in the human fore-arm when applied to raise an object. The fulcrum is at the elbow; the Power is exerted by a muscle which comes from the upper part of the arm, and is inserted in the fore-arm near the elbow; the Weight is the object raised in the hand. The muscle is a strap capable of extension and contraction, after the manner of an india-rubber band.

700. The pressure of the atmosphere plays an important part in keeping together the mechanism of the joints. Thus the head of the thigh bone cannot be separated by the mere weight of the limb from the surface of the cavity in the adjacent bone to which it is accurately fitted; in all motions the contact is maintained by the pressure of the atmosphere: the muscles which surround the hip joint may be divided, but still the weight of the

limb does not move the head of the thigh bone from the cavity. But if the cavity be exposed to the air by boring a small hole, or if the pressure of the atmosphere be removed by the aid of an air pump, the separation takes place. In ascending high mountains the pressure of the atmosphere is much diminished, and thus more stress is thrown on the muscles in order to maintain the contact between the convex and concave bones: this appears to be one cause of the peculiar fatigue felt in a laborious ascent. Dr Arnott seems to have been the first to draw attention to this example of the pressure of the atmosphere; he estimated the pressure at the knee joint to be about 60 pounds.

701. It is owing to the pressure of the atmosphere that various animals can sustain their bodies in opposition to the force of gravity. A fly on the ceiling of a room is an obvious example. The feet of the creature are furnished with a contrivance like a boy's sucker; so that a vacuum can be formed at the extremity of each foot, and the pressure of the atmosphere retains the foot in contact with the ceiling. It is said that the structure can be perceived "by looking at the movement of the feet of any insect upon the inside of a glass tumbler through a common magnifying glass; the different suckers are readily seen separately to be pulled off from the surface of the glass, and reopposed to another part." The same contrivance is found in the feet of other creatures, especially in the feet of the walrus, where it can be easily examined on account of the large size of the animal.

702. There are two kinds of motion of animals on the land. In one the effort consists in pressing the ground in the direction opposite to that in which the motion is to take place; the pressure is produced by internal muscular effort, and the reaction of the ground yields the force necessary to give forward motion: this is the mode of walking of man and quadrupeds. The other kind of motion may be called *creeping*, and is seen in the case of a snail: the animal here lays hold of an external fixed point, and clings to it by a part of his body; then he drags the mass of his body towards this

point. The motion of a snail may be watched by putting the animal on a piece of transparent glass, and looking through it from below.

703. The motion of birds is produced by the reaction of the air which they beat with their wings; so that the resistance of the air is essential to them, and they could not fly in a vacuum. The most arduous part of a bird's motion is the rising from the ground; the bird often runs for a short distance, or throws itself into the air by a sudden leap: the process resembles that of starting a boy's kite. Long-winged oceanic birds appear to use the tips of their wings as levers to raise their bodies. Birds which have a large surface of wing, as eagles, give only slight strokes in their flight. Birds on the contrary which have little wings, as pigeons, move them to a great extent, and thus compensate for the slight resistance which they experience from the air.

704. The motion of a fish is usually produced by lashing the water with its tail. The cuttle fish compresses forcibly its pouch which is full of liquid, drives out this liquid in one direction, and thus propels itself in the opposite direction. Fishes are furnished with an air-bladder which they can compress by muscular action; this accordingly they do when they want to sink, for so they render themselves heavier than water, bulk for bulk: when they want to rise they allow the air-bladder to expand. As a fish is nearly of the same specific gravity as the medium in which it moves, there is no need for constant exertion, as in the case of the bird to prevent sinking; all that is necessary is to overcome the resistance of the medium.

705. The structure of the wings of insects has received much attention. It appears that under all modifications two elements are essential, namely a rigid main-rib, and a flexible membrane. If the rigidity of the former is destroyed flight is prevented; and so also it is if the membrane be covered with a varnish which hardens as it dries.

706. The subject of animal locomotion has been discussed in two works recently published, namely *Animal Mechanism* by E. J. Marey, and *Animal Locomotion* by Dr Pettigrew. One object which is sought by both works is to prepare the way for the construction of flying machines, by a careful study of the motion of birds. From Dr Pettigrew's work some interesting facts may be borrowed. The Albatross, which weighs about 17 pounds, can sail for about an hour at a time without once flapping his wings. The great velocity and consequently great momentum which an animal can acquire is illustrated by the following statements: A sword-fish has been known to thrust his tusk through the upper sheathing of a vessel, a layer of felt, four inches of deal, and fourteen inches of oak plank. A wild duck terminated its career by coming violently into contact with one of the glasses of the Eddystone Lighthouse. The glass, which was fully an inch in thickness, was completely smashed. Advantage is taken of this circumstance in killing sea-birds, a bait being pinned on a board and set afloat with a view to breaking the neck of the bird when it stoops to seize the bait.

707. One of the earliest writers on the subject of the motion of fishes was Borelli; he gave an explanation and a diagram which have been since generally adopted. The half of a fish's body which contains the head is supposed to remain in a straight form, while the half which contains the tail moves to and fro like a pendulum; and this tail part by striking the water produces a reaction which urges the fish forward. According to Dr Pettigrew the fish really bends both the halves of its body, so as to form a figure like two sickles turned in opposite directions. This is the simplest case; in the long-bodied fishes, like eels, instead of two such curved portions there may be several; and the fish alternately straightens and bends them all.

LIX. WATER.

708. As water discharges so many functions in the life of man we may well devote a few pages to tracing the various forms which it assumes.

709. It is the same substance which constitutes seas, rivers, springs, clouds, rain, snow, hail and ice. The sun heating the surface of the oceans and rivers, raises from them vast quantities of water in the form of vapour; this ascends into the atmosphere and remains invisible as long as the temperature is high enough. But the temperature declines as the distance from the earth is increased, owing to the atmosphere being too rare to retain the sun's heat; and thus vapour at some height above the ground becomes mist or cloud. The formation of mists may be observed very frequently along the course of a river, or in a damp valley, in the evening; the vapour raised by the sun continues as vapour during the day, but when the temperature declines it is condensed into mist. There is at all times much vapour present in the air, but especially during the hot summer days, and then if any cold surface is presented to the air some of the vapour is immediately condensed on it; thus for instance, if we fill a tumbler with very cold water the outside of the tumbler on a hot summer day becomes covered with moisture, which arises from the condensation of vapour. In like manner the insides of window panes become covered with moisture when the air outside is suddenly chilled. In ball rooms in Russia and Norway it has sometimes happened that on the sudden opening of a window a slight shower of snow has fallen in the room.

710. Clouds are supported by the atmosphere, and they rise in it until they reach a stratum of air of about the same density as their own; there they remain in equilibrium until disturbed by some change in the temperature. The greatest height which clouds are known to have reached is about 10000 feet. Clouds may be suddenly condensed so that the particles unite and form drops of water, and these descend to the earth as rain. Sometimes the process is accompanied by thunder and lightning, and the

rain is then unusually violent. Snow and hail are forms which the descending water takes when the temperature becomes diminished below the freezing point of water; in the case of snow the diminution takes place *before* the condensation of the vapour into drops, and in the case of hail *after* the condensation.

711. The quantity of rain which falls at any specified place depends much on local circumstances. In the British Islands the western side catches the clouds which have passed over the Atlantic ocean, and have become laden with vapour; thus this side has a much greater rainfall than the eastern side has. Rains in England are also often introduced by a south-east wind. Vapour brought to us by such a wind must have been raised in countries to the south and east of us; and we may accordingly attribute it to the valleys watered by the Meuse, the Moselle, and the Rhine, and to some extent to the more remote regions of the Elbe, the Oder, and the Weser. It has been calculated that the quantity of rain which falls in England is thirty-six inches a year on the average; that is to say if we suppose the rain to be uniformly distributed it would amount to a volume having the area of England for a base, and a yard for height. Of this quantity it is supposed that thirteen inches flow off to the sea by rivers, and that the remaining twenty-three inches are raised again from the ground by evaporation. The thirteen inches which flow into the sea are restored by evaporation from the sea, and are carried back to the land through the atmosphere.

712. The vicinity of mountains exercises considerable influence on the supply of rain, and often gives rise to special phenomena. Killarney in Ireland is noted for its luxuriant vegetation. The south-west wind is checked by the Kerry mountains, tilted up, and carried over them; the vapour which the wind has brought from the Atlantic is expanded on reaching this height: this causes a reduction of the temperature, which produces condensation of the vapour, and incessant rain. Again, a traveller sometimes descends from the Alps amidst a heavy fall of rain or snow, while the plains of Lombardy *from* which the wind is blowing are blue and cloudless. The wind is hot enough to

keep the vapour in the atmosphere in its transparent state over the plains; but when the vapour rises among the mountains the fall of temperature, owing to their cold summits and its own expansion, produces a condensation and the fall of rain or snow.

713. A striking phenomenon of a similar kind is frequently seen at the Cape of Good Hope, "where the south or south-easterly wind which sweeps over the Southern Ocean; impinging on the long range of rocks which terminate in the Table Mountain, is thrown up by them,...... makes a clean sweep over the flat table-land which forms the summit of that mountain (about 3850 feet high), and thence plunges down with the violence of a cataract, clinging close to the mural precipices that form a kind of background to Cape Town, which it fills with dust and uproar. A perfectly cloudless sky meanwhile prevails over the town, the sea, and the level country, but the mountain is covered with a dense white cloud, reaching to no great height above its summit, and quite level, which, though evidently swept along by the wind, and hurried furiously over the edge of the precipice, dissolves and completely disappears on a definite level, suggesting the idea, (whence it derives its name) of a *Table-cloth*." Herschel's *Meteorology*.

714. One form in which water presents itself to our notice is that with which we are familiar under the name of *Dew*. The air always contains vapour of water floating in it; and the higher the temperature of the air the more vapour can it support in an invisible state. But if the temperature of the air is lowered then some of this vapour is condensed and takes the form of globules of water. This condensation frequently happens at night, especially after a warm day; and then at early morning the fields and gardens are found plentifully covered with this moisture which is called *Dew;* if the temperature is below the freezing point of water the moisture becomes frozen, and is called *Hoar Frost.*

715. The principal facts noticed with respect to dew are the following: (1) Surfaces on which dew is deposited are always colder than the neighbouring air. (2) Dew is

not deposited on a cloudy night; but if the clouds withdraw .even for a few minutes, and leave an open sky, the deposition of dew begins: and on the other hand if in a clear night a large cloud passes suddenly over-head the deposition of dew is checked. (3) Dew is not deposited in a sheltered situation. (4) Dew is most copiously deposited on surfaces which part with their heat readily, and regain it slowly; and dew is very slightly deposited on surfaces which part with their heat slowly, and regain it readily: thus much more dew is deposited on grass and plants than on the bare earth or the stones. (5) Dew is not deposited when there is much wind.

716. All the facts stated in the preceding Article agree well with the principle that dew is the vapour in the atmosphere condensed by contact with surfaces colder than the air in which it floated. The earth gains heat from the sun during the day; during the night this heat escapes from the earth again into the air: thus the surface of bodies near the ground becomes colder than the surrounding air, and so dew is deposited. But if the sky is covered with clouds the heat is sent back to the earth, and so prevents the fall of temperature and the consequent deposition of dew. Again if a spot be sheltered the heat is prevented from escaping, and so the temperature is maintained too high to allow the deposition of dew. It belongs to the science of Heat to distinguish the two classes of bodies referred to in (4) of the preceding Article; and it is found that bodies behave with respect to dew precisely as might have been anticipated from the facts established in the science of Heat. Thus, for example, dew is plentifully deposited on such substances as cloth, wool, velvet, and cotton; now these substances are much used for clothing, because they have the property of impeding the passage of heat from the body; they allow their outer surfaces to be very cold while they remain warm within: so that these outer surfaces part with their heat readily and regain it slowly. A wind checks the deposition of dew on a surface, because it perpetually brings warm air over the surface, and so keeps up the temperature.

717. As we have seen in Art. 709 there are various well-known phenomena which depend on the same principle as the deposition of dew. Another example may often be seen in winter time; during a long frost the walls of a house may become very cold, and then when a warm moist thaw occurs the vapour is condensed on the walls, and runs copiously down them. The circumstances connected with the deposition of dew were first carefully studied and explained by Dr Wells; his book on the subject is strongly commended by Sir J. Herschel as a beautiful specimen of experimental enquiry.

718. Water, as we have said, rises from the sea and the land in the form of vapour, and descends in the forms of rain, snow, and hail. Some of the descending water falls at once into seas or oceans; another part falls first into lakes or rivers, and from them passes on to seas or oceans. Part falls on the land, and is absorbed into the structure of trees and vegetables, or is drained off into rivers, or sinks into the ground. That which sinks into the ground after proceeding for some distance arrives at strata which it cannot penetrate, as for instance rock or clay, and then it collects into subterranean cavities. These cavities may find some natural outlet through springs; and the store of water collected in them is often by the aid of wells and pumps made useful to man. In many places a stratum of clay occurs near the surface of the ground which prevents the water from sinking deeper; so that by digging a well water is reached at a very slight depth. The well is in fact a cavity into which the water collects from the neighbourhood, and from which it can be easily raised.

719. Rain-water as it falls from the clouds is very pure, and insipid to the taste. The water which is obtained from springs and wells usually contains in it particles of substances dissolved which it has received from the soil around; thus it is in general more pleasant to the taste. In some cases, as is well known, the water from springs contains a very large quantity of foreign matter, with which it acquires a peculiar flavour and

smell; springs of such a character are called *medicinal.* The water from wells, especially in large towns, is often charged with noxious matter, owing to the fact that refuse which ought to be conveyed away is allowed to sink through the ground, and to contaminate the contents of the well; the water then becomes dangerous to drink, and the more so because there is sometimes nothing unpleasant to the taste to give a warning of the evil. The supply of water is so important for health that in most large towns it is brought from distant reservoirs into which it is collected from springs or from rivers. The reservoirs are placed at as high a level as it is necessary for the water to reach, so that by virtue of the principle that liquids stand at a level the water can be conveyed in pipes from the reservoir to the top of the highest houses: see Art. 390.

720. There is a species of well called *Artesian well,* from having been first adopted at Artois in France. The earth is pierced with a bore of a few inches in diameter, and by carrying this down low enough water is sometimes reached which will rise to the surface of the ground, or even spout out above that level. The water thus reached is contained in a stratum resting on another which the water cannot penetrate. The water-bearing stratum may come to the surface of the earth many miles from the place at which the well is dug, and it may be as full of water as it can hold: thus when the bore is made the water rises in it, on the general principle that liquids stand at a level. A famous Artesian well at Grenelle near Paris is 1860 feet deep; it gives 656 gallons a minute, and the temperature of the water is about 80 degrees of Fahrenheit's thermometer. There are Artesian wells in Cambridge, which reach a water-bearing stratum from 100 to 150 feet below the town; this stratum comes to the surface of the earth in the form of loose sandy beds in Bedfordshire and West Cambridgeshire, and is there supplied with water by rain-fall. Formerly when these wells were few in number the water used to spout out at the surface; but now owing to the increased demand the wells do not overflow except after unusually wet seasons. Bonney's *Cambridgeshire Geology.*

721. At various places on the earth's surface we have springs of warm or hot water. There are many such in the district of the Pyrenees. Four occur at Bath in England; the hottest of these yields 128 gallons every minute at the temperature of 117 degrees of Fahrenheit's thermometer. Such springs may come from spots at some depth below the surface of the earth, for it is well ascertained that the temperature rises as we descend below the surface; on an average the rise seems to be one degree of Fahrenheit's thermometer for about 55 feet in descent.

722. Some springs flow plentifully for a period, then cease for another period, and then flow again as before; these are called *intermittent springs*. It has been suggested as an explanation of these springs that the outlet from the internal cavity is of the nature of a siphon. Imagine a large cavity in the inside of a hill, and suppose that a channel in the form of a siphon proceeds from about the middle, or near the bottom of one side, of the cavity, and that the water thus escapes and issues in the form of a spring from the face of the hill. During a long dry season the surface of the water in the cavity may sink below the point where the siphon enters the side of the cavity. Then the spring ceases; and it will not flow again until the water in the cavity has attained such a height as to stand on a level with the *top of the siphon outlet*, and it is obvious that this may require a considerable prevalence of wet weather before a sufficient store is drained into the cavity.

723. A very slight inclination is sufficient to give the motion to water which rivers require. It is found that a fall of three inches a mile in a smooth straight channel will give a velocity of three miles an hour. The Ganges at the distance of 1800 miles from its mouth is about 800 feet above the level of the sea; and the water takes nearly a month to pass over this course. Rivers bring down with them solid materials which are deposited near their mouths, and thus form bars which obstruct the entrance; the quantity of this matter is so great as in time to form large additions of low land to the coast near the mouth of the river. In a similar way the river Rhone rushes into

the lake of Geneva with its stream turbid owing to the substances which it brings down from the mountains; it issues clear and blue, having deposited in the lake the matter with which it was charged. In all probability the lake will thus be finally filled up.

724. The quantity of water which passes through a vertical section of a river at any point can be ascertained when we know the dimensions of the section and the rate at which the river is flowing. Thus, suppose for example that the breadth of a river is 100 feet, the depth on an average 12 feet, and the rate of the stream 5 feet in a second; then the water flows at the rate of $100 \times 12 \times 5$ cubic feet, that is 6000 cubic feet, a second. It is found that a river is usually navigable if the discharge amounts to 1400 cubic feet in a second. The Seine at Paris is estimated to flow at the rate of about 4500 cubic feet in a second; and the Rhone at Lyons at the rate of about 21000 cubic feet in a second.

725. We must not overlook the important office which *heat* performs in the various changes of water. It is the sun that draws up from the earth and sea the vapour which passing on through the atmosphere augments the polar snow and feeds the glaciers of Switzerland. It has sometimes been hastily assumed that if the sun's heat were diminished greater glaciers would be produced than those now existing; but it must not be forgotten that any diminution of the sun's heat would be followed by a less supply of vapour, and thus the growth of the glaciers would be arrested. It has been said that the "earth and its atmosphere constitute a vast distilling apparatus in which the equatorial region plays the part of the boiler and the chill regions of the poles the part of the condenser." Professor Tyndall's *Forms of Water*.

726. Fresh water as it cools contracts until the temperature is 40 degrees of Fahrenheit's thermometer; in cooling further it expands, and when cooled to 32 degrees it freezes. Hence the greatest density of water occurs at the temperature of 40 degrees, and water of this temperature will lie at the bottom of a reservoir with cooler water

or ice floating above it. Moreover in the act of freezing a further and very considerable expansion takes place; thus ice is lighter than water, bulk for bulk, and so floats on the surface. Hence rivers and lakes do not become frozen throughout in one solid mass; the ice as it is formed rises to the surface until it makes a stratum there thick enough to protect the rest of the water from extreme cold.

727. The fact of the great expansion of water in the act of freezing has been established by repeated experiment. Shells filled with water and well stopped, have been burst by the freezing of the water. In a severe winter the metal pipes which convey water through a house are sometimes cracked by the force of water freezing in them; and when the temperature rises the crack is discovered by the leaking of the water.

728. Ice presents itself in a very interesting and instructive form in the well-known *glaciers* of Switzerland, which may be described briefly as rivers frozen but yet in motion. For an account of these phenomena and for the explanation of them we must refer to special works on the subject. Two important facts may be mentioned which have been used in the explanation of glaciers. The freezing point of water is lowered by pressure; the amount however is slight, being about one seventieth of a degree of Fahrenheit's thermometer for a pressure equal to that of the atmosphere. When two pieces of thawing ice are placed in contact they freeze together; this fact is expressed by the word *regelation.*

729. Vast masses of ice are frequently found floating in the sea, which are called *Icebergs.* Sometimes they rise to the height of hundreds of feet above the surface of the sea; and taking the specific gravity of sea-water at 1·027, and of ice at ·926, it follows that the volume below the surface must be about nine times that above. These icebergs are believed to have come from Arctic glaciers; some run aground on the shores and often maintain themselves for years; while others wander 2000 miles from the place of their origin until finally they are dissolved in the warm waters of the Atlantic Ocean.

LX. THE ATMOSPHERE.

730. We have spoken of the atmosphere as giving rise to many important consequences by reason of the pressure which it exerts; but there are various other ways in which the atmosphere exercises great influence on the life and condition of men, especially by reason of the winds which occur in it, and accordingly we will now pay some attention to the subject.

731. The atmosphere presses with a weight of about 15 pounds on each square inch of the surface of the earth at the level of the sea; hence we can calculate the entire weight, estimated at the surface of the earth, of the air which surrounds us. The result is about eleven millions of millions of millions of pounds, after making some allowance for the fact that a portion of the land is above the level of the sea, so that there is a corresponding diminution of the air. About 99½ per cent. of the atmosphere consists of a mixture of two gases, oxygen and nitrogen. The proportion of these two gases in weight is that of 23 to 77, and in volume, under the same pressure, is that of 21 to 79. Of the remaining half per cent. of the atmosphere, not consisting of oxygen and nitrogen, about one tenth consists of carbonic acid, and the rest of aqueous vapour. The amount of aqueous vapour however is not always the same at a given place; it may be sometimes more and sometimes less than the average: the fluctuations in the amount of vapour at any place, and the transference of vapour from one place to another, give rise to the various phenomena of which we have treated in Chapter LIX.

732. It is found that over that part of the globe which extends from the equator to about 30 degrees of North latitude the wind blows nearly constantly in a direction which may be described as from the North East; and over the corresponding portion of the Southern hemisphere the wind blows nearly constantly in a direction which may be described as from the South East: these remarkable winds are called *trade winds*, and we proceed to explain how they arise. Winds are caused chiefly by the action of the sun

on the atmosphere. Suppose that great heat prevails at a certain place on the earth's surface; the air is expanded, and consequently it rises and overflows into the adjacent parts of the atmosphere. Thus the pressure is diminished on the surface of the earth at the place where the great heat prevails, and increased at the adjacent places on the surface; the result is that air is driven in towards the heated place by this difference of pressure. Hence we have two currents of air, one in the higher parts of the atmosphere *outwards* from the heated place, and one in the lower parts of the atmosphere *inwards* towards the heated place.

733. Consider now the Northern hemisphere of the earth. By the preceding Article we may expect to have a current of air constantly flowing in the higher parts of the atmosphere from the hot equatorial regions towards the cold polar regions; and another current in the lower parts of the atmosphere from the polar regions towards the equatorial. Hence we should have a prevailing North wind in the Northern hemisphere. But we must now examine the effect which will be produced by the *rotation* of the earth on its axis, a circumstance which as yet we have not regarded. By reason of this rotation every place on tho earth describes a circle in 24 hours; the nearer the place is to the equator, the greater this circle is, and consequently the greater the velocity. Now a mass of air coming from the North towards the equator, begins its motion with a certain velocity towards the East, as well as the velocity towards the South; namely, the Eastward velocity which belongs to the starting place. As this mass of air moves towards the equator it is perpetually coming into contact with parts of the earth's surface which are travelling towards the East with greater velocity than its own. Thus the relative motion of the air with respect to the surface of the earth is *from* the East; and this combined with the motion which it has towards the equator, that is *from* the North, gives the wind an intermediate direction, which we describe roughly as coming from the North East. In like manner the prevailing wind in the Southern hemisphere will be one in the direction which we may describe roughly as coming from the South East. This explains the general character of the *trade-winds*.

734. We have seen that a current of warm air travels in the higher parts of the atmosphere from the equator towards the poles. The temperature of this air gradually declines, and the air comes to the surface at some distance from the equator. When the air reaches the surface it is moving towards the East, with the velocity in that direction which it had at the equator; for since it has travelled in the higher parts of the atmosphere it has lost very little of its velocity by friction. Thus the Eastward velocity is greater than the velocity in the same direction of the place with which the air comes in contact, so that the relative motion of the air with respect to the place is towards the East. This will be combined with the motion which the air has from the equator, that is towards the North if we suppose the place in the Northern hemisphere; and the result in this case is that the wind takes an intermediate direction, which we may describe roughly as coming from the South West. Thus we see the origin of the Westerly and South Westerly gales, so prevalent in our latitude.

735. In order to carry our explanation further, and bring out a closer agreement between theory and fact, we must assume that the reader possesses some knowledge of astronomy. If the earth were entirely covered with water, and the axis of rotation were perpendicular to the plane in which the earth moves round the sun, the general character of the winds would be that which we have described; but these two conditions do not hold, and thus the facts do not exactly correspond with our theory. It will be sufficient to examine one case of exception, which is the most important; let us consider then that part of the Indian ocean which is near the continent of Asia. Here the wind actually experienced is one blowing from the North East, during the period comprised between the beginning of October and the end of March; and one blowing from the South West, during the period comprised between the beginning of April and the end of September: these are called *monsoons*. The former monsoon is in fact what our theory gives under the name of trade-wind; the latter monsoon however is decidedly contrary to this, and has to be accounted for: that is, we have to shew why the wind blows from the South West during the period comprised between the be-

ginning of April and the end of September. During this period the sun's heat is given much more to the Northern hemisphere than to the Southern; the sun is vertical on every day to all the places in the North torrid zone which have a certain latitude: this latitude changes from day to day, increasing for the first three months, and decreasing for the last three months. Hence the heat is very great over the region extending from Arabia to Cochin China; consequently the air becomes expanded and flows over. An under current sets in from the equatorial regions which are less strongly heated; this current as it moves from the equator comes to places which have a less Eastward velocity than its own, so that it has a relative Eastward velocity. The motions towards the North and towards the East give rise to a wind in a direction which we may describe roughly as coming from the South West.

736. A curious law has been established by observation with respect to the direction in which the wind shifts in Europe and North America; namely, that the wind has a tendency to pass round the compass in the order North, East, South, West. The wind often makes a complete revolution in this direction, or even more than one revolution; while it seldom moves in the contrary direction, and very rarely continues that motion through a complete revolution. The fact has been known from the time of Lord Bacon, but it is now called *Dove's law of rotation of the wind*, as that writer was the first to give an explanation of it: see Herschel's *Meteorology*.

737. Instruments are constructed by which we ascertain the *direction* and the *force* of the wind at any time; they are called *anemometers*. A common weathercock illustrates the manner by which the *direction* of the wind is ascertained; it should be placed so as to be clear from any obstacles which would impede the free circulation of the wind in its neighbourhood. Arrangements can be made by which the instrument itself shall register the direction of the wind, so as to require inspection only at fixed intervals. The *force* of the wind may be ascertained by observing how far the pressure which it exerts on a square foot of surface will urge back a spring of known elasticity; or it may be determined by allowing the wind to drive round a light

vane, and observing the number of revolutions in a given time: in both cases the instrument may be made self-registering.

738. The temperature and moisture at any place on the earth's surface are much affected by the direction of the prevailing winds. Thus in England a South West wind is warm and rainy, while a North East wind is cold and dry; the former comes to us charged with excessive moisture from the Atlantic ocean, and the latter comes to us from Sweden and Russia, and thus is deficient in heat and in moisture. The perpetual exchange of heat and moisture between one place and another, through the agency of the system of circulation established by the winds, is one of the most striking operations of nature; and the atmosphere has in consequence been called by Dr Whewell *a great watering engine*, and by Sir J. Herschel *a great distilling apparatus*.

739. Leaving the subject of the winds we will now consider the *pressure* of the atmosphere. We have spoken of this as being equal to 15 pounds on a square inch, and as measured by the height 30 inches of the mercury barometer; these may be regarded as average values for a place near the level of the sea, but there are various fluctuations in this pressure which have been detected by observation, and in some degree explained.

740. In a voyage between the tropics it is found that the height of the barometer is diminished in going from a tropic to the equator, the amount being nearly a quarter of an inch. This is attributed to the copious evaporation which is always taking place in the warm intertropical seas. The formation of vapour at any spot faster than it is carried off would be attended by an *increase* of pressure; but if vapour be carried off as soon as it is formed the pressure may remain unchanged: while if the vapour in passing away carries some air with it there will be a *decrease* of pressure. In the intertropical seas the vapour, carrying air with it, is raised up from the surface, and flows over and spreads itself out in the higher regions of the atmosphere: this causes a decrease of pressure as compared with that which is experienced beyond the tropics.

741. There is an annual fluctuation in the pressure of the atmosphere the amount of which has not been settled, except for a few places; it may be described in general terms as consisting in the fact that the average pressure is greater in winter than in summer. Thus, for example, at Calcutta the average height of the barometer is about half an inch greater in January than in July. At the Cape of Good Hope, where our seasons are reversed, the average height is about ·29 of an inch greater in July than in January. In the Northern hemisphere during July the heat is greater than in the Southern hemisphere; hence arise a more copious evaporation and the transfer of air and vapour from the Northern to the Southern hemisphere, and a decrease of pressure in the Northern hemisphere: the process is like that decrease of pressure, in a voyage from the tropics to the equator, explained in the preceding Article.

742. There is also a daily fluctuation in the pressure of the atmosphere; on an average this pressure is *greater* at about nine hours before noon and nine hours after noon than at any other time; and is less at about three hours before noon and three hours after noon than at any other time. This fluctuation occurs with such regularity in some tropical countries, according to Humboldt, that in day-time the hour may be inferred from the height of the barometer without a greater error on an average than 15 or 16 minutes.

743. Besides the regular fluctuations in the height of the barometer there are others which at present must be regarded as facts quite unexplained. First, with regard to *place:* there exist extensive tracts throughout which the barometer is permanently lower than its average height, to the amount of an inch or more; the portion of the Antarctic ocean from 63^0 to 78^0 of South latitude, and from 7^0 to 8^0 of West longitude is said to exhibit this peculiarity. Secondly, with regard to *time:* it is found that occasionally a great atmospheric wave passes over a large extent of country, and the total depth of a wave from crest to trough may be measured by a difference in the height of the barometer of as much as three quarters of an inch. It seems to have been made

out that a vast wave of this kind passes annually over Great Britain and adjacent regions in the month of November. It occupies about 14 days in passing over a place, moving at the rate of about 19 miles per hour, so that its total breadth is not less than 6000 miles; the total depth of the wave from crest to trough corresponds to a difference in the height of the barometer little short of an inch.

LXI. MOLECULES.

744. A few paragraphs may be devoted to a statement of the modern views with respect to molecules; we shall give the *results* which have been obtained, with more or less confidence, though the *methods* employed for obtaining them are not of a suitable character for an elementary work.

745. Take any portion, say a drop, of water; divide it into two, then each portion seems to retain all the properties of the original drop, and among others that of being divisible: the parts are like the whole in every respect except size. Now divide each of the parts into two, each of the new parts again into two, and so on. We shall soon arrive at the stage in which the separate portions are too small to be perceived or handled; but we have no doubt that if our senses and our instruments were more delicate the process might be carried further. Then arises the question, whether this subdivision can be continued for ever. According to the prevalent belief it could not; after a certain number of operations the drop would be separated into parts which could not be further subdivided. We should thus arrive in imagination at the *atom*, which, as the word signifies, is something that cannot be cut into pieces.

746. Now let us introduce the word *molecule*. A drop of water may be divided into a certain number, and no more, of portions which are all similar; each of these is called a *molecule* of water. But the molecule of water is not an atom, for it contains two different

substances, namely oxygen and hydrogen; and by a certain process it may be separated into the two. Whether these two are really atoms or not may be left undecided.

747. Every substance, simple or compound, has its own molecule; if this molecule be divided its parts are molecules of a substance or of substances different from that of which the whole is a molecule. An atom, if there be such a thing, must be a molecule of an elementary substance.

748. The molecules of all bodies are in motion, even when the body itself appears to be at rest. These motions in the case of a solid body are confined within a very narrow range and are quite imperceptible. Each molecule of a solid body has a certain mean position about which it vibrates, and from which it never departs to an appreciable extent, being retained near it by the action of the surrounding molecules. But the molecules of liquids and gases are not confined within any definite limits; they diffuse themselves through the whole mass.

749. Air, or any other gas, when enclosed in a vessel presses upon the sides of the vessel; this is due to the motion of the molecules, which strike against the sides and thus communicate a series of impulses, following each other so rapidly that they produce an effect not to be distinguished from a continuous pressure.

750. Suppose the velocity of the molecules to be given, but the number of them to admit of being varied. Since each molecule on an average strikes the sides of the vessel the same number of times, and with the same impulse, each will contribute an equal share to the whole effect. Thus the pressure in a vessel of given size is proportional to the number of molecules in it, that is to the quantity of the air or gas in the vessel. This is consistent with the well-known fact that the pressure of air is proportional to its density; see Art. 497.

751. Next suppose the velocity of the molecules increased. Each molecule now strikes the sides of the vessel a greater number of blows in a second, and moreover the strength of each blow is also increased in the

same proportion; thus the pressure increases in the same proportion as the *square* of the velocity. The increase of velocity corresponds to a rise of temperature; and in this way we can explain the effect of warming the gas, and also the fact that under a constant pressure all gases expand equally between given temperatures.

752. If molecules of different masses are mixed together the greater masses will move more slowly than the smaller, but on an average every molecule whether great or small will have the same momentum. In a cubic inch of any gas at a standard pressure and temperature there is the same number of molecules.

753. At the temperature of the freezing point it is calculated that the average velocity of the molecules of hydrogen is about 2033 yards a second; that of the molecules of oxygen is one-fourth of this. The mass of a molecule of oxygen is 16 times the mass of a molecule of hydrogen. The velocity of the molecules of air in a room may be taken to be about 17 miles a second. The relative masses of the molecules of other gases have also been determined, and their velocities; and these together with the results already given are held to be very accurate.

754. The molecules moving in every possible direction are perpetually striking against each other. Every time two molecules come into collision the paths of both are changed, and they go off in new directions. Thus each molecule is continually having its course altered, and so, in spite of its great velocity, a long time may elapse before it reaches any considerable distance from its starting point. Ammonia is a gas easily recognisable by its smell; its molecules have a velocity of 656 yards in a second. This velocity is so great that if there were no obstacle as soon as a bottle of ammonia was opened the smell would pervade a large room; but owing to the perpetual collisions of the molecules of ammonia with the molecules of air the smell makes slow progress, and takes a long time to reach the most distant parts of the room.

755. Calculations have been made as to the average distance travelled over by a molecule between one col-

lision and another, and from this and the known average velocity the number of collisions in a second can be inferred. The results however are to be regarded as only rough approximations until the methods of experimenting are greatly improved. For hydrogen the following are the results: the average length of path between two consecutive collisions is about four-millionths of an inch, and the number of collisions in a second eighteen thousand millions.

756. The principal difference between a gas and a liquid seems to be that in a gas each molecule spends the greater part of its time in describing its free path, and is for a very small portion of its time engaged in encounters with other molecules; whereas in a liquid the molecule has hardly any free path, and is always in a state of close encounter with other molecules. Hence in a liquid the diffusion of motion from one molecule to another takes place much more rapidly than the diffusion of the molecules themselves.

757. Calculations have been made in order to determine the mass of a molecule, its diameter, and the number of molecules in a given volume. The results however do not claim to be accurate like those of Art. 753, nor even to be approximate like those of Art. 755, but only to be probable conjectures. Thus some calculations by Professor Clerk Maxwell give the following results: the size of the molecules of hydrogen is such that about fifty millions of them in a row would occupy an inch; and a million million million million of them would weigh about seventy grains; in a cubic yard of any gas at standard pressure and temperature there are about twenty-five million million million molecules. The following result is given by Sir William Thomson: imagine a single drop of water to be magnified until it becomes as large as the earth, having a diameter of 8000 miles; and let all the molecules be magnified in the same proportion; then a single molecule will appear under these circumstances to be larger than a shot and smaller than a cricket ball.

758. A molecule seems to be always the same, incapable of growth or decay, of generation or destruction. Moreover the sun and the stars appear to be built up of molecules of the same kind as those which we find on the earth. "None of the processes of Nature, since the time when Nature began, have produced the slightest difference in the properties of any molecule. We are therefore unable to ascribe either the existence of the molecules or the identity of their properties to the operation of any of the causes which we call natural. On the other hand, the exact equality of each molecule to all others of the same kind gives it, as Sir John Herschel has well said, the essential character of a manufactured article, and precludes the idea of its being eternal and self-existent....Natural causes, as we know, are at work, which tend to modify, if they do not at length destroy, all the arrangements and dimensions of the earth and the whole solar system. But though in the course of ages catastrophes have occurred and may yet occur in the heavens, though ancient systems may be dissolved and new systems evolved out of their ruins, the molecules out of which the systems are built—the foundation stones of the material universe—remain unbroken and unworn. They continue this day as they were created—perfect in number and measure and weight, and from the ineffaceable characters impressed on them we may learn that those aspirations after accuracy in measurement, truth in statement, and justice in action, which we reckon among our noblest attributes as men, are ours because they are essential constituents of the image of Him who in the beginning created, not only the heaven and the earth, but the materials of which heaven and earth consist."

This chapter is derived from a lecture delivered by Professor Clerk Maxwell at Bradford on Sept. 22, 1873.

LXII. PERPETUAL MOTION.

759. The search after perpetual motion has been in Mechanics like the attempts at squaring the circle in Geometry, or at the transmutation of metals in Chemistry,

equally fascinating and delusive. It is probable that many who have proposed to themselves to seek for perpetual motion have had no clear idea of what they were aiming at, and a few remarks on the subject may be conveniently placed in an elementary work like the present.

760. We are familiar with the performance of work by men and animals; and we know that the strength of such agents has to be maintained by a constant supply of food. The steam-engine is a more economical instrument, because the fuel required to produce a certain amount of work costs far less than the food which would be necessary to enable men or animals to attain an equivalent result. Thus naturally speculative persons might be led to enquire if a machine could be constructed infinitely more economical than any yet known; a machine in fact which should be always working, and which should cost absolutely nothing to maintain it in operation. Such a machine would be a source of inexhaustible wealth, far surpassing in value the power of changing the baser metals into gold. For the value of gold depends largely on its scarcity, and not on its intrinsic utility; while on the other hand an unlimited supply of force would reduce the cost of almost every article which is manufactured, and practically annihilate the labour consumed in every department of human life.

761. There are many applications of natural forces by which we may continually *do work;* but at the same time these are not solutions of the problem of perpetual motion and do not even pretend to be such. For example, if a person lives near an unfailing waterfall, he may construct a waterwheel, and thus have a machine always working or fit to work, and requiring no assistance to maintain its action. But this is not a self-supporting machine; the force arises from the constant flow of water. On the other hand all who make attempts at perpetual motion avail themselves of one or more of the powers of nature, such as gravity or magnetism; thus it is necessary to draw some distinction by which we may discriminate between what is to be considered as a solution of the problem and what is not. Perhaps those who speculate on the subject would say that the powers of nature must be such as act on the

definite mass of the machine, and do not continually introduce fresh matter, as the waterfall does; or they would say that any natural agent might be used which is everywhere available in an unlimited quantity and at no cost.

762. A machine may be easily constructed which, after once being started, shall continue in action for a long time without any fresh assistance. Thus a common clock when wound up will go for 15 days, and it might be made to go for a much longer period, as a year. So also a chemical or galvanic action might be adjusted which should be very slow in its process, and thus continue for a long time in operation. But in both these cases the motion ceases after some time, so that we do not obtain perpetual motion.

763. The *perpetual motion* which enthusiasts seek is something more than the words strictly imply: the desired machine is not one that merely moves perpetually, it must be able to *do work;* so that in fact if it were not to do work its motion would be for ever increasing in speed. It is very easy to obtain theoretically the *useless* perpetual motion which is just kept up, but produces no practical effect. Put a grindstone in rotation; the motion theoretically would last for ever, setting aside friction and the resistance of the air: these impediments might by ingenious contrivances be so far alleviated as to enable the motion to be continued for a long time, but even if they could be entirely removed so that the motion would be literally perpetual, still it would be what we have called *useless;* for the grindstone would be soon arrested if we attempted to make it do work, such as that of sharpening an axe, without any new application of force to it.

764. Many of the projects for perpetual motion really are like the example just selected; even if all friction and consequent waste of power could be removed the most that would be obtained would be the useless result of a continual repetition of the same motion without any increase, so that it would be retarded and finally stopped if the attempt were made to obtain any *work* out of it. Two schemes which have been frequently suggested under various modifications may be noticed.

765. Imagine a wheel with spokes, like a cartwheel, turning round a horizontal axis; let a heavy weight, in the form of a ring, slide on each spoke, or the spoke may be made hollow and the weights in the form of balls, may be confined within them. As the wheel turns round the weights move along the spokes; some of them will help the rotation of the wheel and the others will retard it: if the wheel is turning round in the same direction as the hands of a watch, then those weights which are at any instant on the right-hand side, to a person looking at the wheel, will assist the motion, and those weights which are at the same instant on the left-hand side will retard the motion. It is not difficult to see that nothing is gained by the use of these moveable weights; those which help are just balanced by those which retard. But instead of *straight* spokes sanguine speculators have proposed to use spokes *curved* in a particular manner, so that the weights which help the motion should remain as remote as possible from the axis, and therefore exert greater influence, while those which retard the motion should linger near the axis, and therefore exert less influence. Such attempts however are in vain; theory proves distinctly that, setting aside friction and resistance, the same motion will perpetually recur without any increase; and experiment soon shews that by reason of friction and resistance even this useless perpetual motion cannot be secured.

766. Again, imagine a screw of Archimedes employed to raise water, and let the water when raised fall on a waterwheel suitably adjusted so as to turn the screw itself. Here again, on the most favourable estimate all that could be obtained would be *useless* perpetual motion; but even this could never be secured owing to the waste of power by friction and resistance. In fact the *modulus* of a good waterwheel is not greater than $\frac{7}{10}$; so that the waterwheel could not do more than $\frac{7}{10}$ of the work it ought theoretically to do. And the *modulus* of the screw of Archimedes could not be more than $\frac{7}{10}$. Thus on the whole the

machine instead of perpetually raising again all the water supplied would not raise more than $\frac{7}{10} \times \frac{7}{10}$, that is $\frac{49}{100}$, of it.

767. Both these contrivances were suggested at a very early period, and Bishop Wilkins, more than two centuries ago, speaking of two substantially coincident with them, says "till experience had discovered their defect and insufficiency, I did certainly conclude them to be infallible." After describing them he says; "Thus have I briefly explained the probabilities and defects of these subtle contrivances, whereby the making of a perpetual motion hath been attempted. I would be loth to discourage the enquiry of any ingenious artificer by denying the possibility of effecting it with any of these mechanical helps; but yet (I conceive) if those principles which concern the slowness of the power in comparison to the greatness of the weight were rightly understood and thoroughly considered, they would make this experiment to seem, if not altogether impossible, yet much more difficult than otherwise, perhaps, it will appear. However, the enquiring after it cannot but deserve our endeavours, as being one of the most noble amongst all these mechanical subtilties. And, as it is in the fable of him who dug the vineyard for a hid treasure, tho' he did not find the money, yet he thereby made the ground more fruitful, so, tho' we do not attain to the effecting of this particular, yet our searching after it may discover so many other excellent subtilties as shall abundantly recompence the labour of our enquiry." The Bishop then adverts to "the pleasures of such speculations which do ravish and sublime the thoughts with more clear angelical contentments": this he illustrates by the examples of Archimedes, Thales, and Pythagoras.

768. The mathematician knows that with such forces as experience shews to be actually operating in nature useful perpetual motion is impossible. There are indeed *conceivable* laws of force for which this assertion does not hold; but forces never actually present themselves which

conform to such laws. So firmly has the conviction of the impossibility of a useful perpetual motion impressed itself on men of science that they extend their belief even to the mechanical applications of heat and electricity; and they have thus in some cases been led to anticipate with confidence the results of untried experiments.

769. A history of attempts to produce perpetual motion was published in 1861, entitled *Perpetuum Mobile; or, search for self-motive power*...by Henry Dircks, C.E. In the long list of projects which this volume records there appear to be only two which carry any special interest with them; and in both the alleged discovery was kept secret. One of these is due to the Marquis of Worcester, who published a curious book entitled the *Century of Inventions*, in which among other things he claims to have contrived a machine which bears some resemblance to a modern steam-engine. In the fifty-sixth Article of his book he gives a brief account of a wheel which he invented and exhibited in the Tower before King Charles I. and several members of his court. He says it was "a most incredible thing if not seen"; and that "The wheel was 14 foot over, and 40 weights of 50 pounds apiece." It would seem to have been of the nature of the contrivance noticed in Art. 765. The other discovery is claimed by Orffyreus, who was born in Alsace. He is said to have constructed for the Landgrave of Hesse Cassel a wheel which revolved 25 or 26 times in a minute; and continued in motion for two months. A letter on this wheel is in print which was addressed by 's Gravesande to Newton, speaking favourably of the contrivance and its inventor. The most probable conjecture is that the rotation was produced by clockwork concealed in the wheel. Orffyreus is said to have destroyed his wheel in dissatisfaction with the treatment which he received.

770. Amusing instances are on record of the confidence of sanguine speculators in the success of their projects. Thus we find one person whose anxiety was as to whether he should ever be able to stop his machine when once in motion. Another person proposed a modification of the waterwheel and pump scheme of Art. 766, and being in

alarm lest his machine should pump up more water than required for itself, suggested the use of a *wastepipe* by which the superfluous water could in fact be thrown away. In a case which was submitted to the late Dr Whewell the only misgiving of the projector arose from moral considerations; he feared that the labouring classes having no longer any necessity for toiling would sink into idleness and vice.

771. The history of the subject reproduces perpetually the same features. Accident brings the idea of perpetual motion to the attention of a man who can handle tools, and who has some inventive faculty, a quality by no means rare but which is of little use except it be combined with a knowledge of what has been done before. Such a person however is in general ignorant of all the preceding failures and also of the principles on which a well-trained mathematician relies for his conviction of the impossibility of attaining the proposed result. He soon convinces himself that he has succeeded, and having probably exhausted his own funds he applies to some person who can furnish capital for constructing models and engines, and for making the invention known. Loss and perhaps ruin ultimately fall on the sanguine but undisciplined contriver and the credulous patron.

EXAMPLES.

IV. MOTION. FALLING BODIES.

1. A railway train performs a journey of 45 miles in 2 hours: find the velocity in feet per second.

2. A train is moving at the rate of 270 yards per minute: express this velocity in feet per second.

3. The distance of the Moon being about 240000 miles find the uniform velocity of a body which would pass from the Earth to the Moon in 400 days.

4. The minute-hand of a watch is twice as long as the second-hand: shew that the end of the second-hand moves thirty times as fast as the end of the minute-hand.

5. Find the space described in the fifth second by a falling body.

6. A body falls for six seconds: find the space described in the last two seconds of the fall.

7. Find the space described by a falling body in one tenth of a second beginning at the end of four seconds.

8. Find the space described by a falling body in one twentieth of a second beginning at the end of two seconds.

9. If a body fall for a quarter of a minute shew that it would then be moving at the rate of 480 feet per second; and ascertain what this velocity will be, expressed in miles per hour.

10. Shew that a falling body acquires in the seventh of a second a velocity of about three miles per hour.

11. Find the velocity of a falling body at the end of two seconds, and also at the end of two seconds and a twentieth: and shew that the space actually described in the twentieth of a second is the same as if the body had moved uniformly with a velocity equal to half the sum of the two velocities.

12. A stone dropped into a well is heard to strike the water in two seconds and a quarter: find the depth of the well.

13. Shew by Art. 88 that the spaces described by a falling body in the first, second, third, fourth... seconds are in the proportion of the successive odd numbers 1, 3, 5, 7,...

14. A falling body is observed to describe 336 feet in one second: find how long it has been falling altogether.

15. A body has been falling for a certain number of seconds: shew that the number of feet described in the next two seconds is the product of 64 into the number of seconds increased by unity.

16. A falling body is observed to describe 144 feet in two seconds: find how long it had been falling when it was first observed.

17. Shew by numerical examples that the velocity of a falling body at the *middle* of any interval is half the sum of the velocities at the beginning and the end of the interval. For instance, half the sum of the velocities at the end of three seconds and at the end of three seconds and a half is the velocity at the end of three seconds and a quarter.

18. Shew that the space described by a falling body between the end of three seconds and the end of three seconds and a half is the same as would be described by a body moving uniformly with the velocity which the falling body has at the end of three seconds and a quarter. Take other examples of a similar kind.

19. Two balls are dropped at the same instant from two different points, one vertically above the other: shew that the balls as they fall always keep at the same distance from each other.

20. Two balls are dropped from the same point at different instants: shew that as they fall the distance between them increases continually with the time.

V. RELATIVE MOTION. COMPOUND MOTION.

1. A steamer is moving at the rate of 20 feet per second: find the resultant velocity of a ball which is shot from the stern to the bow of the steamer with a velocity of 15 feet per second.

2. Find also the resultant velocity of the ball if it is shot from the bow to the stern.

3. Find also the resultant velocity of the ball if it is shot from side to side.

4. One steamer moves from West to East at the rate of 12 miles per hour, and another moves from South to North at the rate of 16 miles per hour; the steamers start from the same point at the same time: shew that one will separate from the other at the rate of 20 miles per hour.

5. An express train 66 yards long moving at the rate of 40 miles an hour meets a slow train 110 yards long moving at the rate of 20 miles an hour. Find how long a man in the express train takes to pass the slow train, and how long the express train takes in completely passing the slow train.

6. A railway train is moving at the rate of 60 miles per hour, and a ball is dropped from a point at the end of the train which is 16 feet above the ground: shew that if the ball did not partake of the motion of the train when it reached the ground it would be 88 feet behind the train.

7. A river one mile broad is running downwards at the rate of four miles an hour; a steamer can go up the river at the rate of six miles per hour: find at what rate it can go down the river.

8. A stone after falling for one second strikes a plane of glass in breaking through which it loses half its velocity: find how far it will fall in the next second.

9. Through what space must a heavy body fall from rest in order to acquire a velocity of 160 feet per second? If it continue falling for another second after having acquired this velocity find through what space it will fall.

10. A heavy particle is dropped from a given point, and after it had fallen for one second another particle is dropped from the same point. Find the distance between the particles when the first has moved for five seconds.

VI. MOTION CAUSED BY FORCE.

1. A body in motion is observed to move over 45 feet in the first three seconds, and over 80 feet during the next five seconds: shew that it must have been acted on by some force during the motion.

2. A body falls through 169 feet: find its velocity: find also the time it takes to fall through the next 120 feet.

3. An arrow is shot upwards and at the end of six seconds reaches the ground again: find the height to which it ascended and the velocity at starting.

4. A body falls down through 289 feet: find the time of motion and the velocity acquired.

5. An arrow is shot upwards with a velocity of 112 feet per second: find the height to which it will rise.

6. A body is thrown upwards with a velocity of 96 feet per second: find after how many seconds it will be moving downwards with a velocity of 48 feet per second.

7. A balloon is moving upwards with a certain velocity; a weight hangs from the balloon by a string: if the string be cut what will be the motion of the weight?

8. A heavy particle is dropped from a height of 169 feet above a level plane, and while falling it is carried horizontally with a uniform velocity of 8 feet per second. At what distance from the starting point will it strike the ground?

9. A ball is allowed to fall to the ground from a height of 64 feet, and at the same instant another ball is thrown upwards with just sufficient velocity to carry it to the height from which the first one falls: shew when the balls will pass each other.

10. Find in the Example of Art. 124 the height of the body at the end of two seconds and a half.

VII. MASS AND MOMENTUM.

1. A body has a certain momentum after falling for a certain time: shew that in order to gain a double momentum it must fall for double the time.

2. A body has a certain momentum after falling through a certain space: shew that in order to gain a double momentum it must fall through four times the space.

3. Shew that the momentum of a ball weighing three pounds two ounces, moving with the velocity of 20 feet per second is the same as that of a ball weighing two pounds and a half, moving with the velocity of 25 feet per second.

4. An arrow shot upwards from a bow reaches to a certain height: shew that if the weight of the arrow be doubled, other circumstances remaining the same, the height reached will be one-fourth of its former value.

5. A body weighing 5 pounds moves uniformly over 300 feet in the same time as another body weighing 3 pounds moves uniformly over 500 feet: shew that the momentum is the same in the two cases.

6. A certain force can give to a body weighing a pound a velocity of 10 feet per second: shew that it could give to a body weighing a ton a velocity of $\frac{3}{56}$ of an inch in a second.

7. A boat with its crew weighs 9 cwt.; and the crew can row it at the rate of 3 miles an hour. If the boat be fastened to a vessel weighing 120 tons, shew that the crew will be able to pull the vessel along at the rate of about a foot in a minute.

8. A body is known to be under the action of a constant force; it is observed to move from rest and in the first second to describe 8 feet: shew that the force is equal to half the weight of the body.

9. A body known to be acted on by a constant force moves from rest and is found to describe a space of 36 feet in the first three seconds of its motion: find with what velocity it will be moving at the end of the sixth second of its motion.

10. A body containing 50 pounds of matter is set in motion by a constant force which acts for five seconds; the force then ceases to act, and the body, now acted on by no force, moves over 64 feet in the next two seconds; shew that the force is equivalent to a weight of 10 pounds.

11. A moving body is observed to increase its velocity by a velocity of 8 feet per second in every second: find how far the body would move from rest in 5 seconds.

12. A body under the action of a constant force describes in three successive seconds spaces of 12 feet, 18 feet, and 24 feet respectively: find what proportion the force producing the motion bears to the weight of the body.

13. A body moves from rest and at the end of 8 seconds has a velocity of 40 feet per second; its velocity is known to have been uniformly accelerated: find how far the body went in the 8 seconds. Supposing the motion to continue under the same circumstances find how far the body will go in the next 8 seconds.

14. The velocity of a train is known to have been increasing uniformly; at one o'clock its velocity was 12 miles an hour, at ten minutes past one o'clock its velocity was 36 miles an hour: find the velocity at $7\frac{1}{2}$ minutes past one o'clock.

15. Find the proportion which the force acting on the train in the preceding Example bears to the weight of the train.

16. A body is moving at a given instant with a velocity of 40 feet per second: from this instant a constant force is made to act on it in a direction opposite to that of the motion which brings it to rest after it has described 20 feet: find the proportion which this force bears to the weight of the body.

17. Two bodies whose weights are 2 cwt. and 96 lbs. respectively move from rest under the action of constant forces; the former is found to describe 8 feet in the first three seconds of its motion, and the latter to describe 7 feet in the first two seconds of its motion. Find the proportion of the former force to the latter.

18. A body moving from rest under the action of a constant force acquires in each second an additional velocity of 12 feet per second: find the distance it passes over in the first five seconds of its motion, and the velocity it has after passing over 96 feet from its starting point.

19. A stone is let fall from the top of a tower. A second after another stone is thrown downwards after it, and overtakes the first stone in a second: find the velocity with which the second stone was projected.

20. If a body is projected upwards with a velocity of 120 feet per second find the greatest height to which it will rise; and find when it is moving upwards with a velocity of 40 feet per second.

VIII. THIRD LAW OF MOTION.

1. Explain the *kick* of a gun.

2. Suppose in Art. 139 that the heavier body weighs 9 ounces, and the lighter body 7 ounces: find the space which each body describes in two seconds, and the velocity acquired.

3. Find the tension of the string in the preceding Example.

4. In Atwood's machine one of the two bodies is heavier than the other by half an ounce: find the weight of each body, so that the heavier may fall through one foot in the first second.

5. In Atwood's machine the heavier weight is $4\frac{1}{4}$ ounces, and the lighter is $3\frac{3}{4}$ ounces: find the velocity acquired at the end of a quarter of a minute. If the string be then cut find after what interval the ascending weight will be for an instant at rest.

6. Shew that if in Atwood's machine the lighter of the two weights is $\frac{3}{5}$ of the heavier the velocity gained in any time is $\frac{1}{4}$ of that of a falling body in the same time; if the lighter weight is $\frac{4}{6}$ of the heavier the velocity is $\frac{1}{5}$ of that of a falling body; if the lighter weight is $\frac{5}{7}$ of the heavier the velocity is $\frac{1}{6}$ of that of a falling body; and so on.

7. Find the proportion of the weights in Atwood's machine in order that the heavier body may fall through one foot in the first second.

8. Find also the proportion of the weights in order that the heavier body may fall through one inch in the first second.

9. Shew that in Atwood's machine if the sum of the weights is 3 pounds, and their difference half an ounce, the motion will be such that instead of the number 32 feet of Art. 92 we shall have four inches.

10. A man jumps suddenly off a platform with a twenty pound weight in his hand: find the pressure of the weight on his hand while he is in the air.

IX. COMPOSITION OF FORCES.

1. Find the resultant of three forces of 3, 5, and 8 pounds respectively, acting all in the same direction in a straight line.

2. Find the resultant of three forces of 3, 5, and 8 pounds respectively, acting in the same straight line, but the largest force opposite in direction to the other two.

3. Forces equal to 40 pounds and 9 pounds respectively act at right angles: find the magnitude of their resultant.

4. Two forces equivalent to 36 pounds and 48 pounds act at a point (1) in the same direction, (2) in opposite directions, (3) at right angles. Find their resultant in each case.

5. Shew by a diagram that if two equal forces act in directions which include an angle of 120 degrees the resultant is equal to each component.

6. Three equal forces act in one plane in such a way that each of them makes an angle of 120 degrees with each of the other two: shew that the three forces will balance.

7. Employ the foregoing proposition to shew that the resultant of the forces 7 pounds and 14 pounds acting at an angle of 120 degrees is the same as the resultant of forces of 7 pounds, and 7 pounds acting at an angle of 60 degrees.

8. $ABCD$ is a square: find the resultant of forces represented by AB, AC, and AD.

9. Forces represented by 4, 5, 10 pounds respectively act on a particle: shew that they cannot keep it at rest.

10. $ABCD$ is a square. Forces of 10 pounds each act from A to B, from B to C, and from C to D respectively: shew that they can be balanced by a force of 10 pounds acting along a certain straight line.

11. *ABCD* is a square. A force of 10 pounds acts from *D* to *A*, a force of 10 pounds from *B* to *C*, and a force of 20 pounds from *A* to *B*: find their resultant.

12. *ABCD* is a square. A force of 4 pounds acts from *A* to *B*, a force of 6 pounds from *B* to *C*, and a force of 10 pounds from *C* to *D*: find their resultant.

13. *ABCD* is a square: a force of 100 pounds acts from *A* to *C* and is balanced by two forces acting respectively from *B* to *A*, and from *D* to *A*: find these forces.

14. Draw a rectangle *ABCD*, such that the side *AB* is three-fourths of the side *BC*; forces of 3, 9, and 5 pounds act from *B* to *A*, from *B* to *C*, and from *D* to *B* respectively: find their resultant.

15. It is required to substitute for a given vertical force two others, one horizontal and one inclined at an angle of 45 degrees to the vertical: determine by a diagram the magnitude of these two forces.

16. A body rests on a smooth horizontal plane, and is acted on by a force of six pounds in a direction inclined obliquely downwards at an angle of 45 degrees to the horizon. Find by a diagram the magnitude of the horizontal force required to prevent the motion of the body.

17. Three strings are tied in a knot; the ends of two of them are fastened to pegs, and the third has a known weight attached to it: give a construction for finding the forces pulling the pegs; and from the construction shew to what the two forces respectively become almost equal when one of the supporting strings is almost long enough to allow the other to hang in a vertical position.

18. A weight of 24 pounds is suspended by two strings, one of which is horizontal, and the other is inclined at an angle of 45^0 to the vertical direction: find by a diagram the tension of each string.

19. Six vertical smooth posts are fixed in the ground at equal intervals round the circumference of a circle, and a cord without weight is passed twice round them all in a horizontal plane, and pulled together with a force of 100 pounds. Find the magnitude and direction of the resultant pressure on each post.

20. When a horse is employed to tow a large barge along a canal the tow-rope is usually of considerable length: give a reason for using a long rope instead of a short one.

Shew whether the same considerations hold good in relation to the length of the rope when a steam-tug is used instead of a horse.

X. PARALLEL FORCES. CENTRE OF GRAVITY.

[The parallel forces are supposed *like*.]

1. Two parallel forces of 5 and 7 pounds act in straight lines which are one foot apart: find their resultant.

2. Three forces of 2, 10, and 12 pounds act along parallel lines in a body: shew how they may be adjusted so as to produce equilibrium.

3. At two opposite corners of a square act two forces each of one pound, and at the other two corners act two forces each of two pounds; all the forces are parallel: find the magnitude of the resultant and the point where it acts.

4. The resultant of two parallel forces of 18 pounds and 54 pounds is distant 2 feet from the former force: find its distance from the latter.

5. Three equal parallel forces act at three points of a straight line A, B, C; if $AB=BC$ find the position of the centre of the parallel forces.

6. Four equal parallel forces act at four points of a straight line A, B, C, D; if $AB=BC=CD$ find the position of the centre of the parallel forces.

7. A rod AB weighs 10 pounds and is found to balance about a point 8 feet from A; a weight of 6 pounds is fastened to A: find about what point the rod will now balance.

8. Two balls of uniform density and 6 inches in radius are placed side by side in contact; one weighs 120 pounds and the other weighs 360 pounds; find how far the centre of gravity of the two balls is from the centre of the heavier.

9. Two thin circular discs of the same material are placed in contact; if the radius of one be double the radius of the other shew that the centre of gravity of the two discs is at a distance from the centre of the larger circle equal to one fifth of the distance of the centres.

10. A uniform rod weighs 10 pounds; a weight of 10 pounds is fastened to one end and a weight of 20 pounds to the other: find about what point the whole will balance.

11. A rod whose weight can be neglected rests on two points 12 inches apart; a weight of 18 pounds hangs on the rod between the points and 4 inches from one of them: find the pressure on each point.

12. A uniform rod 6 feet long has a weight of 10 pounds fastened to one end; it will balance on a point 6 inches from that end: find the weight of the rod.

13. A straight rod is bent at right angles, so that one part is twice as long as the other: shew how the centre of gravity of the bent rod can be determined.

14. Equal weights are placed at the corners of a triangle: find the centre of gravity of the three weights, and shew that it is the same point as the centre of gravity of the triangle.

15. Weights of one pound each are placed at two of the corners of a triangle, and a weight of two pounds at the third corner: find the centre of gravity of the three weights.

16. A cylindrical vessel weighing 4 pounds and the internal depth of which is 6 inches will just hold 2 pounds of water; the centre of gravity of the vessel when empty is 3·39 inches from the top: find the centre of gravity of the vessel and its contents when full of water.

17. Equal weights are placed at the angular points of a heavy triangular plate, and also at the middle points of the sides: find the centre of gravity of the plate and the weights.

18. Two uniform cylinders of the same material are joined together end to end so that their axes are in the same straight line; one cylinder is 9 inches long and 2 inches in diameter, and the other is 6 inches long and 3 inches in diameter: find the centre of gravity of the combination.

19. Four heavy particles of the relative weights 2, 3, 4, 5 are placed at the corners of a square board in order: find the centre of gravity of the four particles.

20. Find the centre of gravity of three equal rods AB, AC, AD, diverging from a common point A.

XI. PROPERTIES OF THE CENTRE OF GRAVITY.

1. Shew that a cylinder if placed on its flat end will be in stable equilibrium, but if placed on its curved surface in neutral equilibrium.

2. A uniform rod has at one end a small heavy ball; the rod is pushed gently along a table with the ball foremost and falls off when a quarter of the length of the rod is beyond the edge of the table: shew that the ball is as heavy as the rod.

3. Two balls weighing 4 pounds and 1 pound respectively have their centres connected by a rod without weight 50 inches long; they are supported at a point of the rod and can turn freely round: find in what position they will rest if the point is 8 inches from the centre of the heavier ball.

4. A triangular board is hung by a string attached to one corner: find what point in the opposite side will be in a line with the string.

5. If the force of gravity instead of acting vertically were to act horizontally from East to West would this affect the position within a body of its centre of gravity?

6. *ABCDE* is a borad of irregular figure; and it is found that when the board is hung from *A* the point *C* is in the vertical line through *A*; and that when it is hung from *B* the point *D* is in the vertical line through *B*: if the board is hung from the point *E* find what point in the perimeter will be vertically below *E*.

7. A uniform equilateral triangle has a sphere of the same weight as the triangle attached to it so that the centre of the sphere is at an angular point of the triangle: if the triangle be suspended by a string attached to the middle point of one of the sides which passes through the centre of the sphere shew by a diagram the situation of the sides of the triangle in equilibrium.

8. *ABCD* is a quadrilateral figure such that the sides *AB* and *AD* are equal, and also the sides *CB* and *CD* are equal: find the centre of gravity of the figure.

9. A short circular cylinder of wood has a hemispherical end; when placed with its curved end on a smooth table it rests in any position in which it is placed: determine the position of the centre of gravity.

10. A piece of uniform paper in the form of a regular hexagon has one of the equilateral triangles obtained by joining the centre to two consecutive angular points cut out: determine the position of the centre of gravity of the remainder of the paper.

XII. THE LEVER.

[The lever is supposed to be without weight unless the contrary is stated.]

1. In a lever of the first kind the force at one end is 3 pounds, and its distance from the fulcrum is 4 inches; if the distance of the force at the other end from the fulcrum is 6 inches, find the force.

2. A lever 10 feet long has a weight of 11 pounds at one end; the fulcrum is 10 inches from this end: find what weight at the other end the 11 pounds will balance.

3. Find where the fulcrum must be placed that 2 pounds and 8 pounds may balance at the extremities of a lever 5 feet long.

4. A child weighing 56 pounds is at one end of a plank and a child weighing 72 pounds at the other end; the plank is 16 feet long: find the distance of each child from the fulcrum when the plank is used for a see-saw.

5. The arms of a lever are respectively 15 and 16 inches: find what weight at the end of the short arm will balance 30 pounds at the end of the long arm, and what weight at the end of the long arm will balance 30 pounds at the end of the short arm.

6. In a pair of nut-crackers if the nut be placed at a distance of one inch from the hinge, and the hand presses at a distance of eight inches, shew that for every ounce of pressure exerted by the hand the nut undergoes a pressure of half a pound.

7. A force of 1 pound 14 ounces acts on a lever at the distance of 3 feet 4 inches from the fulcrum; another force of 2 pounds 8 ounces acts at the distance of 2 feet 6 inches from the fulcrum and tends to turn the lever in the contrary direction: shew that the lever will remain in equilibrium.

8. A straight lever 6 feet long, and heavier towards one end than the other, is found to balance on a fulcrum 2 feet from the heavier end, but when placed on a fulcrum at the middle it requires a weight of 3 pounds hung at the lighter end to keep it horizontal: find the weight of the lever.

9. A straight lever 20 inches long weighs 10 ounces: find where the fulcrum must be placed in order that the lever may be in equilibrium with a weight of 16 ounces hung at one end and a weight of 9 ounces at the other.

10. A pole 10 feet long weighing 24 pounds rests with one end against the foot of a wall; from the other end a cord runs horizontally to a point in the wall 8 feet from the ground: find the tension of the cord.

11. A man whose weight is 160 pounds wishing to raise a rock leans with his whole weight on one end of a horizontal crow-bar 5 feet long which is propped at the distance of 4 inches from the end in contact with the rock: find what force he exerts on the rock, and what pressure the prop has to sustain.

12. Two men A and B carry a weight of 200 pounds on a pole between them; the men are 5 feet apart and the weight is at a distance of 2 feet from A: find the weight which each man has to bear.

13. Two weights are carried on a pole which rests at M and N on the shoulders of two men; one weight is 40 pounds and is put at a point C such that MC is to CN as 3 is to 2; the other weight is 56 pounds and is put at a point D such that MD is to DN as 5 is to 2: find the weight which each man must bear.

14. ACB is a *bent lever* with its fulcrum at C; the angle ACB is a right angle; the arms AC and BC are 10 inches and 7 inches, and AC is in a vertical position; a horizontal force of 21 pounds acting at A is balanced by a vertical force acting at B: find the force at B, and shew in what direction the pressure on the fulcrum acts.

15. AB is a straight lever acted on at A and B by two equal forces whose directions contain an angle of 60 degrees; the force at A acts at right angles to AB: find where the fulcrum must be taken for equilibrium, and the pressure on the fulcrum.

XIII. THE BALANCE.

1. The arms of a balance instead of being equal are respectively 12 and 13 inches long; a body which really weighs 9 pounds 12 ounces is weighed on this balance: find the apparent weight first when it is put at the end of the long arm, and next when it is put at the end of the short arm.

2. Suppose that a body which really weighs one pound appears in a balance to weigh one pound one ounce: find the proportion of the lengths of the arms.

3. The arms of a balance instead of being equal are respectively 15 and 16 inches long; a body which really weighs 30 pounds is weighed on this balance: find the apparent weight first when it is put at the end of the long arm, and next when it is put at the end of the short arm.

4. Shew by Examples 1 and 3 and others of the like kind that the sum of the two apparent weights is greater than twice the real weight.

5. Shew by Examples 1 and 3 and others of the like kind that the product of the two apparent weights is equal to the square of the real weight, the weights being all expressed in terms of the same unit.

6. A substance is weighed from both arms of a false balance, and its apparent weights are 9 pounds and 4 pounds: find the true weight.

7. A uniform bar 20 inches long and weighing two pounds is used as a common steel-yard, the fulcrum being 5 inches from one end: find the greatest weight which can be weighed with a moveable weight of 4 pounds.

8. Shew that in a common balance it makes no difference at what point of the scale-pan the weight is put, whether at the centre or nearer to the edge.

9. A man in the act of being weighed in a balance of the ordinary kind pushes with a walking stick the beam of the balance at a point between the point of suspension of the scale-pan in which he is and the fulcrum: determine whether any effect will be produced on his apparent weight.

10. In the preceding case if the scale in which the man is be kept from moving laterally by a horizontal string attached to a fixed point, find the effect.

XIV. THE WHEEL AND AXLE. THE TOOTHED WHEEL.

1. If the radius of the Axle is 3 inches, and that of the Wheel 2 feet, find what Power will support a Weight of 112 pounds.

2. The radius of the Axle is $1\frac{1}{2}$ inches: find what the radius of the Wheel must be, so that a Weight of any number of pounds may be supported by a Power of as many ounces.

3. The radius of the Axle is 4 inches, and that of the Wheel 3 feet; a Weight of 18 pounds is hung from the Axle: find the Power required for equilibrium, and the whole pressure on the axis of the machine.

4. A weight is to be raised by means of a rope passing round a horizontal cylinder 10 inches in diameter, turned by a winch with an arm $2\frac{1}{2}$ feet long: find the greatest weight which a man could so raise without exerting a pressure of more than 50 pounds on the handle of the winch.

5. The radius of the Axle is 3 inches, and that of the Wheel 10 inches: if the Power be 4 pounds and the Weight 13 pounds, shew that there will not be equilibrium but that the Power will prevail.

6. Find the dimensions of a Wheel and Axle by means of which a Power of 40 pounds will suffice to raise a Weight of 5 cwt.

7. It is found that by means of a Wheel and Axle a Weight of 15 pounds is supported by a Power of 2 pounds. If the Power be slightly increased so as to raise the Weight, find how far the Power must descend to raise the Weight through one foot.

8. The radius of the Axle of a capstan is 1 foot; if four men push each with a force of 100 pounds on spokes 5 feet long, shew that on the whole a tension of 2000 pounds can be produced on the rope which passes round the Axle.

9. A Wheel and Axle is used to raise a bucket from a well; the circumference of the Wheel is 60 inches, and while the Wheel makes three revolutions the bucket, which weighs 30 pounds, rises one foot: find the smallest force which can turn the Wheel.

10. Explain how it is that in drawing up a bucket of water from a deep well the difficulty increases slightly as the bucket ascends.

XV. THE PULLY.

[The pullies are supposed to be without weight unless the contrary is specified.]

1. In the first system of Pullies if there are five Pullies and the Power is 3 pounds, find the Weight.

2. In the same system if there are four Pullies and the Weight is 48 pounds, find the Power.

3. In the same system if the Power is 7 pounds and the Weight 112 pounds, find the number of Pullies.

4. In the second system of Pullies if there are 6 strings at the lower block and the Power is 5 pounds, find the Weight.

5. In the same system if there are 8 strings at the lower block and the Weight is 56 pounds, find the Power.

6. In the same system if the Power is 5 pounds and the Weight is 25 pounds, find the number of strings, and draw the diagram.

7. In the third system of Pullies if there are five Pullies and the Power is 2 pounds, find the Weight.

8. In the third system of Pullies if there are four Pullies and the Weight is 60 pounds, find the Power.

9. In the same system if the Power is 3 pounds and the Weight is 45 pounds, find the number of Pullies.

10. Suppose there are four Pullies in the third system, each weighing 1 pound, and that the Power is 5 pounds, find the Weight.

XVI. THE INCLINED PLANE, THE WEDGE, AND THE SCREW.

1. Suppose the Power to act parallel to the Plane, and that the height of the Plane is to its length as 3 is to 5; if the Power is 12 pounds, find the Weight.

2. Suppose the Power to act parallel to the Plane, and that the height of the Plane is to its base as 5 is to 12; if the Weight is 65 pounds, find the Power.

3. Find the resistance of the Plane in the two preceding Examples.

4. Suppose the Power to act horizontally, and that the height of the Plane is to its base as 3 is to 8; if the Power is 12 pounds, find the Weight.

5. Suppose the Power to act horizontally, and that the height of the Plane is to its length as 7 is to 25: if the Power is 14 pounds, find the Weight.

6. Find the horizontal force required to support a Weight of 28 pounds on a smooth Plane which is inclined at half a right angle to the horizon.

7. Find the pressure on the Plane when a Weight of 12 pounds is supported by a horizontal Power of 9 pounds.

8. The circumference of the circle described by the end of the Power-arm of a screw is 24 inches; and by one turn of the screw the head advances half an inch; if the Power is 3 pounds, find the Weight.

9. In a Screw used to raise a load of 10 tons the Power is 50 pounds acting by an arm 4 feet long: find the distance between two consecutive threads.

10. Find the relation between the Power and the Weight in a Screw which has ten threads to an inch, and is moved by a Power acting at right angles to an arm at the distance of one foot from the centre.

XVII. COMPOUND MACHINES.

1. In the machine of Art. 262 if the advantages of the levers separately are expressed by 4, 5, and 6 respectively, and the Weight is 240 pounds, find the Power.

2. If the advantage of a Lever is expressed by the number 7, find the advantage of a combination of three such levers.

3. A machine is formed by the combination of three Wheels and Axles; the radius of each Wheel is four times that of the corresponding Axle: if the Power is 4 pounds, find the Weight.

4. In the Chinese Wheel the length of the Power-arm is 30 inches, the radius of the larger cylinder is $3\frac{1}{2}$ inches, and the radius of the smaller is 3 inches: if the Power is 4 pounds, find the Weight.

5. In Hunter's Screw the circumference of the circle which has the Power-arm for radius is 6 feet; the threads of one Screw are half an inch apart, and those of the other one third of an inch: shew that the advantage of the machine is 432.

XVIII. COLLISION.

1. A ball weighing 5 pounds, moving with a velocity of 12 feet per second *overtakes* another ball weighing 4 pounds, moving with a velocity of 3 feet per second: find the velocity after the collision, the balls being inelastic.

2. A ball weighing 6 pounds moving with a velocity of 8 feet per second *meets* another ball weighing 9 pounds and moving with a velocity of 2 feet per second: find the velocity after collision, the balls being inelastic.

3. A ball falls vertically and strikes a horizontal plane; if the index of elasticity is $\frac{1}{2}$, shew that the ball will rise to one fourth the height from which it fell.

4. A ball weighing one pound moving at the rate of 15 feet per second *overtakes* another ball which weighs two pounds and moves at the rate of 10 feet per second: find the velocity after collision, the balls being inelastic.

5. Equal spherical inelastic balls are placed at intervals in a smooth horizontal groove; the first ball is projected from the end along the groove with the velocity of 120 feet per second: find the velocity after successive impacts.

6. A ball strikes obliquely an equal ball which is at rest: shew that if the balls are perfectly elastic the directions of motion after collision will be at right angles.

7. A and B are two equal perfectly elastic balls which come into collision under the following circumstances; A is moving with a given velocity along the line of centres, and B is moving with an equal velocity at right angles to the line of centres: determine the motions after the collision.

8. A and B are two perfectly elastic balls; A moving at the rate of 10 feet per second impinges directly on B at rest: if the mass of A is nine times that of B, find the velocity at the end of the *first part* of the impact. Find also the velocities at the end of the *second part*.

9. If in Example 1 the bodies instead of being inelastic have $\frac{3}{4}$ for their index of elasticity, find the velocities at the end of the second part of the impact.

10. If in Example 2 the bodies instead of being inelastic have $\frac{1}{2}$ for their index of elasticity, find the velocities at the end of the second part of the impact.

XIX. MOTION DOWN AN INCLINED PLANE.

1. The height of an Inclined Plane is 5 feet, and the length is 16 feet: find the velocity gained in every second by a body sliding down the Plane.

2. The height of an Inclined Plane is 25 feet, and the length is 80 feet: find the time which a body takes to slide down the Plane.

3. Find the velocity which the body in the preceding Example has on reaching the end of the Inclined Plane.

4. Shew that the velocity just found is the same as a body would gain in falling freely through 25 feet.

5. The height of an Inclined Plane is one foot: shew, by various examples, that the time of sliding down the plane in seconds is one fourth of the number of feet in the length of the plane.

6. The base of an Inclined Plane is 24 feet and the height is 7 feet: find the velocity acquired in a second by a body sliding down the Plane.

7. The intensity of the attraction of Jupiter at its surface is about 2·6 times as great as that of the Earth at its surface: find approximately the time which a heavy body would take in falling through 260 feet at the surface of Jupiter.

8. The hanging weight in Art. 291 is 6 pounds, and that on the table is 4 pounds: find the velocity acquired in 10 seconds.

9. A weight of 3 ounces as in Art. 291 draws another weight from rest along the table over a distance of 2 feet 6 inches in 5 seconds: find the weight on the table.

10. Shew that if the weight on the table in Art. 291 is eleven times the hanging weight the motion is like that of a falling body, but on the scale of one inch to a foot.

XX. PROJECTILES.

1. A rifle bullet is shot vertically downwards from the top of a high tower at the rate of 40 feet per second: find how many feet it will fall through in two seconds, and the velocity at the end of four seconds.

2. From the top of a high tower two bodies are projected at the same instant, one vertically upwards with the velocity of 10 feet per second and the other vertically downwards with the velocity of 12 feet per second: find their distance apart at the end of 2, 3, 4, and 5 seconds respectively.

3. A projectile is discharged in an oblique direction upwards; if there were no gravity it would have reached a height of 1000 feet above the Earth's surface at the end of one second: state its actual height at that instant.

4. A rifle is pointed horizontally with its barrel 4 feet above a lake; when discharged the ball is found to strike the water 400 feet off: find approximately the velocity of the ball.

5. A body is projected so that the vertical part of the velocity of projection is 80 feet per second, and the horizontal part 60 feet per second: find the position of the body at the end of half a second, and at the end of a second.

6. In the preceding Example find the whole velocity at starting; find also the vertical velocity and the whole velocity at the end of one second.

7. In the same Example find the time the projectile takes to reach its highest point, and the height of that point.

8. In the same Example find the whole time before the projectile comes to the ground again; find also the vertical velocity and the horizontal velocity on reaching the ground.

9. Treat in the manner of the four preceding Examples the case in which at starting the vertical velocity is 64 feet per second, and the horizontal velocity 48 feet per second.

10. A body is thrown in a direction inclined to the horizon at an angle of 45 degrees, and strikes the hori-

zontal plane passing through the point of projection after 5 seconds: find the velocity of projection.

11. A body has given to it simultaneously an upward vertical velocity of 48 feet per second, and a horizontal velocity of 25 feet per second: find the greatest height, the whole time of flight, and the range.

12. In Art. 291 if while the system is in motion the string be cut, state what will then be the motions of the two bodies.

13. If the Moon's motion round the Earth were stopped, so that it began to fall towards the Earth, shew that in the first minute it would fall through about 16 feet.

14. From a point 25 feet above the ground a body is discharged horizontally with a velocity of 30 miles per hour: find where the body strikes the ground and with what velocity.

15. At what point of its path is the velocity of a projectile least?

XXI. MOTION IN A CIRCLE.

1. A ball weighing 2 pounds describes a circle of radius 4 feet with a uniform velocity of 16 feet per second: find the force.

2. A ball describes a circle of radius 8 feet with uniform velocity: find the velocity if the force is four times the weight.

3. A body weighing 112 pounds describes in 8 seconds the circumference of a circle of which the radius is 14 feet: find the force.

4. A body whose weight is 5 pounds moves with a uniform velocity of 80 feet per second in a circle whose radius is 10 feet: find the force.

5. A locomotive engine of 10 tons weight passes on level ground round a curve 800 yards in radius at a rate of 30 miles in an hour: find the horizontal pressure on the rails.

6. Shew with the figures given in Art. 311 that the velocity of the Moon round the Earth is about 3400 feet per second; hence shew that the force which would keep a

body revolving round the Earth like the Moon does, at her distance, is about $\frac{1}{3600}$ of the weight of the body.

7. Shew that a body removed from the surface of the Earth to the distance of the Moon would weigh about $\frac{1}{3600}$ as much as at the Earth's surface.

8. A body describes uniformly a circle of radius 21160000 feet; if the force be equal to the weight, shew that the velocity is about 26000 feet per second.

9. Shew in the preceding Example that the circle is described in about 5100 seconds.

10. If a small satellite revolved round the Earth close to its surface, find the time of revolution.

XXII. PENDULUM.

1. Find the length of a simple pendulum which would oscillate in half of a second.

2. Find the time of oscillation of a simple pendulum 50 feet long.

3. A simple pendulum is 13 feet long; it is drawn aside until the heavy particle is distant 5 feet from the vertical straight line which passes through the fixed point, and then allowed to move: find the velocity of the particle at its lowest point.

4. Shew that the length of the seconds pendulum increases slightly as we pass from the equator to the pole.

5. Shew that the length of the seconds pendulum decreases as the fixed point is taken more distant from the surface of the Earth in a given latitude.

6. Shew that if the fixed point be taken at two miles from the surface of the Earth, the length of the seconds pendulum must be diminished about one-thousandth part.

7. A certain pendulum is made first to describe excessively small arcs, and next to describe arcs of 2 degrees each: shew that 50001 oscillations in the first case are made in about the same time as 50000 oscillations in the second case.

8. A certain pendulum is made first to describe excessively small arcs, and next to describe arcs of 20

degrees each: shew that 501 oscillations in the first case are made in about the same time as 500 oscillations in the second case.

9. A pendulum oscillates in a second; if the length is increased by an inch, shew that the time of oscillation is increased by about one-eightieth of a second.

10. A pendulum is $4\frac{1}{2}$ feet long; if it is shortened 2 feet, shew that the time of an oscillation becomes three-fourths of what it was originally.

XXIII. FRICTION.

1. An Inclined Plane is 13 feet long, and the base is 12 feet; a body weighing 39 pounds is placed on the plane and just remains without sliding: find the coefficient of friction and the resistance of the Plane.

2. If the *angle* of friction is 45 degrees, determine the *coefficient* of friction.

3. A body weighing 20 pounds is placed on a horizontal table, and it is found that it can be just drawn along the table by a force of 5 pounds: determine the coefficient of friction. If a force of 12 pounds act vertically *upwards* on the heavy body, find what horizontal force will be sufficient to move the body along the plane.

4. The length of an Inclined Plane is 25 feet and its height is 7 feet; a weight of 50 pounds is placed on the Plane: find what force must act along the Plane to keep the weight at rest, when the coefficient of friction is $\frac{1}{4}$.

5. A beam rests on the rough ground at one end, and on a roller at the other end: draw the directions of all the forces which act on the beam and the roller in two distinct diagrams.

6. A body placed on a rough horizontal plane is found to be capable of being drawn along the plane by a force of 27 pounds applied horizontally; it is known that the coefficient of friction is ·2: find the weight of the body.

7. A solid cube of wood rests by one of its faces on an Inclined Plane in such a manner that the upper and lower edges of its base are horizontal; the inclination of the Plane to the horizon is gradually increased, until it is found

that the cube begins to slide just as it topples over: find the coefficient of friction.

8. The base of a rough Inclined Plane is four-fifths of its length; a body is placed on the Plane and it is found that a force acting parallel to the Plane and equal to three-fourths of the weight of the body will just make it move up the Plane: find the coefficient of friction.

9. A body slides along a horizontal plane; it is known that 12 seconds before it came to rest it was moving at the rate of $7\frac{1}{5}$ feet per second: determine the distance it moved through in the 12 seconds, and the coefficient of friction.

10. The base of an Inclined Plane is 24 feet, and its height is 7 feet: find the velocity acquired in one second by a body sliding down the Plane if the coefficient of friction is $\frac{1}{4}$. Find also the time taken in sliding down the Plane.

XXIV. GENERAL MOTION.

1. A slender rod is 12 feet long: find at what point of the rod it might be suspended so as to oscillate in the same time as if suspended from one end.

2. Find the length of a slender rod so that if suspended from one end the time of oscillation may be half a second.

3. A long slender rod hung up by one end oscillates in 2 seconds: find the length of the rod.

4. A long rope hung up by one end oscillates in 3 seconds: find the length of the rope.

5. A slender rod oscillates about one end: find at what point a small additional weight might be put without altering the time of oscillation.

6. Determine what the effect will be on the time of oscillation of the rod if a small additional weight is placed above or below the point found in the preceding Example.

7. A pendulum consists of two equal heavy particles attached to a rod without weight at the distances of 2 and 3 feet respectively from the axis of rotation on the same side: find the length of the equivalent simple pendulum.

8. A pendulum consists of three equal heavy particles attached to a rod without weight at the distances of 2, 3, and 4 feet respectively from the axis of rotation on the same side: find the length of the equivalent simple pendulum.

9. AB is a rod without weight 12 feet long; a small body weighing one ounce is fixed to the rod 1 foot from A, and a small body weighing three ounces is fixed to the rod 9 feet from A; the system can turn about a point 3 feet from A: find the length of the equivalent simple pendulum.

10. A slender rod 12 feet long is suspended from a point 3 feet from one end: find the length of the equivalent simple pendulum.

11. Find the tension of a rope which draws a carriage of 8 tons weight up a smooth incline of 1 in 16, and causes an increase of velocity of 3 feet per second.

12. If the rope in the preceding Example break when the carriage has a velocity of 48 feet per second, find how long and how far the carriage will move up the incline.

13. A balloon is ascending vertically with a velocity which is increasing at the rate of 4 feet in a second: find the apparent weight of one pound weighed in the balloon by means of a spring balance.

14. A bullet striking a piece of timber with a velocity of 400 feet per second penetrates to the depth of one foot: find the resistance of the timber, supposed uniform, and the time the bullet moved in the timber.

15. A railway carriage detached from a train going at the rate of 30 miles an hour is stopped by the friction of the rails in half a minute: find the coefficient of friction.

XXV. FLUIDS.

1. Shew that at the depth of 33 feet below the surface the bulk of a given mass of water is reduced about $\frac{1}{20000}$ part of its bulk at the surface.

2. Shew that at the depth of 2800 fathoms the bulk is reduced about $\frac{1}{40}$ part.

3. It is found by experiment that under the pressure of a column of water 33 feet high, the bulk of a given mass of mercury is reduced about $\frac{3}{1000000}$ part: find at what depth below the surface of water mercury would be reduced in bulk $\frac{1}{1000}$ part.

4. It is found by experiment that under the pressure of a column of water 33 feet high the bulk of a given mass of alcohol is reduced about $\frac{1}{10000}$ part: find the reduction of bulk under the pressure of a column of water one mile high.

5. For chloroform the reduction of bulk is $\frac{1}{16000}$ part under the pressure of a column of water 33 feet high: find the reduction of bulk under the pressure of a column of water one mile high.

XXVI. PRESSURE TRANSMITTED IN ALL DIRECTIONS.

1. Suppose in Art. 351 that the pistons at E and F are circular, the former one inch in diameter, and the latter two inches in diameter: if the former is pushed down by a force of 7 pounds, find the force by which the latter must be pushed down for equilibrium.

2. Suppose in Art. 351 that the pistons at E and F are circular, and that the diameter of the latter is four times that of the former: shew that for every ounce by which the former is thrust down the latter must be thrust down by a pound for equilibrium.

3. Let the piston at E be a square of which the side is 8 inches long, and that at F a square of which the side is 1 inch long: if the former is thrust down by a weight of one ton, find the weight by which the latter must be thrust down for equilibrium.

4. In the *hydrostatic paradox* find the force by which the piston must be pushed down in order to support one pound on the board, the dimensions being as in Art. 356.

5. Find also the force by which the piston must be pushed down to support five hundred weight on the board.

XXVII. PRESSURE FROM THE WEIGHT OF LIQUIDS.

1. A vessel has a square base, each side of which is 18 inches long; the depth is 28 inches: if the vessel is filled with water find the whole pressure on the base.

2. The height of a cylindrical vessel is 7 feet; the vessel is filled with water: find the pressure on each square inch of the base.

3. Find the pressure on a square inch at the bottom of a vessel 14 feet deep filled with water.

4. A piston, the area of which is 6 square inches, is inserted in the side of a vessel containing water at the average depth of 3 feet 6 inches below the surface: find the force which must be exerted on the piston to keep it from being thrust out by the water.

5. Suppose in Art. 354 that the pistons at E and G are each of 4 square inches in area, and that the depth of G below DC is 21 inches: if the piston at E is pushed down by a force of 5 pounds, find the force which must be exerted on the piston at G to keep it in its place.

XXVIII. VESSELS OF ANY FORM.

1. The depth of water in a vessel is 14 feet, the base of the vessel is a square with a side $1\frac{1}{2}$ feet long: find the pressure on the base of the vessel.

2. A vessel in the form of a pyramid 7 feet high, with a base 4 feet square, is filled with water: find the pressure on the base.

3. A cube whose edge is 21 inches long is placed in water, the top face being exactly in the surface: find the pressure on its lowest face.

4. A vessel has two opposite sides vertical and two opposite sides inclined, as in Art. 367, the base being a rectangle; water is put into it: if CD is three times AB, shew that the pressure on the base is equal to half the weight of the water contained in the vessel.

5. Again, supposing that AB is three times CD, shew that the pressure on the base is equal to 1·5 times the weight of the water contained in the vessel.

6. In Art. 369 suppose that the area of the base is a square foot, and that the height of the shallow part is $3\frac{1}{2}$ inches: find the pressure on the base when the shallow part is just full of water.

7. Suppose that water is now poured in till it occupies the height of 14 inches in the neck, find the *additional* pressure on the base.

8. A vessel is in the shape of a cylinder; the radius of the base is 10 inches, and the height 3 feet; a slender vertical tube reaches to the height of 4 more feet: suppose the cylinder and the tube filled with water, find the pressure on the base.

9. A vessel in the form of a hollow cone or pyramid with its vertex upwards is full of water: shew that the pressure on the base is three times the weight of the water in the vessel.

10. A vessel in the form of a hollow cone or pyramid with its vertex upwards is full of water: shew that if one-eighth of the water is removed, the pressure on the base becomes half what it was before.

XXIX. PRESSURES ON THE SIDES OF VESSELS.

1. A vessel in the shape of a cube without a lid is filled with water: shew that the resultant pressure on a side is half the pressure on the base.

2. Find the pressure on a vertical rectangle 10 inches long and 6 inches broad, immersed in water, with its larger sides horizontal, and with the upper one 2 inches below the surface.

3. A cubical tank can just hold ten tons of water: when the tank is filled with water find the resultant pressure on one of the sides, and its point of application.

4. One end of a vessel full of water is a rectangle 6 feet high and 4 feet broad: find the pressure on this end. Find, also, the pressure on the upper half, and on the lower half of this end.

5. In the preceding Example determine the position of the centre of pressure for the whole end, and for the upper half of it by Art. 378; and then deduce the position of the centre of pressure for the lower half.

6. The diameter of a hollow cylinder is 10 inches; the cylinder is filled with water, and placed with its axis horizontal: find the pressure on an end.

7. A reservoir has one of its walls vertical; a circle a yard in diameter is described on that wall: when the water just covers the circle find the pressure on the portion of the wall within the circle.

8. Consider in a mass of water a cube such that its upper face is horizontal, and 32 feet below the surface of the mass; determine the pressure on the six faces of the cube, supposing an edge to be a foot long.

9. A square, $ABCD$, is placed in a liquid with its plane vertical, and the side AB horizontal and fixed; when the square is below AB the fluid pressure on it is twice what it would be if the square were turned round so as to be above AB: find the depth of AB below the surface.

10. The side of a vessel is a rectangle; a diagonal is drawn dividing the rectangle into two triangles: shew that if the vessel is full of liquid the pressure on one triangle is twice the pressure on the other.

XXX. LIQUIDS STAND AT A LEVEL.

1. State some of the practical applications of the principle that liquids stand at a level.

2. In a town it is found that water rises to the height of 50 feet above the level of the pipes which pass down the street: find the pressure caused by the water on a square inch of the surface of the pipes in the street.

3. Shew that, in order to supply water in the usual way to the tops of high houses, the pipes in the street must be very strong.

4. A lock-gate is 8 feet broad; on one side the water is 12 feet deep, and on the other side 4 feet deep: find the pressure on each side.

5. Shew what form the ocean appears to take when viewed from a lofty point, as the top of the Peak of Teneriffe.

XXXI. SOLIDS IMMERSED IN LIQUIDS. VOLUMES.

1. A vessel containing water has four vertical sides, and its base is a rectangle 8 inches long and 4 inches broad; a stone is put in and sinks to the bottom, and the surface of the water rises half an inch: find the volume of the stone.

2. The base of a cylindrical vessel is a circle, which is 20 inches in diameter; a solid body is put inside and covered with water, and when the body is removed the surface of the water sinks an inch: find the volume of the body.

3. Find how much the surface of the water in Example 1 would rise if a heavy body 24 cubic inches in volume were put in.

4. Find how much the surface of the water in Example 2 would rise if a circular slab of marble of 10 inches diameter and 2 inches thickness were put in.

5. Find how much the surface of the water in Example 2 would rise if a heavy sphere of radius 6 inches were put in.

XXXII. SOLIDS IMMERSED IN LIQUIDS. WEIGHTS.

1. A heavy body supported by a string is immersed in a vessel full of water, and half a pint of water runs over: find the apparent diminution of weight.

2. In the Example XXXI. 1 find the diminution of weight if the stone be attached to a string and just immersed in water.

3. In the Example XXXI. 2 find the diminution of weight if the solid body be attached to a string and just immersed in water.

4. A cork is placed on the water in a full vessel, and twelve cubic inches of the water run over: find the weight of the cork.

5. A person plunges his hand into a vessel partly full of water: determine the change made in the vertical part of the pressure of the water on the vessel.

6. Explain the fact that in drawing water from a well the full tension of the rope is not felt until the bucket is quite out of the water.

7. Find what weight must be placed within a hollow sphere weighing 500 pounds to enable it just to sink in water, when its radius is fifteen inches.

8. The diameter of a sphere is 2 inches, and its weight is 10 ounces: find its apparent weight when wholly immersed in water.

9. A piece of wood weighing 120 pounds floats on water with four-fifths of its volume immersed: find its whole volume.

10. A rectangular box open at the top is made of sheet iron half an inch thick; a cubic foot of the iron weighs 490 pounds. The internal dimensions of the box are 36 inches in length, 24 inches in width, and 12 inches in depth. Find what weight placed in the box will just sink it in water.

XXXIII. APPLICATIONS.

1. Find the apparent weight of a cubic inch of gold if immersed in mercury.

2. Find the apparent weight of a cubic inch of ivory if immersed in water.

3. A ship with its cargo weighs 2000 tons: find how many cubic feet of fresh water it would displace.

4. The area of the section of a ship made by the plane of floatation is 12,000 square feet: find what addition to her cargo will make the ship sink three inches deeper in the water.

5. Suppose that at the level of the plane of floatation the average length of a ship is 200 feet, and the average breadth 40 feet: find the depth through which the ship will descend if 200 tons more weight is put on it.

6. If the ship in Example 3 floats on *salt* water, find how many cubic feet it displaces.

7. Suppose a ship with its cargo to weigh 2000 tons, and that the dimensions at the level of the plane of floatation are as in Example 5: find how much the ship rises in passing from fresh water to salt water.

8. A ship passes from the sea to a river; when 20 tons of cargo have been removed it is found that the ship sinks in the river to the same mark as in the sea: find the weight of the ship and cargo.

9. A ship passes from the sea to a river, and sinks 12 inches deeper; when 20 tons of cargo have been removed the ship rises 8 inches: find the weight of the ship and cargo.

10. It is found that a piece of very smooth wood will continue at the bottom of a vessel of mercury without any tendency to rise: explain this.

XXXIV. DIFFERENT LIQUIDS.

1. The sides of a vessel are vertical, and the area of the base is a square foot; the vessel contains sea-water to the height of 2 feet, and then fresh water to the height of 2 more feet: find the pressure on the base of the vessel.

2. In Art. 420 suppose there to be water from AB to GH, and oil from GH to CD: if EA be a yard, find GC.

3. In Art. 420 suppose there to be mercury from AB to GH, and water from GH to CD: if EA be a foot, find GC.

4. In Art. 420 suppose there to be mercury from AB to GH, and oil from GH to CD: if EA be 18 inches, find GC.

5. If a column of liquid consist of a certain height of water and an equal height of oil, shew that the pressure at the bottom of the column is the same as if the whole column consisted of liquid, the specific gravity of which is $\frac{19}{20}$ of that of water.

XXXV. EQUILIBRIUM OF FLOATING BODIES.

1. A hollow sphere weighs half as much as a sphere of water of the same radius: if the sphere float on water determine its position of equilibrium.

2. A cylinder floats on water with its axis horizontal: shew that if the cylinder be tilted parallel to its ends the *metacentre* is on the axis, and the equilibrium stable.

3. A cylinder 12 inches long floats vertically on water with 3 inches out of the water: find how many inches will be out of the water if the cylinder floats vertically on oil.

4. A cube floats on water: shew that there are other positions of equilibrium besides that in which a face is horizontal.

5. A cone floats on a liquid with its axis vertical and vertex downwards: if half the axis is immersed, shew that the specific gravity of the liquid is eight times that of the cone.

XXXVI. SPECIFIC GRAVITY. SOLIDS.

1. A piece of silver weighs 159 grains, and in water it weighs 144 grains: find the specific gravity.

2. A piece of flint weighs 6 ounces, and in water it weighs 4 ounces: find the specific gravity.

3. A piece of copper weighs 1100 grains, and in water it weighs 975 grains: find the specific gravity.

4. A piece of elm weighs 3 ounces; a weight of 2 ounces, applied as in Art. 442, will just keep the elm totally immersed: find the specific gravity.

5. A piece of cork weighs 7 ounces; a weight of 28 ounces, applied as in Art. 442, will just keep the cork totally immersed: find the specific gravity.

6. A cylindrical cork floats vertically with 1 inch above the water, and $\frac{3}{10}$ of an inch below: find the specific gravity.

7. The volume of a body is one cubic foot, and its specific gravity is ·9: find how many cubic inches the body will displace when floating on water.

8. The specific gravity of a piece of cork is ·26, and its weight is an ounce; the cork is fastened by a thread to the bottom of a vessel so as to keep it totally immersed: find the tension of the thread.

9. A body weighing 12 pounds, and having a specific gravity ·75, is fastened by a thread to the bottom of a vessel; water is poured in so that the body is completely covered: find the tension of the thread.

10. The specific gravity of a body is 17: find the volume of 89 ounces of it.

11. Find the volume of a ton of gold of the specific gravity 19·36.

12. A stone of specific gravity 2·5, which weighs 5 cwt., is sunk in water: find the force which will be required just to lift it from the bottom.

13. A body whose specific gravity is 7 weighs 18 ounces in water: find the weight of the body.

14. A body whose specific gravity is 3·5 weighs 4 pounds in water: find the weight of the body.

15. A piece of metal whose specific gravity is 9·8 weighs in water 55 grains: find the weight of the body.

16. A lump of bees-wax weighing 2895 grains is stuck on to a crystal of quartz weighing 795 grains, and the whole when suspended in water is found to weigh 390 grains: find the specific gravity of bees-wax, that of quartz being 2·65.

17. A piece of cork weighing one ounce is fastened to a sinker weighing $3\frac{1}{2}$ ounces; it is found that they will just sink when placed in water: find the specific gravity of the sinker, that of the cork being ·25.

18. Standard gold consists of eleven parts by weight of gold mixed with one part of copper; taking the specific gravity of pure gold as 19·26 and that of copper as 8·85; shew that the weight lost in water by 12000 grains of standard gold is theoretically 684 grains, and therefore the specific gravity of standard gold 17·54.

19. A new sovereign weighs 123·374 grains, and loses in water 7·18 grains: find the specific gravity.

20. The specific gravity of a certain compound of silver and copper is 9·2: find the volumes of silver and copper in 48 cubic inches of it, taking the specific gravities of silver and copper as in Art. 403.

21. The specific gravity of a certain compound of gold and silver is 17: find the volumes of gold and silver in 89 cubic inches of it, taking the specific gravities of gold and silver as in Art. 403.

22. Find the weights of gold and silver in the compound of Example 21.

23. A nugget of gold mixed with quartz weighs 10 oz.; the specific gravity of gold is 19·35, of quartz 2·65, and

of the nugget 6·45: find the volumes and the weights of the gold and the quartz contained in the nugget.

24. A vessel contains mercury of specific gravity 13·6, on which floats a cube of iron of specific gravity 7·2; water is poured into the vessel until the cube is completely immersed: find what portion of the cube is below the surface of the mercury.

25. An accurate balance is totally immersed in a vessel of water; in one scale pan some glass of specific gravity 2·5 is being weighed, and exactly balances a one pound weight of specific gravity 8 which is placed in the other scale: find the real weight of the glass.

XXXVII. SPECIFIC GRAVITY. LIQUIDS.

1. A cup when empty weighs 6 ounces; when full of water it weighs 16 ounces; when full of petroleum it weighs $14\frac{3}{4}$ ounces: find the specific gravity of petroleum.

2. A bottle holds 1500 grains of water; when filled with alcohol it weighs 1708 grains, and when empty 520 grains: find the specific gravity of alcohol.

3. A glass flask being filled with mercury weighed 1346 grains more than when empty; when filled with water it weighed 99 grains more than when empty: find the specific gravity of mercury.

4. A body weighs 80 grains; in water it weighs 56 grains, and in another liquid 46 grains: find the specific gravity of the liquid.

5. A body weighs 2300 grains; in water it weighs 1100 grains, and in spirit 1300 grains: find the specific gravity of the spirit.

6. A body weighs 100 grains; in water it weighs 85 grains, and in another liquid 88 grains: find the specific gravity of the liquid.

7. A piece of metal weighs 211·6 grains; in water it weighs 187·32 grains, and in another liquid 182·37 grains: find the specific gravity of the liquid.

8. A glass ball immersed in water loses 803 grains, and in milk loses 811 grains: find the weight of a cubic foot of milk.

9. A body weighs 8 ounces; in olive oil it weighs 6·17 ounces, and in sea-water 5·948 ounces: compare the specific gravities of olive oil and sea-water.

10. A body weighs 3 ounces; in oil of turpentine it weighs 1·86 ounces: find the specific gravity of the body, that of oil of turpentine being ·88.

11. A body weighs 346 grains; in alcohol it weighs 210 grains: find the specific gravity of the body, that of alcohol being ·85.

12. Find the weight of a gallon of olive oil.

13. A glass ball weighs 3000 grains, and has a specific gravity 3: find its apparent weight when immersed in a liquid of specific gravity ·92.

14. Given that a pint of water weighs 20 ounces, and that the specific gravity of proof spirit is ·915, find what fraction of a quart of proof spirit will weigh 30 ounces.

15. A solid body of specific gravity ·8 floats on a liquid of specific gravity ·9: find how much of the body will be immersed.

16. If half a pint of water is added to a pint of liquid of the specific gravity ·52, shew that the specific gravity of the mixture is ·68.

17. A pint of water is mixed with three pints of a liquid of the specific gravity ·8: find the specific gravity of the mixture.

18. Equal volumes of alcohol and water are mixed, the specific gravity of the alcohol being ·8; the volume of the mixture after it has returned to the original temperature is found to fall short of the sum of the volumes of the constituents by 4 per cent.: find the specific gravity of the mixture.

19. A mixture is made of 7 cubic inches of Sulphuric Acid of specific gravity 1·843, and 3 cubic inches of water; the specific gravity when cold is found to be 1·615: determine the contraction which has taken place.

20. A Nicholson's Hydrometer weighing 250 grains requires 50 grains to sink it to a given depth in naphtha, and 150 grains to sink it to the same depth in water: find the specific gravity of naphtha.

XXXVIII. SPECIFIC GRAVITY. GASES.

1. A room is 25 feet long, 12 feet wide, and 8 feet high: find the weight of the air which it contains.

2. Find the weight of 100 cubic inches of oxygen.

3. Find the weight of 100 cubic inches of hydrogen.

4. Find the specific gravity of a gas composed of one cubic foot of oxygen with four cubic feet of nitrogen.

5. Suppose the volume of a balloon to be 5000 cubic feet, find the weight of the air which would fill it: find also the weight of the hydrogen which would fill it.

XXXIX. EFFLUX OF LIQUIDS.

1. A hole is made 4 feet below the surface of the water in a large vessel: find how long it takes for a cubic foot of water to pass through the hole, supposing the section of the *contracted vein* to be one square inch.

2. A vessel 10 feet high is kept full of fluid: find the velocity with which fluid issues from a hole which is one foot *above* the middle point.

3. Find the time which the fluid in the preceding Example takes to pass from the hole to the ground on which the vessel stands; and hence determine the distance from the vessel of the point where the fluid reaches the ground.

4. Determine the distance from the vessel of the point where the fluid reaches the ground when the orifice is one foot *below* the middle point.

5. Determine the distance from the vessel of the point where the fluid reaches the ground when the orifice is *two* feet above or *two* feet below the middle point.

6. Determine also the distance when the orifice is *three* feet above or *three* feet below the middle point.

7. Also vary the Examples 3...6 by supposing the height of the vessel to be 12 feet.

8. Shew from the preceding and similar Examples that the point on the ground which the fluid reaches is the same for two holes equally distant above and below the middle point.

9. Shew in like manner that the distance reached is equal to twice the square root of the product of the distances of the hole from the top and bottom of the vessel.

10. Shew in like manner that the greatest distance reached is when the hole is at the middle of the vessel.

XL. RESISTANCE OF LIQUIDS.

1. Calculate the amount of resistance on a board a foot square, which is moved through water in a direction at right angles to its surface with a velocity of $21\frac{9}{11}$ miles an hour.

2. It is found that a board a foot square can be kept moving uniformly through water in a direction at right angles to the surface by a force of 250 pounds: determine the velocity.

3. Shew that the resistance on a board 15 inches square, moving with a velocity of 11 feet per second, is the same as that on a board 11 inches square moving with a velocity of 15 feet per second.

4. Shew by trial that the result of Example 3 is true also when any other pair of numbers is used instead of 11 and 15.

5. A board a foot square is made to move uniformly through water with a certain velocity by a force of 200 pounds: find the force which will make a board a yard square move uniformly with double the velocity.

XLI. GASEOUS BODIES.

1. Explain why the terms solid, liquid, and gaseous apply rather to difference of state than to difference of substance.

2. Explain what is meant by latent heat.

3. A pound of ice at the temperature 32 degrees is mixed with a pound of water at the temperature 212 degrees: shew that the temperature of the mixture when the ice is melted is about 51 degrees.

4. A pound of ice at the temperature 32 degrees is mixed with thirteen pounds of water at the temperature 212 degrees: find the temperature of the mixture when the ice is melted.

5. Alcohol, which passes readily into the gaseous state, when dropped on the hand produces a sensation of *cold;* explain this.

XLII. AIR A SUBSTANCE.

1. Find the weight of the air in a room which is 30 feet long, 18 feet broad, and 12 feet high.
2. Shew that the air which would fill a tube one square inch in section, and five miles long, would weigh about fifteen pounds.
3. Explain why smoke *ascends* in the air.
4. Cork is weighed in a balance, the weights being made of brass: shew that the true weight of the cork is somewhat greater than the apparent weight.
5. A piece of gold is in equilibrium with a piece of cork in the scales of a very delicate balance; afterwards the cork is removed, and tin is put in to keep the gold in equilibrium: if the gold is now removed, and the cork put in its place, determine if there will be equilibrium.

XLIII. PRESSURE OF THE ATMOSPHERE.

1. If instead of mercury in Art. 489 water is used, find how high the tube must be. See Art. 504.
2. If a board one foot square be placed horizontally in a mass of water 34 feet below the surface, find the whole pressure on one side of the board, taking into account the pressure of the atmosphere.
3. In Art. 489 suppose that the tube is cylindrical and two inches in diameter, and the end A flat: find the pressure of the atmosphere on this end.
4. If in Art. 489 the height of G above EF increases by half an inch, find the increase of the pressure of the atmosphere on each square inch of area, taking the specific gravity of mercury as 13·6.
5. If the process of Art. 489 were performed at the bottom of a deep mine, find what effect would be produced.

XLIV. PRESSURE AND VOLUME.

1. A vessel contains a quantity of air which weighs 8 grains, and exerts a pressure of $16\frac{1}{2}$ pounds on each square inch: if 3 more grains of air are introduced into the vessel, find what pressure the air will now exert.

2. Air under the ordinary pressure of the atmosphere occupies a cube of which the edge is 12 inches: find the additional pressure which must be applied to compress the air into a cube of which the edge is 6 inches.

3. A vertical tube is closed at the bottom, and has a moveable piston at the other end; if the area of the piston is a square inch, and its weight five pounds, shew that the density of the air in the tube must be $\frac{4}{3}$ of that of the ordinary atmosphere when there is equilibrium.

4. If the piston in the preceding Example be loaded with a weight of 10 pounds, find the density of the air in the tube when there is equilibrium.

5. Suppose that the equilibrium in Example 3 subsists at the temperature of 50 degrees of the common thermometer: shew that if the temperature be raised to 150 degrees, and the piston loaded with 4 pounds, the equilibrium will subsist with the same density of the air.

6. A certain quantity of air forms a small spherical bubble of a given radius when 5 feet below the surface of water: find at what depth the same quantity of air would form a bubble of half the given radius.

7. A tube of uniform bore is bent as in Art. 494; when the height of the barometer is $28\frac{1}{2}$ inches, the mercury stands at the same level in the two branches, and the enclosed air occupies 8 inches; the barometer rises, and then the difference of level of the mercury in the two branches is half an inch: find the height of the barometer.

8. A certain volume of air weighs 327 grains when the temperature is at 32 degrees of the common thermometer, and the pressure 30 inches of mercury: shew that at the temperature of 77 degrees, and under the pressure of 32 inches of mercury, an equal volume of air would weigh about 320 grains.

9. A tumbler full of air is placed mouth downwards under water at such a depth that the surface of the water inside it is at a depth of $25\frac{1}{2}$ feet: compare the weight of

a cubic inch of the air in the tumbler with that of a cubic inch of the air outside, the barometer standing at 30 inches, and the specific gravity of mercury being 13·6.

10. A barometer tube has an internal section of $\frac{1}{5}$ of a square inch; there are 30 inches of mercury standing in it, and 6 inches of vacuum above; a bubble of air containing $\frac{4}{27}$ of a cubic inch of external air is introduced: shew that there will be equilibrium when the mercury falls 4 inches.

XLV. BAROMETER.

1. A faulty barometer has some air instead of a vacuum above the mercury, in consequence of which, in a certain state of the atmosphere, the height of the mercury is 29 inches, while that of a correct barometer is 30 inches: find the density of the air above the mercury.

2. The length of the tube of a barometer is 32 inches; some air has got into the space above the mercury, and in consequence the mercury stands at 29 inches when it would stand at 30 inches if the instrument was uninjured: find the true height of the barometer when the injured instrument reads 28·4 inches.

3. If the tube and the vessel in Art. 489 are cylinders, and the diameter of the vessel eleven times that of the tube, find how the graduation should be made in the manner of Art. 502.

4. Let a bottle of air be closely stopped on the summit of a mountain, and be brought in this state to the plain below; then let the mouth be inserted in a vessel of water and the stopper withdrawn, and it will be found that a certain portion of water will enter the bottle: explain this statement.

5. Find by the rule of Art. 507 how high a balloon must ascend to leave one quarter of the mass of the air below the level of the balloon.

6. The heights of the barometer at a lower and a higher station are 29·93 and 24·78 inches respectively; the temperatures at the two stations are 73 and 48 degrees respectively of the common thermometer: find the height of the higher station above the lower in feet.

7. If a layer of water one inch deep be placed on the upper surface of the mercury in the cistern of the barometer, find how the difference of level between the surfaces of mercury in the tube and cistern will be affected. Find also what would be the effect of causing a piece of iron to float on the surface of the mercury in the cistern.

8. Assuming the height of the homogeneous atmosphere to be 26000 feet when $g=32$, find the height when $g=32{\cdot}2$: see Art. 288.

9. Find the height of a barometer of alcohol corresponding to the height 30 inches of the mercury barometer.

10. A cylinder 3 feet long floats on water with its axis vertical and two-thirds immersed when the height of the barometer is 30 inches: shew that when the barometer falls to 28 inches, the cylinder will sink about one-thousandth of an inch in the water, supposing water 800 times as heavy, bulk for bulk, as the air, when the barometer is at 30 inches.

XLVI. BAROMETER FOR COMMON USE.

1. If in Art. 512 the radius of the large wheel is twelve times that of C, and the circumference of the large wheel is 24 inches, shew that when the pressure of the atmosphere changes by an inch of mercury, the end of the pointer on the large circle must pass over 6 inches.

2. Explain why a pair of bellows will not work if the board with the valve is placed *above*, and the other board *below*.

3. Find the amount of the pressure on every square inch of the surface of a fish 102 feet below the surface of the water.

4. The mercury standing at a certain point G in the experiment of Art. 489, state what would take place if the pressure of the atmosphere were suddenly destroyed and then restored.

5. A barometer is lowered into a vessel of water so that the surface of the water is finally six inches above the cistern of the barometer: find what kind of change will take place in the reading of the column of mercury of the instrument.

6. Find what change in the pressure of the atmosphere on a square inch is indicated by a fall of one inch in the height of the barometer, taking the weight of a cubic inch of water as 253 grains, and the specific gravity of mercury as 13·6.

7. Taking 34 feet as the height of a water-barometer, find at what depth in a lake the pressure will be three times what it is at the depth of two feet.

8. If the tube of a barometer is inclined to the vertical, does this affect the height of the mercury?

9. If a bladder containing 300 cubic inches of air under a pressure equal to that of 30 inches of mercury be sunk to 170 feet below the surface of water, find the volume to which the air in the bladder will be compressed, the height of the barometer at the time being 28·5 inches, and the specific gravity of mercury 13·6.

10. In the experiment of Art. 519, if the mercury stands at C at the height of 20 inches above the level in the vessel, shew that the space denoted by BC is just twice the volume of all the pores.

XLVII. AIR PUMP.

1. Shew in the Example of Art. 523, that after *seven* operations the density of the air in the receiver will be less than half that of the atmosphere.

2. Suppose that the volume of the receiver and the pipe together is four times that of the cylinder; then find by trial the number of operations necessary to reduce the mass of the air in the receiver to one-third of the original mass.

3. In the Example of Art. 523, suppose that the area of a section of the piston is 5 square inches; then find the force which must be applied to move the piston upwards when it is almost at the end of its second ascent.

4. Shew that the labour of working the air pump, as described in Art. 521, increases as the number of strokes increases.

5. If we wish the air in the receiver to have a density not greater than $\frac{1}{100}$ of the original density, find what must be the indication of the siphon gauge in Art. 526 at which we must arrive.

XLVIII. AIR-PUMP EXPERIMENTS.

1. If instead of air being taken from the receiver, as in Art. 528, air be forced into the receiver, find how the guinea and feather experiment will be affected.

2. If in Art. 530 air be forced into the receiver instead of taken from it, find how the bladder will be affected.

3. If the barometer falls two inches in ascending to an elevated station, find the temperature at which water will boil at the station.

4. Suppose that in a tube like the barrel of an air gun a pressure double that of the atmosphere could be maintained behind a ball: find the velocity with which a ball would issue from the tube, if its weight is one third of an ounce, the tube a square inch in section, five feet long, and the ball just fitting the tube.

5. If the pipe and the receiver are ten times as great as the cylinder in Art. 534, find how many operations are necessary to make the density of the air three times as great as at first.

XLIX. PUMPS.

1. Find the greatest length admissible for CE in the diagram of Art. 537 at a place where the height of the barometer is 25 inches.

2. Find the greatest height the pipe of a common pump can have to raise a liquid of specific gravity 1·7, when the barometer stands at 29 inches, the specific gravity of mercury being 13·6.

3. If the pumps of Arts. 537 and 540 were employed to raise salt water instead of fresh water, shew that the extreme admissible length of CE must be somewhat diminished.

4. State the result which would follow if a hole were made in the pipe CE; also, if a hole were made in the cylinder AB.

5. The diameter of the piston of common pump is 2 inches, and the height of the top of the water in the pump above the well is 18 feet: find what pressure the piston bears.

6. The area of the piston of a forcing pump is 8 square inches: find what force must be exerted to push it down when the water in the ascending tube is at a level 30 feet above the piston.

7. Will it make any difference in the labour of working a force pump whether the ascending pipe is wide or narrow?

8. The area of the piston of a forcing pump is a square inch: find what force must be exerted on the piston in order to raise water to the height of 40 feet above the piston.

9. If in Art. 541 the density of the air in the upper part of the receiver is twice that of the atmosphere, find to what height above the level of *G* the water can be raised.

10. In a forcing pump the area of the piston is 3·5 square inches, and a power of 77 pounds is employed in forcing the piston down: find the pressure of the air within the air chamber.

L. VARIOUS INSTRUMENTS.

1. In Bramah's press suppose that the diameter of the large piston is twenty times that of the small piston: find the pressure on the small piston if that on the large piston is one ton.

2. In the Bramah's press used for raising the Britannia bridge over the Menai straits, the diameter of the large piston was 20 inches; the machine was capable of raising 2000 tons: find the pressure on each square inch of the piston.

3. In a Bramah's press the diameter of the large piston is 2 inches, and of the small piston $\frac{1}{2}$ an inch; the length of the lever handle is 2 feet, and the distance from the fulcrum to the end of the rod of the small piston is 2 inches: find the force exerted by the large piston when 10 pounds is hung at the end of the lever.

4. State what would take place if a hole were made in a siphon while in use.

5. If a balloon is to be filled with hydrogen, find what weight it will raise, supposing its volume 2000 cubic feet.

6. Find the volume of a balloon filled with hydrogen so that it may raise 400 pounds.

7. In a diving bell the surface of the water inside the bell is just 17 feet below the surface of the water outside the bell: shew that the water inside the bell occupies one third of it.

8. A cylindrical diving bell 4 feet deep, whose volume is 20 cubic feet, is lowered into water until its top is 14 feet below the surface of the water; and air is forced in until it is three-quarters full; find what volume the air would occupy under the pressure of the atmosphere.

9. Find the reading of Fahrenheit's thermometer corresponding to 20 on the Centigrade thermometer.

10. Find the reading of the Centigrade thermometer corresponding to 122 on Fahrenheit's thermometer.

LI. FAMILIAR APPLICATIONS.

1. A plank 14 feet long is used for a see-saw by two boys, one weighing 60 pounds and the other 80 pounds: find where the fulcrum must be situated.

2. Considering the swing as a simple pendulum, find the time of oscillation if it be about 8 feet long.

3. The bore of a pop-gun is half an inch in diameter, and the pellet leaves it when the piston has been pushed half-way down: find the resultant pressure of the air on the pellet at the instant of expulsion.

4. If a sucker is round and 4 inches in diameter, find what weight could be raised by it.

5. A wheel 13 inches in radius has to be pulled over an obstacle 1 inch high by a horizontal force through the centre of the wheel: find this force if the wheel be pressed down through its centre by a vertical force of 120 pounds.

6. The weight of a window-sash 3 feet wide is 5 pounds; each of the weights attached to the cords is 2 pounds: if one of the cords be broken, find the magnitude and position of the force which will be just sufficient to keep the window-sash at any height at which it may be placed.

7. A boat is rowed by eight men, each using one oar; the distance of the rowlock from the end of the oar grasped by the hand is $1\frac{1}{2}$ feet, and from the part of the blade

which serves as fulcrum 6 feet: if each man can pull his oar with a force of 80 pounds, find the whole force to urge the boat on.

8. In a wheelbarrow the distance from the axle to the end of the arms is 4 feet, the weight of the trough part and the load is 120 pounds, and the vertical line through the centre of gravity of the trough part and load passes at the distance of $1\frac{1}{2}$ feet from the axle: find the pressure on the ground of a man who weighs 160 pounds and has just raised the feet of the wheelbarrow slightly from the ground, friction being neglected.

9. Suppose that the centre of gravity of the trough part and load in the preceding Example is *above* the plane passing through the axle and the ends of the arms: then shew that the higher the man raises his end of the wheelbarrow, the less force he has to exert to support the barrow.

10. A weight is attached to a pole the ends of which are supported on the shoulders of two men, and the pressures on them are equal: shew that if the weight is *hung* from the pole the pressures will still be equal when the men stand on a slope, one above the other, but that if the weight is *fastened* to the pole this is not necessarily the case. The pressures on the men are supposed to be *parallel*.

LII. WORK.

1. Find how many units of work are performed in raising a ton of coals from a mine 40 fathoms deep.

2. Find how many cubic feet of water an engine of 50 horse power can raise in an hour from a mine 40 fathoms deep.

3. Find how many bricks a labourer could raise to the height of 30 feet in a day of 6 hours, by the aid of a cord and pully, supposing a brick to weigh 8 pounds.

4. A bricklayer's labourer with his hod weighs 160 pounds; he puts into the hod 20 bricks weighing 7 pounds each; he then walks up a ladder to the height of 30 feet: find how many units of work he does. And if he can do 1350000 units of work in a day, find how many bricks he takes up the ladder in a day.

5. Find the horse power of an engine which is to raise

30000 pounds of water from the depth of a furlong in five minutes.

6. If a man can do 900000 units of work in a day of 9 hours, find at what fraction of a horse power he works on an average.

7. Find how many gallons of water a steam-engine of ten horse power can raise from a depth of 200 fathoms in an hour.

8. A man works on a machine in such a manner as to do 1000000 units of work in a day of 8 hours; the machine is so arranged that he can lift a weight of 5 cwt.: find how long it will take him, working at his average daily mean rate, to lift the weight through a height of 100 feet.

9. A weight of 8 cwt. is raised from a depth of 100 fathoms by means of a rope weighing one pound per foot: find how many units of work are expended.

10. Find how many units of work must be expended in raising from the ground the materials for building a uniform column 50 feet high and 10 feet square; a cubic foot of the materials weighing one hundred weight.

11. A mass of 5 tons moves at the rate of 10 feet per second: find the number of units of work accumulated in it. If the mass is acted on by a force of 20 pounds in the direction opposite to that of motion, find how far it will move before being brought to rest.

12. A train weighing 120 tons runs on a level road, and the resistances to be overcome are 8 pounds per ton: find how many units of work must be expended in making a run of 40 miles.

13. A body weighing 100 pounds moves at the rate of 15 miles per hour: find the number of units of work accumulated in it. If from the instant under consideration the body slides along a rough horizontal plane, find how far it will go before coming to rest, the coefficient of friction between the body and the plane being ·05.

14. An inclined plane is 130 feet long and 50 feet high: find the number of units of work expended in drawing a body weighing 56 pounds *up* the plane, the coefficient of friction being ·5.

15. Find in the case of the preceding Example the number of units of work required to draw the body *down* the plane.

16. Find the horse power of an engine which is to move at the rate of 25 miles an hour, the weight of the engine and load being 30 tons, and the resistance from friction 16 pounds per ton.

17. Find the horse power of an engine which is to move at the rate of 10 miles an hour up an incline which rises 1 foot in 100, the weight of the engine and load being 30 tons, and the resistance from friction 12 pounds per ton.

18. A railway truck weighs 12 tons; it is drawn from rest by a horse through a distance of 50 feet and is then moving at the rate of 3 miles per hour: if the resistances are 8 pounds per ton, find how many units of work the horse must have done on the truck.

19. It is said that a horse can do about 1630000 units of work per hour, walking at the rate of $2\frac{1}{2}$ miles per hour: find what force in pounds this assumes the horse continually to exert.

20. The drum of a capstan is 18 inches in diameter, and the handspikes are 10 feet long. Four men are employed to raise weights from a depth of 50 feet by the machine. Assuming that when a man exerts a continuous force in pushing or pulling of 27 pounds he can do the greatest amount of daily work, namely 1500000 units, find the load that should be raised at each lift and the total load raised daily.

LIII. ENERGY.

1. A body weighing 112 pounds is moving at the rate of 15 miles per hour: determine the Energy.

2. Shew that a body weighing 64 pounds, and moving with a velocity of 5 feet per second, has the same Energy as a body weighing 25 pounds and moving with a velocity of 8 feet per second.

3. A rifle weighs 12 pounds and discharges a ball of one ounce weight with a velocity of 960 feet per second: shew that at the instant of leaving the rifle the ball has 192 times more Energy than the rifle.

4. A ball moving with any velocity strikes directly a ball of half the mass at rest, the two being inelastic: shew

that the Energy of the system after collision is two-thirds of that before collision.

5. There are two balls whose masses are 15 pounds and 20 pounds respectively; the former moves at the rate of 12 feet per second and impinges directly on the latter moving in the same direction at the rate of 6 feet per second: find the Energy of the system before impact, and at the end of compression.

6. A ball moving at the rate of 4 feet per second overtakes an equal ball moving in the same direction at the rate of 3 feet per second, the two being inelastic: shew that the Energy after impact is $\frac{49}{50}$ of its value before impact.

7. A ball weighing 6 pounds moving at the rate of 7 feet per second overtakes a ball weighing 7 pounds moving in the same direction at the rate of 6 feet per second, the two being inelastic: shew that $\frac{1}{169}$ of the Energy is lost by the collision.

8. In the Example XVIII. 9, find the Energy before collision, at the end of the first part of the impact, and at the end of the second.

9. Also in the Example XVIII. 10, find the same quantities.

10. If a ball of iron weighing 9 pounds falls through 193 feet into a vessel containing 8 pounds of water, shew that the temperature of the ball and of the water will be raised one quarter of a degree of Fahrenheit's thermometer.

LIV. ELASTICITY.

1. If lead be compressed by a force of one pound per square inch the shortening is $\frac{1}{720000}$ of the original length: find the amount of shortening produced by the pressure of 15 cwt. on a cylinder of lead a foot long, and a square inch in section.

2. If glass be compressed by a force of one pound per square inch the shortening is $\frac{1}{8000000}$ of the original

length: find the amount of shortening produced by the pressure of 15 cwt. on a cylinder of glass a foot long and a square inch in section.

3. A bar of wrought iron is 100 feet long; the section of it is a square a side of which is a quarter of an inch; a weight of 2 tons stretches it 3 inches: find how much a weight of 5 tons will stretch a bar 80 feet long, the section being a square the side of which is half an inch.

4. Find the weight necessary to stretch the first bar of the preceding Example to $\frac{1}{1000}$ of its length.

5. A bar of iron will expand to about $\frac{1}{1000}$ of its length as the temperature changes from the freezing point to the boiling point of water: find the weight which would produce the same amount of lengthening in the second bar of Example 3.

LV. STRENGTH OF MATERIALS.

1. A bar of cast iron 20 feet long and a square inch in section is stretched by a weight of 5 tons: find how much it is lengthened.

2. If the bar in the preceding Example has a section which is a square of half an inch in side, find how much it is lengthened.

3. Find the strain that a bar of cast iron will bear if the section is a square of three quarters of an inch in side.

4. A cylindrical column of cast brass has for section a circle of one inch radius: find the weight it will bear without being crushed.

5. Find the height of a column of granite such that it would be sufficient to crush the lower parts, taking the specific gravity of granite as 2·6, and the *endurance* of granite as 5500.

LVI. STRENGTH OF BEAMS.

1. A beam of oak 20 feet long, 4 inches broad, and 3 inches deep supports a weight of 252 pounds in the manner of Art. 666: find what weight would be sup-

ported by a beam of oak 25 feet long, 5 inches broad, and 4 inches deep.

2. A beam of oak of the same kind as in the preceding Example is supported at its ends: find what weight can be put at the middle without breaking the beam; supposing the beam 24 feet long, 3 inches broad, and $4\frac{1}{2}$ inches deep.

3. Find the deflection of a beam of oak 20 feet long, 4 inches broad, and 5 inches deep, which carries a load of 1120 pounds at its middle point. See Art. 678.

4. A beam of oak is 12 feet long, 3 inches broad, and 3 inches deep; a weight of 315 pounds is placed at the middle: find the deflection.

5. Shew that if the length and depth of a beam be changed in the same proportion while the weight and the breadth are unchanged, the deflection is unchanged.

LVII. CAPILLARY PHENOMENA.

1. Find the capillary elevation in a tube of ·03 of an inch in diameter for water, and for nitric acid: see Arts. 685 and 686.

2. Assuming that the capillary depression in a tube of ·08 of an inch in diameter for mercury is ·15 of an inch, find the capillary depression in a tube of ·05 of an inch in diameter.

3. Two plates of glass are joined along a common edge so as to include a small angle; they are placed in water with this edge vertical: indicate by a diagram the form which the water will take between the two plates.

4. If instead of water the plates are placed in mercury draw the diagram.

5. The sides of a vessel containing water are composed of a material of half the specific gravity of water: shew by reasoning that we may expect the water to be exactly level near the sides.

ANSWERS.

IV. 1. 33. 2. $13\frac{1}{2}$. 3. $36\frac{2}{3}$ feet per second. 5. 144 feet. 6. 320 feet. 7. $12\frac{24}{25}$ feet. 8. $3\frac{6}{25}$ feet. 9. $327\frac{3}{11}$. 12. 81 feet. 14. 11 seconds. 16. $1\frac{1}{4}$ seconds.

V. 1. 35 feet per second. 2. 5 feet per second. 3. 25 feet per second. 5. $\frac{1}{16}$ of a minute; $\frac{1}{10}$ of a minute. 7. 14 miles per hour. 8. 32 feet. 9. 400 feet; 176 feet. 10. 144 feet.

VI. 2. 104 feet per second; one second. 3. 144 feet; 96 feet per second. 4. $4\frac{1}{4}$ seconds; 136 feet per second. 5. 196 feet. 6. $4\frac{1}{2}$ seconds. 7. It will start upwards with the velocity which the balloon has when the string was cut; and afterwards it will fall. 8. 26 feet. 9. At the end of a second. 10. 220 feet.

VII. 9. 48 feet per second. 11. 100 feet. 12. $\frac{3}{16}$ of the weight. 13. 160 feet; 480 feet. 14. 30 miles per hour. 15. $\frac{11}{6000}$ of the weight. 16. $1\frac{1}{4}$ times the weight. 17. 32 to 27. 18. 150 feet; 48 feet per second. 19. 48 feet per second. 20. 225 feet; after $2\frac{1}{2}$ seconds.

VIII. 2. 8 feet; 8 feet per second. 3. $7\frac{7}{8}$ ounces. 4. $4\frac{1}{4}$ ounces; $3\frac{3}{4}$ ounces. 5. 30 feet per second; $\frac{15}{16}$ of a second. 7. 15 to 17. 8. 191 to 193. 10. No pressure.

IX. 1. 16 pounds. 2. No force. 3. 41 pounds. 4. 84 pounds; 12 pounds; 60 pounds. 8. The direction is AC, and the magnitude is twice AC. 11. Pro-

duce CB through B to E, so that BE is half of BC; then the resultant is a force of 20 pounds through E parallel to AB. 12. Produce DC through C to E, so that CE is two-thirds of DC; then we get a force of 6 pounds at E along EC, and a force of 6 pounds at E at right angles to EC; and the resultant of these two is obvious. 13. They must be equal and in the same proportion to 100 pounds as the side of a square is to the diagonal. 14. Five pounds from B to C. 19. On each post the resultant pressure is 200 pounds, acting towards the centre of the circle; see Example 5.

X. 1. 12 pounds at a distance of 5 inches from the force of 7 pounds. 2. The force of 12 pounds must act in the direction opposite to that of the two other forces; and its distance from the smallest must be five times its distance from the other. 3. A force of 6 pounds at the centre of the square. 4. 8 inches. 5. At B. 6. Midway between B and C. 7. 5 feet from A. 8. 3 inches. 10. At three-eighths of the length from the end where the weight of 20 pounds is fastened. 11. 12 pounds and 6 pounds. 12. 2 pounds. 15. Midway between the third corner and the middle point of the opposite side. 16. 3·26 inches from the top. 17. The centre of gravity of the triangle. 18. At the point where the axes meet. 19. Find the centre of gravity of the weights 2 and 5; also of the weights 3 and 4: then the point required is midway between the two. 20. Join the middle points of the three rods; the required point is the centre of gravity of the triangle thus formed.

XI. 3. The rod vertical; for stable equilibrium the lighter ball must be lowest. 4. The middle point. 6. Join E to the intersection of AC and BD; and produce the straight line to meet the perimeter. 7. Let the sphere be at the point A of the triangle ABC, and the string fastened to the middle point of the side AB; let G be the centre of gravity of the triangle; and D the point midway between A and G: the direction of the string will pass through D in equilibrium, and it will be found that it will cut the side AC at right angles. 8. Midway between the centres of gravity of ABC and ADC. 9. At the point where the axis of the cylinder meets the hemispherical part. 10. Let O be the centre of the hexagon,

G the centre of gravity of the triangle before it is removed; join GO and produce it to B, so that OB is one-fifth of OG: then B is the centre of gravity of the remainder.

XII. 1. 2 pounds. 2. One pound. 3. One foot from the weight of 8 pounds. 4. 9 feet; 7 feet. 5. 32 pounds; 28 pounds 2 ounces. 8. 9 pounds. 9. 8 inches from the weight of 16 ounces. 10. 9 pounds. 11. 2240 pounds; 2400 pounds. 12. A bears 120 pounds; and B bears 80 pounds. 13. 32 pounds; 64 pounds. 14. 30 pounds. 15. Let the direction of the forces at A and B meet at O; then AOB is an angle of 60 degrees. Bisect this angle by a straight line which meets AB at C; then C is the required fulcrum, and the pressure on it is equal to the resultant of two equal forces inclined at an angle of 60 degrees.

XIII. 1. 10 pounds 9 ounces; 9 pounds. 2. 16 to 17. 3. 32 pounds; 28 pounds 2 ounces. 6. 6 pounds. 7. 14 pounds.

XIV. 1. 14 pounds. 2. 2 feet. 3. 2 pounds; 20 pounds. 4. 300 pounds. 6. The radius of the wheel must be 14 times that of the axle. 5. $7\frac{1}{2}$ feet. 9. 2 pounds.

XV. 1. 96 pounds. 2. 3 pounds. 3. Four. 4. 30 pounds. 5. 7 pounds. 6. Five. 7. 62 pounds. 8. 4 pounds. 9. Four. 10. 86 pounds.

XVI. 1. 20 pounds. 2. 25 pounds. 3. 16 pounds; 60 pounds. 4. 32 pounds. 5. 48 pounds. 6. 28 pounds. 7. 15 pounds. 8. 144 pounds. 9. About ·7 of an inch. 10. The weight is about 754 times the power.

XVII. 1. 1 pound. 2. 343. 3. 256 pounds. 4. 480 pounds.

XVIII. 1. 8 feet per second. 2. 2 feet per second. 4. $11\frac{2}{3}$ feet per second. 5. 60, 40, 30, 24,...feet per second. 7. A comes to rest; B moves in a direction equally inclined to the original directions of the two balls. 8. 9 feet; 8 feet and 18 feet per second. 9. 5 feet per second; $11\frac{3}{4}$ feet per second. 10. 1 foot per second backwards; 4 feet per second.

XIX. 1. 10 feet per second. 2. 4 seconds. 3. 40 feet per second. 6. $8\frac{24}{25}$ feet per second. 7. $2\frac{1}{2}$ seconds. 8. 192 feet per second. 9. 477 ounces.

XX. 1. 144 feet; 168 feet per second. 2. The distance in feet is the product of the number of seconds into 22. 3. 984 feet. 4. 400 feet per half second. 5. At the end of half a second the height is 36 feet, and the horizontal distance is 30 feet; at the end of a second the height is 64 feet, and the horizontal distance is 60 feet. 6. At starting, 100 feet per second; at the end of a second the vertical velocity is 48 feet per second, and the whole velocity is the square root of the sum of 3600 and 2304, that is the square root of 5904. 7. $2\frac{1}{2}$ seconds; 100 feet. 8. 5 seconds; vertical velocity, 80 feet per second *downwards*, horizontal velocity 60 feet. 10. Composed of a vertical velocity, and a horizontal velocity, each of 80 feet per second. 11. 36 feet; 3 seconds; 75 feet. 14. At the distance of 55 feet; the vertical velocity will be 40 feet per second downwards, and the horizontal velocity 44 feet per second.

XXI. 1. 4 pounds. 2. 32 feet per second. 3. About 30 pounds. 4. 100 pounds. 5. $5\frac{1}{24}$ cwt. 10. 5100 seconds.

XXII. 1. 9·78 inches. 2. Nearly 4 seconds. 3. 8 feet per second.

XXIII. 1. $\frac{1}{12}$; 36 pounds. 2. 1. 3. $\frac{1}{4}$; 2 pounds. 4. 2 pounds. 6. 135 pounds. 7. 1. 8. $\frac{3}{16}$. 9. $43\frac{1}{8}$ feet; $\frac{3}{160}$. 10. $1\frac{7}{25}$ feet per second; $6\frac{1}{4}$ seconds.

XXIV. 1. 8 feet from the end. 2. $\frac{3}{8}$ of 39·1393 inches. 3. About 20 feet. 4. About 45 feet. 5. At two-thirds of the length from the end. 6. If above, the time will be diminished; if below, it will be increased. 7. $2\frac{2}{9}$ feet. 8. $3\frac{5}{9}$ feet. 9. 7 feet. 10. 7 feet. 11. $1\frac{1}{4}$ tons. 12. 24 seconds; 576 feet. 13. $1\frac{1}{8}$ pounds. 14. 2500 times the weight of the bullet; $\frac{1}{200}$ of a second. 15. $\frac{11}{240}$.

XXV. 3. 11000 feet. 4. ·016. 5. ·01.

XXVI. 1. 28 pounds. 3. 35 pounds. 4. 7 grains. 5. ·56 of a pound.

XXVII. [In obtaining answers to XXVII. and XXVIII. use the result at the end of Art. 359.] 1. About 324 pounds. 2. About 3 pounds. 3. About 6 pounds. 4. About 9 pounds. 5. 8 pounds.

XXVIII. 1. About 1944 pounds. 2. About 6912 pounds. 3. About $330\frac{3}{4}$ pounds. 6. About 18 pounds. 7. About 72 pounds. 8. About 942 pounds.

XXIX. 2. $\frac{300000}{1728}$ ounces. 3. 5 tons; for the point of application see Art. 378. 4. 72000 ounces; 18000 ounces; 54000 ounces. 5. The distance from the top of the centre of pressure of the whole is 4 feet, and of the centre of pressure of the upper half is 2 feet; then the distance of the centre of pressure of the lower half will be 4 feet 8 inches: see Example X. 4. 6. $\frac{125 \times 3141{\cdot}6}{1728}$ ounces. 7. $\frac{18 \times 18 \times 18 \times 3{\cdot}1416}{1728}$ ounces. 8. 32000 ounces, 33000 ounces, 32500 ounces. 9. 1·5 times a side of the square.

XXX. 2. About 21 pounds. 4. 36000 pounds; 4000 pounds.

XXXI. 1. 16 cubic inches. 2. 314·16 cubic inches. 3. $\frac{3}{4}$ of an inch. 4. $\frac{1}{2}$ an inch. 5. 2·88 inches.

XXXII. 1. $\frac{5}{8}$ of a pound. 2. $9\frac{7}{27}$ ounces. 3. Nearly 182 ounces. 4. Very nearly 7 ounces. 7. $181\frac{1}{4}$ ounces. 8. About 2·4 ounces less. 9. 2·4 cubic feet. 10. Nearly 1272 ounces.

XXXIII. 1. $\frac{5800}{1728}$ ounces. 2. $\frac{900}{1728}$ ounces. 3. 71680. 4. 187500 pounds. 5. ·896 of a foot. 6. 69727. 7. $\frac{1953}{8000}$ of a foot. 8. For every cubic foot of water displaced 28 ounces more would be supported in the sea than in the river; thus $\frac{20 \times 20 \times 112 \times 16}{28}$ is the number of cubic

feet of sea-water displaced: this is 25600. Hence the weight required is 25600×1028 ounces. 9. If 30 tons were removed the ship would rise 12 inches: then proceed as in Example 8.

XXXIV. 1. 4056 ounces. 2. 40 inches. 3. 13·6 feet. 4. 272 inches.

XXXV. 1. Half immersed. 3. 2 inches.

XXXVI. 1. 10·6. 2. 3. 3. 8·8. 4. ·6. 5. ·2. 6. $\frac{3}{13}$. 7. 1555·2. 8. $2\frac{1}{13}$ ounces. 9. 4 pounds. 10. $\frac{89}{17000}$ of a cubic foot. 11. $\frac{224}{121}$ cubic feet. 12. 3 cwt. 13. 21 ounces. 14. 5·6 pounds. 15. 61·25 grains. 16. ·965. 17. 7. 19. 17·18. 20. 39 cubic inches of copper. 21. 24 cubic inches of silver. 22. $\frac{65 \times 19400}{1728}$ ounces; $\frac{24 \times 10500}{1728}$ ounces. 23. Volume of gold is $\frac{38}{167} \times \frac{1}{645}$ of a cubic foot; volume of quartz is $\frac{129}{167} \times \frac{1}{645}$ of a cubic foot: for the weights in ounces multiply the former by 193500, the latter by 2150. 24. $\frac{62}{126}$. 25. $\frac{35}{24}$ of a pound.

XXXVII. 1. ·875. 2. ·792. 3. Nearly 13·6. 4. $1\frac{5}{12}$. 5. $\frac{5}{6}$. 6. ·8. 7. $\frac{2923}{2428}$. 8. 1008 ounces nearly. 9. As 1830 is to 2052. 10. $\frac{44}{19}$. 11. 2·316 nearly. 12. 9·15 pounds. 13. 2080 grains. 14. $\frac{50}{61}$. 15. $\frac{8}{9}$. 17. ·85. 18. $\frac{15}{16}$. 19. The volume is about 9·84 cubic inches. 20. $\frac{3}{4}$.

XXXVIII. 1. 3120 ounces. 2. 34·1935 grains. 3. 2·1336 grains. 4. 1·0013. 5. 6500 ounces; 447 ounces.

XXXIX. 1. 9 seconds. 2. 16 feet per second.

3. $\frac{\sqrt{6}}{4}$ seconds; $4\sqrt{6}$ feet. 4. $4\sqrt{6}$ feet. 5. $2\sqrt{21}$ feet. 6. 8 feet.

XL. 1. 16000 ounces. 2. 16 feet per second. 5. 7200 pounds.

XLI. 4. 189 degrees.

XLII. 1. 8424 ounces.

XLIII. 1. 13·6 times as high. 2. 144×30 pounds. About 47 pounds. 4. About four ounces.

XLIV. 1. 22 pounds 11 ounces. 2. Seven times the ordinary pressure. 4. Twice the original density. 6. At an additional depth of 7×39 feet. 7. Very nearly 29·92 inches. 9. As 7 to 4.

XLV. 1. $\frac{1}{30}$ of that of the atmosphere. 2. $28·4 + \frac{5}{6}$.

3. The length which is actually 1 must be marked as $1\frac{1}{120}$.

5. $\frac{1}{7}$ of 52428 feet. 6. $4936 + 281$. 8. 25839 feet nearly. 9. About 513 inches.

XLVI. 3. 60 pounds. 6. ·491 of a pound.

7. 74 feet. 8. $\frac{20}{119}$ of 300 cubic inches.

XLVII. 2. Five operations reduce the density to somewhat less than a third of the original density.

3. About 14 pounds. 5. A difference in level of $\frac{3}{10}$ of an inch.

XLVIII. 3. $208\frac{8}{9}^{\circ}$. 4. 480 feet per second. 5. Twenty.

XLIX. 1. 340 inches. 2. 232 inches. 5. About 25 pounds. 6. Nearly 106 pounds. 7. Nearly 18 pounds. 9. 34 feet. 10. 37 pounds per square inch.

L. 1. $5\frac{3}{4}$ pounds. 2. About 14000 pounds per square inch. 3. 1920 pounds. 5. About 150 pounds. 6. About 5300 cubic feet. 8. $22\frac{1}{2}$ cubic feet. 9. 68. 10. 50.

LI. 1. 6 feet from the heavier boy. 3. $\frac{11}{7}$ of a

second. 3. Nearly 3 pounds. 4. Nearly 190 pounds. 5. 50 pounds. 6. 3 pounds upwards, one foot from the broken cord. 7. 160 pounds. 8. 205 pounds.

LII. 1. 537600. 2. 6600. 3. 2340. 4. 9000; 3000 bricks. 5. 120. 6. $\frac{5}{99}$. 7. 1650. 8. $\frac{56}{125}$ of an hour. 9. 717600. 10. 14000000. 11. 17500; 875 feet. 12. 202752000. 13. $756\frac{1}{4}$; $151\frac{1}{4}$ feet. 14. 6160. 15. 560. 16. 32. 17. 27·52. 18. 12931·2. 19. $123\frac{1}{3}\frac{9}{3}$ pounds. 20. 1440 pounds; 120000 pounds.

LIII. 1. 847. 5. 45; $40\frac{5}{28}$. 8. $\frac{189}{16}$, 9, $\frac{2709}{256}$. 9. $\frac{105}{16}$, $\frac{15}{16}$, $\frac{75}{32}$.

LIV. 1. ·028 of an inch. 2. ·00252 of an inch. 3. $1\frac{1}{2}$ inches. 4. ·8 of a ton. 5. $3\frac{1}{3}$ tons.

LV. 1. ·013 of a foot. 2. ·052 of a foot. 3. $\frac{9}{16}$ of 16500 pounds. 4. $\frac{22}{7}$ of 10300 pounds. 5. 4874 feet.

LVI. 1. 448 pounds. 2. $1417\frac{1}{2}$ pounds. 3. $5\frac{1}{3}$ inches. 4. 2 inches.

LVII. 1. 1·6, 1·2 inches. 2. ·24 of an inch.

CAMBRIDGE: PRINTED BY C. J. CLAY, M.A. AT THE UNIVERSITY PRESS.

A Treatise on the Theory of Equations. New Edition. Crown 8vo. cloth. 7*s.* 6*d.*

Plane Trigonometry for Colleges and Schools. With numerous Examples. New Edition. Crown 8vo. cloth. 5*s.*

Key to the Plane Trigonometry for Colleges and Schools. Crown 8vo. cloth. 10*s.* 6*d.*

A Treatise on Spherical Trigonometry for the use of Colleges and Schools. With numerous Examples. New Edition. Crown 8vo. cloth. 4*s.* 6*d.*

A Treatise on Conic Sections. With numerous Examples. New Edition. Crown 8vo. cloth. 7*s.* 6*d.*

A Treatise on the Differential Calculus. With numerous Examples. New Edition. Crown 8vo. cloth. 10*s.* 6*d.*

A Treatise on the Integral Calculus. With numerous Examples. New Edition. Crown 8vo. cloth. 10*s.* 6*d.*

Examples of Analytical Geometry of Three Dimensions. Fourth Edition. Crown 8vo. cloth. 4*s.*

A Treatise on Analytical Statics. With numerous Examples. Fourth Edition. Crown 8vo. cloth. 10*s.* 6*d.*

An Elementary Treatise on Laplace's, Lamé's, and Bessel's Functions. Crown 8vo. 10*s.* 6*d.*

A History of the Mathematical Theory of Probability, from the time of Pascal to that of Laplace. 8vo. cloth. 18*s.*

A History of the Mathematical Theories of Attraction and of the Figure of the Earth from the time of Newton to that of Laplace. In two Volumes. 8vo. 24*s.*

The Conflict of Studies, and other Essays on Subjects connected with Education. 8vo. 10*s.* 6*d.*

MACMILLAN AND CO. LONDON.

Todhunter, I.
Natural philos

NAME	DATE	